CHAPTER 7

First-order Logic
Outline

- Syntax and semantics of FOL
- Wumpus world in FOL
- Fun with sentences
Quantifiers

Equality

\( x \equiv y \equiv \neg \land \lor \lor \) Connectives

\( z, \hat{a}, \hat{q} \) Variables

\( \text{sort, let, lego, } f \) Functions

\( \text{brother, } > \) Predicates

\( \text{King John, 2, UCB} \) Constants

Syntax of FOL: Basic Elements
\[
\begin{align*}
&E.< \text{ Brother(John, Richard, Lionheart)} < \\
&\text{or constant or variable} \\
&\text{Term} = \text{function}(\text{term}_1, \ldots, \text{term}_n) \\
&\text{or term}_1 = \text{term}_2 \\
&\text{Atomic sentence} = \text{predicate}(\text{term}_1, \ldots, \text{term}_n)
\end{align*}
\]
Complex sentences are made from atomic sentences using connectives.
An atomic sentence \( \text{predicate}(\text{term}_1, \ldots, \text{term}_n) \) is true if the objects referred to by \( \text{term}_1, \ldots, \text{term}_n \) are in the relation referred to by \( \text{predicate} \).

Interpretation specifics referents for

Model contains objects and relations among them

Sentences are true with respect to a model and an interpretation

Truth in first-order logic
Models for FOL: Example

Functional Relations: all tuples of objects + "value" object

Relations: sets of tuples of objects
"Everyone is at Berkeley and everyone is smart"

\[
\forall x \; \text{At}(x, \text{Berkeley}) \land \text{Smart}(x)
\]

Common mistake: using \( \land \) as the main connective with \( \forall \).

Typically, \( \forall \) is the main connective with \( \land \).

\[
\ldots \land
\]

\[
\text{At}(\text{Berkeley}, \text{Berkeley}) \land \text{Smart}(\text{Berkeley}) \land \text{Smart}(\text{Richard}) \land \text{Smart}(\text{KingJohn}) \land \ldots
\]

is equivalent to the conjunction of instantiations of \( p \).

\[
\forall x \; \text{At}(x, \text{Berkeley}) \land \text{Smart}(x)
\]

\[
\forall \text{sentence} \langle \text{variables} \rangle
\]

\textbf{Universal quantification}
is true if there is anyone who is not at Stanford:

\[ (x) \text{Smart}(x) \iff \forall x \in \text{At}(x, \text{Stanford}) \]

Common mistake: using \( \exists \) is the main connective with \( \text{At} \).

Typically, \( \forall \) is the main connective with \( \text{At} \).

\[ \ldots \wedge \]

\[ \text{At}(\text{Stanford}, \text{Stanford}) \wedge \text{At}(\text{Richard}, \text{Stanford}) \wedge \text{At}(\text{John}, \text{Stanford}) \wedge \text{At}(\text{Jenny}, \text{Stanford}) \]

is equivalent to the disjunction of instantiations of \( \exists \):

\[ \exists x \in \text{At}(x, \text{Stanford}) \]

Someone at Stanford is smart:

\[ \exists \text{sentence} \in \text{variables} \]

\textbf{Existential quantification}
Quantifier duality: each can be expressed using the other

"Everyone in the world is loved by at least one person"
\[ (\forall x \in \mathcal{H} ) \exists y \in \mathcal{H} \quad \text{loves}(x,y) \]

"There is a person who loves everyone in the world"
\[ (\exists x \in \mathcal{H} ) \forall y \in \mathcal{H} \quad \text{loves}(x,y) \]

\[ x \in \mathcal{H} \quad \forall A \quad x \text{ is not the same as } \exists A \quad x \in \mathcal{H} \]

\[ (\exists y \in \mathcal{H} ) \quad x \in \mathcal{H} \quad \forall E \quad x \text{ is the same as } \exists E \quad x \in \mathcal{H} \]

\[ (\exists y \in \mathcal{H} ) \quad x \in A \quad \forall A \quad x \text{ is the same as } \exists A \quad x \in A \]

Properties of quantifiers
A first cousin is a child of a parent's sibling.

One's mother is one's female parent.

"Sibling" is reflexive.

Brothers are siblings.

Fun with sentences
\[\forall (d', ps', \text{parent}(p', d') \land \text{Sibling}(d', p')) \in p', \text{sibling}(x') \iff (\forall x, \text{Question}(x')) \land \text{parent}(x', p') \]

\[\text{Mother}(x', y) \iff (\forall x, \text{Mother}(x')) \land \text{parent}(x', p') \]

\[\text{Sibling}(x', y) \iff (\forall x, \text{Sibling}(x')) \land \text{parent}(x', p') \]

\[\text{Brother}(x', y) \iff (\forall x, \text{Brother}(x')) \land \text{parent}(x', p') \]
\[ ([h,f]_{\text{parent}} \lor [h,m]_{\text{parent}} \lor (x,f)_{\text{parent}} \lor (x,m)_{\text{parent}} \lor (f = n) \implies f \in x \lor (f = x) \implies) \Rightarrow (\exists x, y. \text{Sibling}(x,y) \land \forall x \in \text{Parent}(x,y)) \] 

\text{E.g. definition of (full) Sibling in terms of Parent:}

\text{E.g.}\ 2 = 2 \text{ is valid}

\text{E.g.}\ x = (\text{x})_{\text{print}} \times (\text{y})_{\text{print}}

\text{if and only if term}_1 \text{ and term}_2 \text{ refer to the same object}

\text{term}_1 = \text{term}_2 \text{ is true under a given interpretation}
ASK(KB, S) returns some all o such that KB |= So

S_o = Smarteer(Hillary, Bill)
{H/Hillary, y/Bill, x} = o
S = Smarteer(x, y)

So denotes the result of plugging o into S: e.g.,
Given a sentence S and a substitution o,

substitution(binding list) → \{Shoot\}

\text{Answer: Yes,}\quad (a/Shoot)

\text{I.e., does the KB entail any particular actions at}\ t = 5?\]

ASK(KB, \exists a Action(a, 5))

\text{Tell}(KB, \text{Perce}t\text{[Breeze, Smell, None,}\ N\text{e}[5, 5])

Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at t = 5:

**Interacting with FOL KBS**
Keeping track of change is essential

\[ \text{Holding}(\text{gold}, t) \text{ cannot be observed} \]

\[ \text{Action}(\text{grab}, t) \subseteq (\text{Holding}(\text{gold}, t) \land \text{At(ACTgold)}(t)) \]

Reflex with internal state: do we have the gold already?

\[ \text{Reflex: At(ACTgold)}(t) \subseteq \text{Action}(\text{grab}, t) \]

A&q, t Percept([s]\text{, smell}(t)) \subseteq \text{At(ACTgold)}(t)

A&q, t Percept([s]\text{, smell}(t)) \subseteq \text{At(ACTgold)}(t)

"Perception"

Knowledge base for the Wumpus world
\[
(\forall y \text{ Breezy}(y) \lor (\forall x \ P(x) \land x \in E) \iff (\forall y \text{ Breezy}(y) \land (y, x) \in \text{Adjacent})
\]

Definition for the Breezy predicate:

Squares far away from pits can be breezy squares far away from pits can be breezy.

Neither of these is complete—e.g., the causal rule doesn't say whether

(\forall y \text{ Breezy}(y) \land (y, x) \in \text{Adjacent}) \iff (\forall y \text{ Breezy}(y) \land (y, x) \in \text{Adjacent})

Causal rule—infer effect from cause

(\forall y \text{ Breezy}(y) \land (y, x) \in \text{Adjacent}) \iff (\forall y \text{ Breezy}(y) \land (y, x) \in \text{Adjacent})

Diagnostic rule—infer cause from effect

Squares are breezy near a pit:

(\forall y \text{ Breezy}(y) \land (y, x) \in \text{Adjacent}) \iff (\forall y \text{ Breezy}(y) \land (y, x) \in \text{Adjacent})

Properties of locations:

\text{Detecting hidden properties}
The result of an action is the situation that results from doing a in s. Situations are connected by the result function.

E.g., \( \text{now in Holding(gold, now)} \) denotes a situation that adds a situation argument to each non-external predicate.

Adding a situation argument is one way to represent change in FOL:

E.g., \( \text{Holding(gold, now)} \) rather than just \( \text{Holding(gold)} \).

Facts hold in situations rather than externally.

Keeping track of change.
Ramiﬁcation problem: real actions have many secondary consequences—
what about the dust on the gold, wear and tear on gloves, what if gold is slippery or nailed down or...

Qualiﬁcation problem: true descriptions of real actions require endless caveats—what if gold is the dirty or nailed down or...

Frame problem: ﬁnd an elegant way to handle non-change

(a) Representation—avoid frame axioms
(b) Inference—avoid repeated „copy-overs” to keep track of state

\( \text{HaveArm}(\text{result} \text{\text{Frame}}) \leftarrow \text{HaveArm}(s) \)  \\
\( \text{Holding}(\text{gold} \text{\text{Frame}}) \leftarrow \text{ActGold}(s) \)
\[[(\text{Holding}(\text{Gold}, s) \land a \neq \text{Release}) \land ((\text{Grad} \land \text{ATGold}(s)) = a)] \Rightarrow ((\text{Grad} \land \text{Result}(a, s)) \land a)\]

For holding the Gold:

\[P \text{ true already and no action made } \land \text{P true } \quad \land \quad \text{P true afterwards } \quad \Rightarrow \quad \text{an action made } \land \text{P true}\]

Each axiom is "about" a predicate (not an action per se): successor-state axioms solve the representational frame problem

**Describing actions II**
This assumes that the agent is interested in plans starting at $s_0$ and that $s_0$ is the only situation described in the KB.

Answer: \{ Result(Grab, Result(Forward, $s_0$)) \}

Query: ASK (KB, \exists Holding(Gold, s))

\[ \forall t \in [1, 2], \forall s_0^0 \] Initial condition in KB:

Making plans
PlanInference systems are special-purpose reasoners designed to do this type of inference more efficiently than a general-purpose reasoner.

\[
\forall a', p', s. \text{PlanResult}(p', \text{Result}(a', s)) = (\forall [d|a]) s = (\forall [\text{PlanResult}] s)
\]

Definition of PlanResult in terms of Result:

\[
\{ [[\text{Forward}\_\text{plan}]/d] \} \text{ has the solution } \]

Then the query ASK \( \forall x. \exists y. \text{Result}(y, x) \) is the result of executing \( p \) in \( s \), \n
\[
\text{PlanResult}(p', s) \] is the result of action sequences \([a_1, a_2, \ldots, a_n]\)

Making plans: a better way
can formulate planning as inference on a situation calculus KB

– conversions for describing actions and change in FOL

Situation calculus:

Increased expressive power: sufficient to define wumpus world

– syntax: constants, functions, predicates, equality, quantifiers
– objects and relations are semantic primitives

First-order logic

Summary