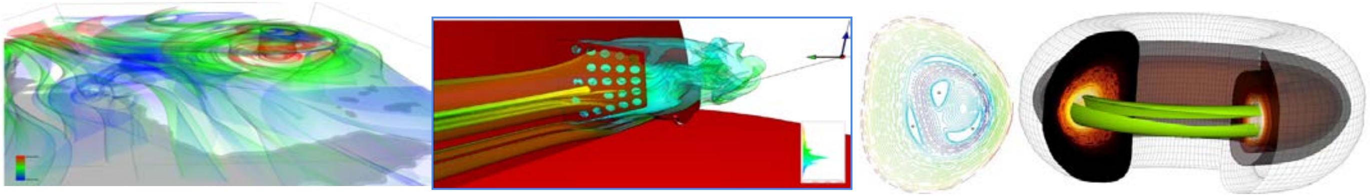
Enhanced Vector Field Visualization via Lagrangian Accumulation

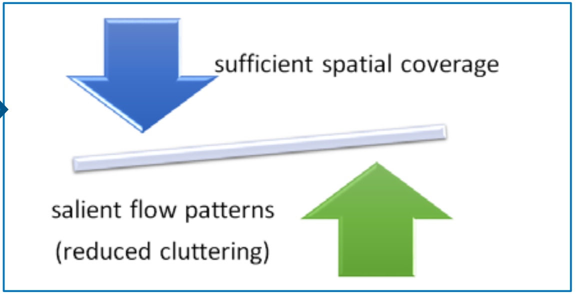
Lei Zhang[1], Duong B Nguyen[1], Robert Laramee[2], David Thompson[3], Guoning Chen[1]

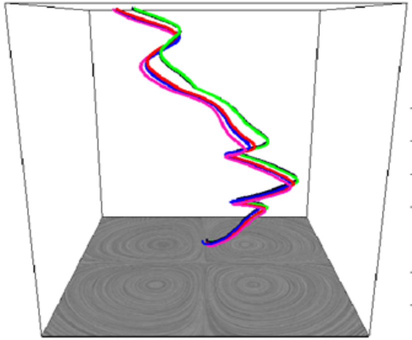
[1] University of Houston
[2] Swansea University
[3] Mississippi State University

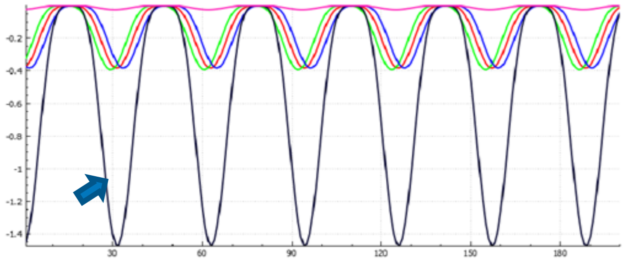
Motivation



- Vector fields and their visualization are important!
- The goals of effective vector field visualization are conflicting
- Previous geometry-based representation needs not reveal important physical properties of the flows







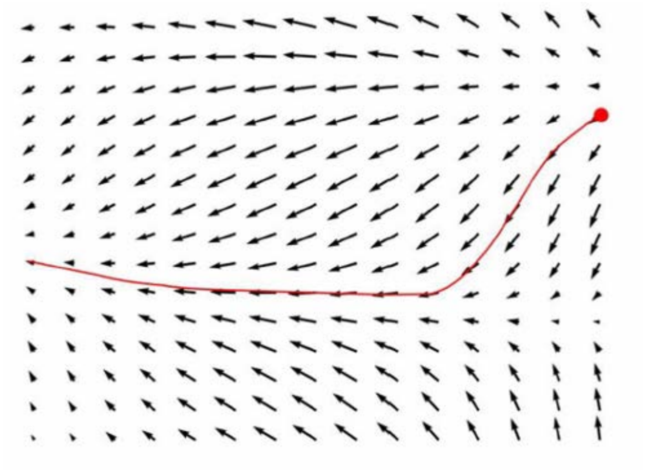
Outline

- Lagrangian accumulation (L.A.) and derived attribute fields
- Attribute field properties
- L.A. based flow exploration
  - Ribbon placement
  - Integral surface seeding
  - Flow segmentation

Vector Field Background

- A Vector Field
  - is a continuous vector-valued function *V(x)* on a manifold *X*
  - introduces a flow $\phi : R \times X \rightarrow X$
  - can be expressed as a system of ODE

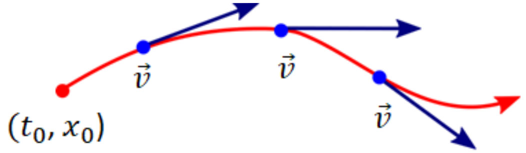
$$\frac{d\varphi(x)}{dt} = V(x)$$



Outline

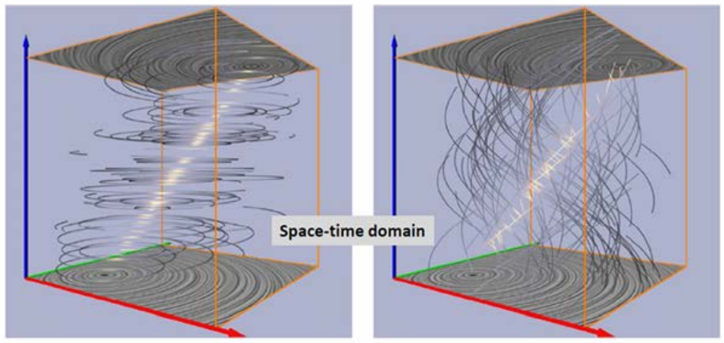
- **Streamline**: a curve that is everywhere tangent to the steady flow

$$\boldsymbol{s}(t) = s_0 + \int_{t0}^{t} \boldsymbol{v}(\boldsymbol{s}(u))du$$



- **Pathline**: a curve that is everywhere tangent to the unsteady flow

$$\boldsymbol{s}(t) = s_0 + \int_{t0}^{t} \boldsymbol{v}(\boldsymbol{s}(u), \boldsymbol{u})du$$



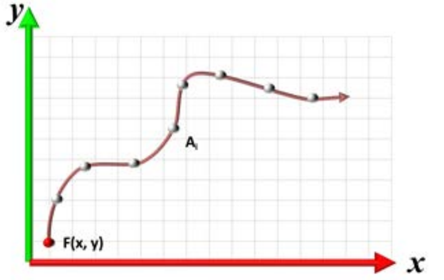
A moving center over time: (left) streamlines; (right) path lines. [Weinkauf et al. Vis10]

Local Attributes

- A number of local attributes are of interest in this work

- $\mathbf{A}_1$: vorticity, $||\nabla \times \mathbf{v}||$.
- $\mathbf{A}_2$: divergence, $tr(\mathbf{J})$, i.e., trace of $tr(\mathbf{J})$.
- $\mathbf{A}_3$: helicity, $\nabla \times \mathbf{v} \cdot \mathbf{v}$.
- $\mathbf{A}_4$: λ_2, the second largest eigenvalue of the tensor $\mathbf{S}^2 + \mathbf{R}^2$ [15].
- $\mathbf{A}_5$: $Q = \frac{1}{2}(\|\mathbf{R}\|^2 - \|\mathbf{S}\|^2)$ [13].
- $\mathbf{A}_6$: local shear rate, the Frobenius norm of $\mathbf{S}$.
- $\mathbf{A}_7$: determinant of $\mathbf{J}$.
- $\mathbf{A}_8$: change of flow direction (also known as winding angle), $\angle(\mathbf{v}(\mathbf{p}_i), \mathbf{v}(\mathbf{p}_{i+1}))$ where $\mathbf{p}_i$ denotes a point on an integral curve. This geometric attribute essentially measures the curvature of the integral curve at $\mathbf{p}_i$.
- $\mathbf{A}_9$: velocity vector $\mathbf{v}$.
- $\mathbf{A}_{10}$, acceleration, $\mathbf{a}(\mathbf{x},t) = \frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}(\mathbf{x},t)}{\partial t} + (\mathbf{v}(\mathbf{x},t) \cdot \nabla)\mathbf{v}(\mathbf{x},t)$.

Definition of Lagrangian Accumulation



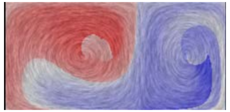
- Consider an integral curve C, starting from a given point ***(x**, t),* the Lagrangian accumulation can be formulated as:

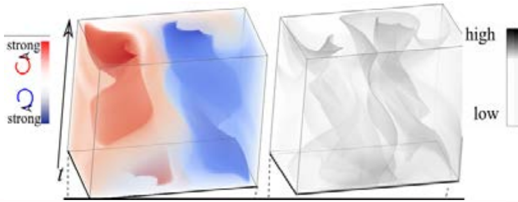
$$A_g((\mathbf{x},t_0),t) = \int_0^t k(\tau)a_l(\mathscr{C}(\tau),t_0+\tau)d\tau$$

- Similarly, the accumulation can also be done within a range along the integral curves with a specified length:

$$A_g(\mathbf{x},s) = \int_0^s k(\eta)a_l(\mathscr{C}(\eta))d\eta$$

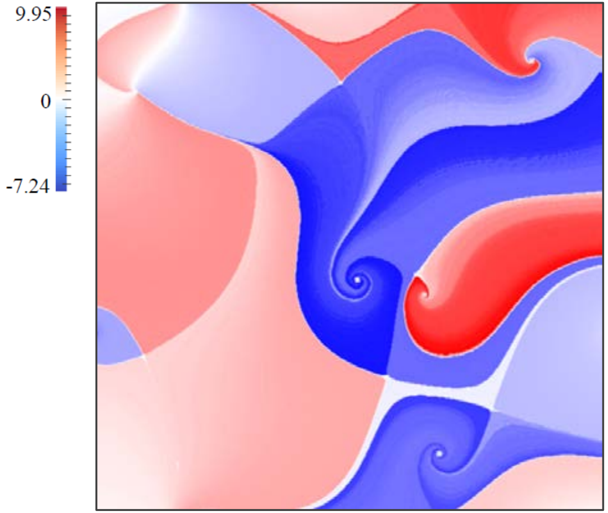
- **Attribute (*A*)** field: a derived **scalar** field obtained from the above convolution.

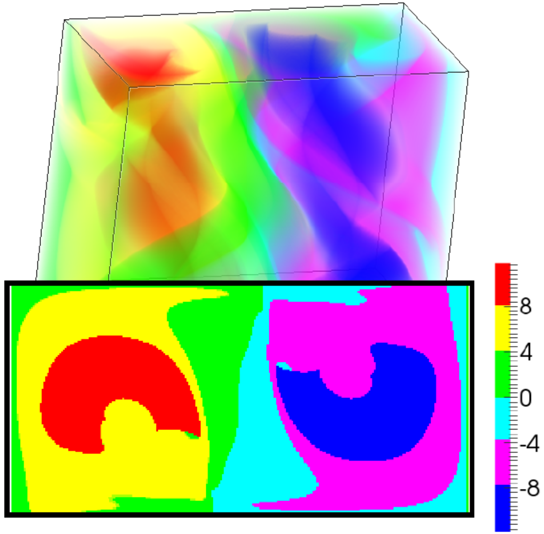




Properties of the Attribute (A) Fields

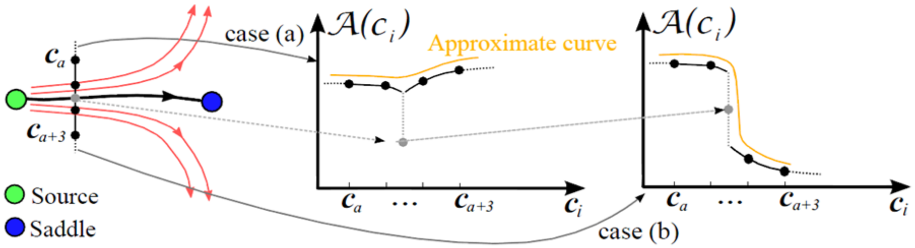
- Property 1: Existence and Uniqueness of **A** value
  - *There is exactly one* **A** *value returned given the specified parameters*

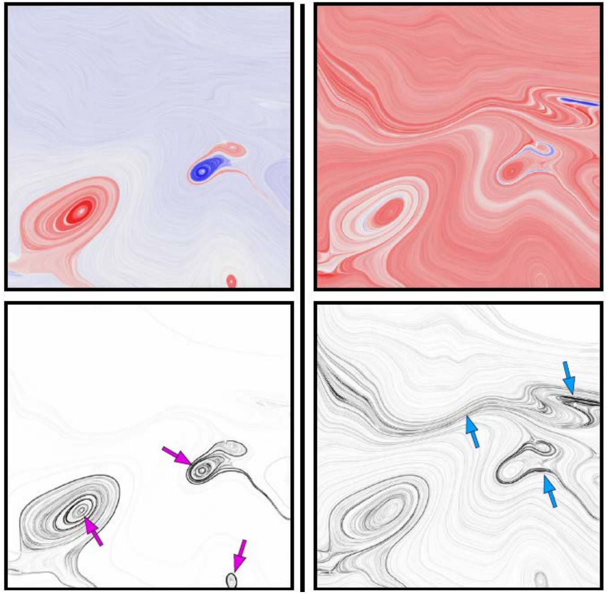




Properties of the Attribute (A) Fields

- Property 2: Discontinuity [Lei et al. TopoInVis15]
  - *This* **A** *field needs not be continuous everywhere in the flow domain*

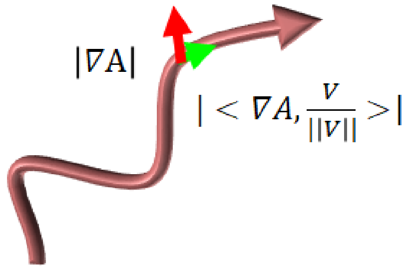




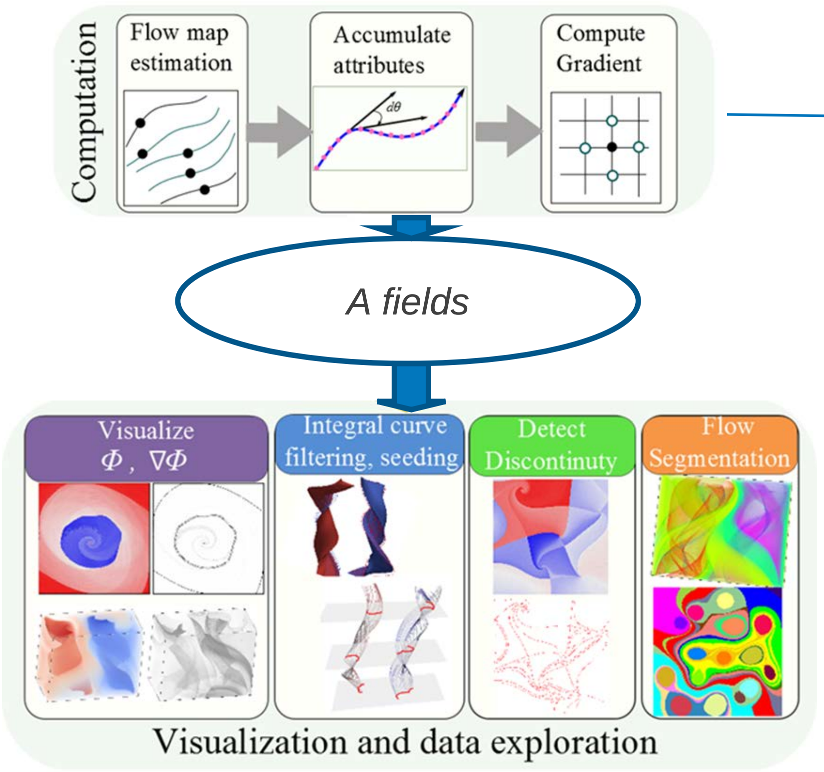
Properties of the Attribute (A) Fields, cont,

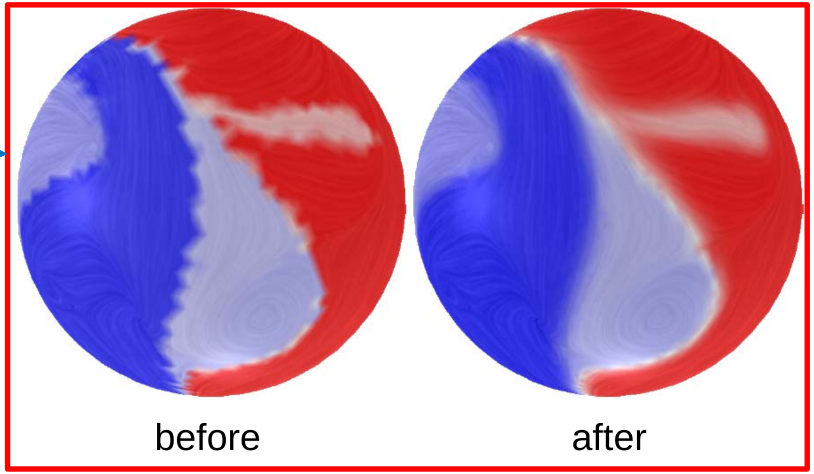
- Property 3: Inequality Property
    - *This inequality depicts that the change of* **A** *along flow direction is smaller than along a direction perpendicular to the flow direction*

$$|\nabla A| > |< \nabla A, \frac{V}{||V||} >|$$

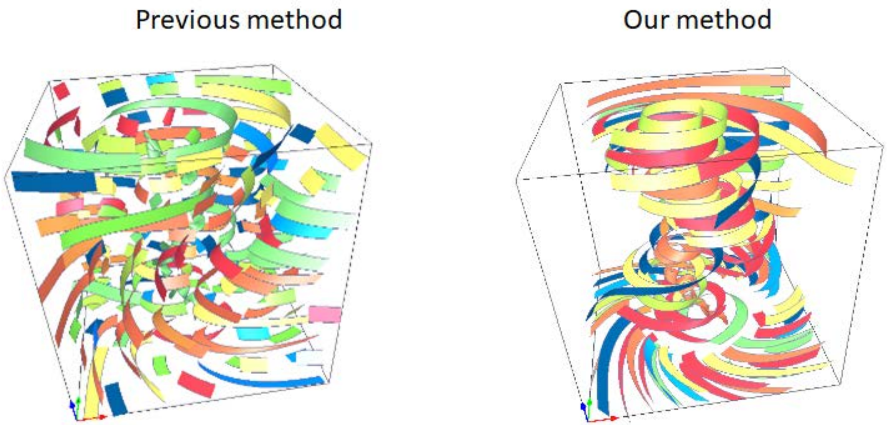


L.A. based Flow Exploration





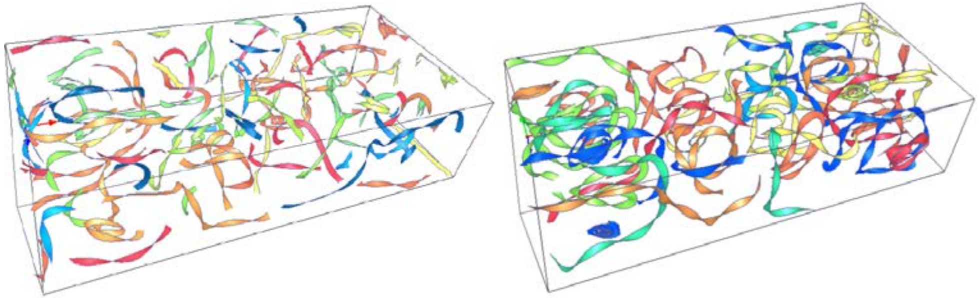
L.A. based Flow Exploration I



Ribbon placement results of the tornado data.

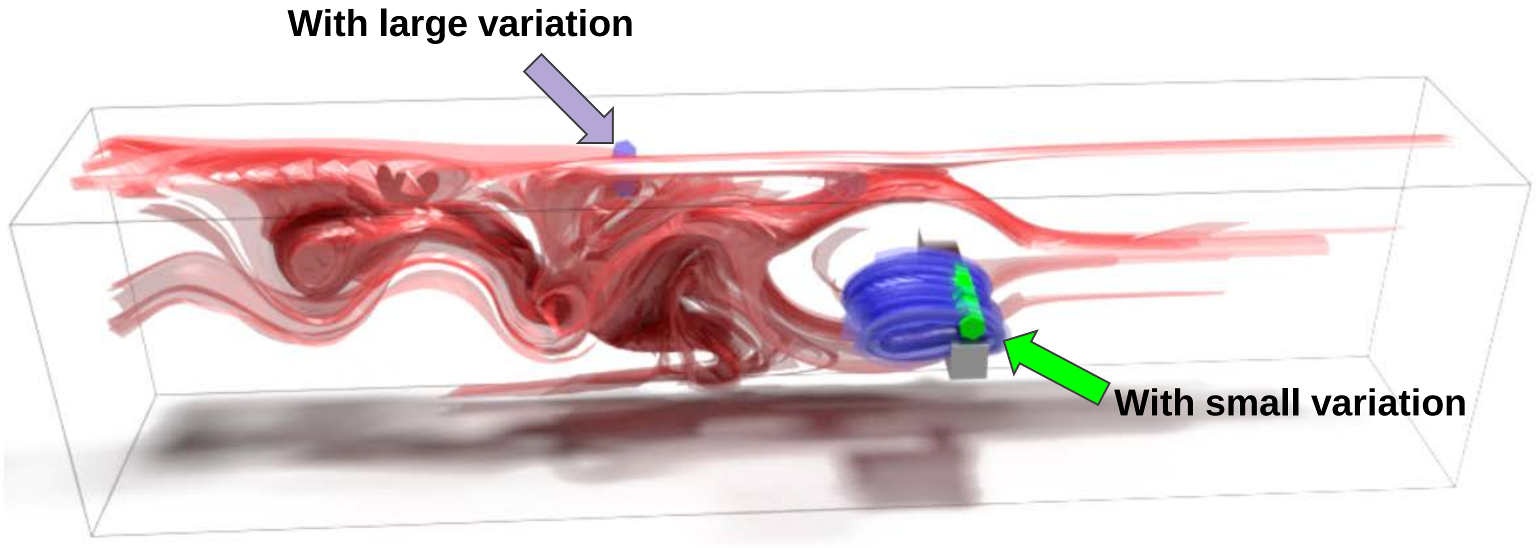
Previous method

Our method

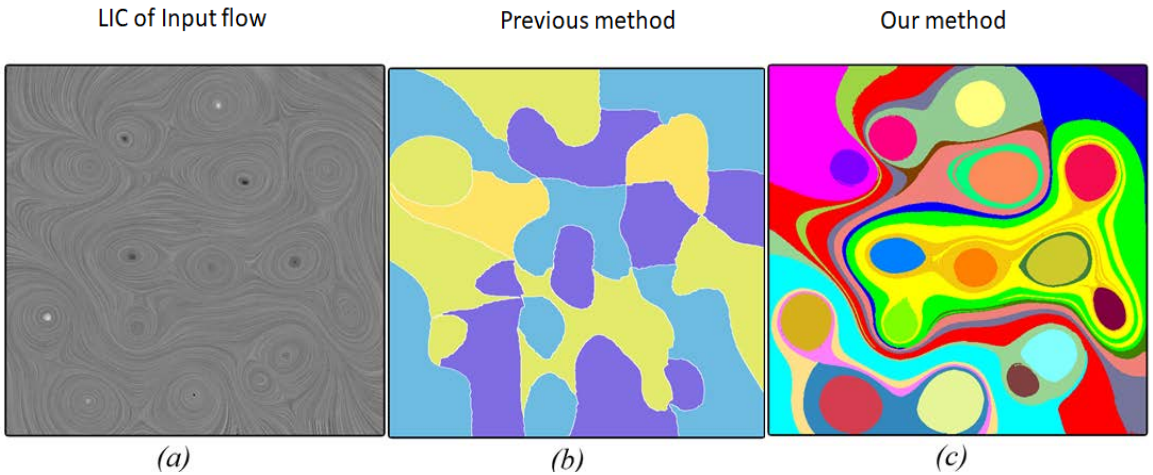


Ribbon placement results of the Bernard data.

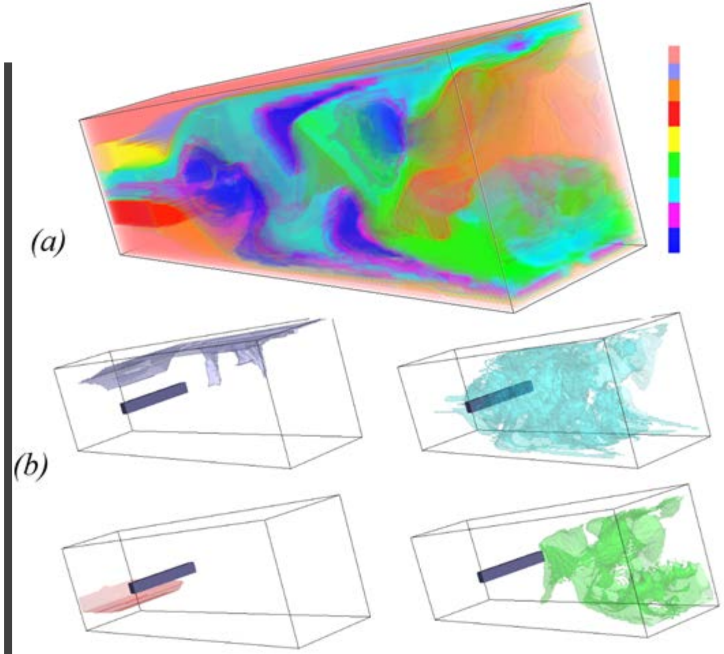
L.A. based Flow Exploration II



L.A. based Flow Exploration III

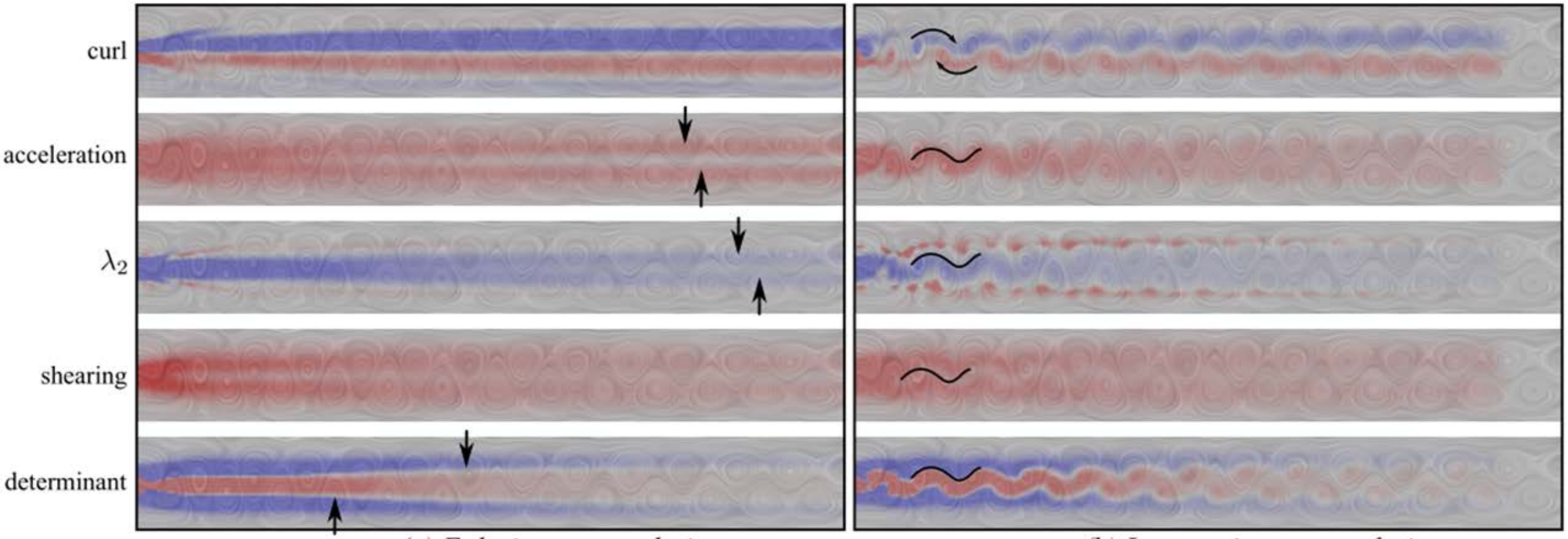


A synthetic 2D flow



3D flow behind cylinder

Comparison with Eulerian Accumulation



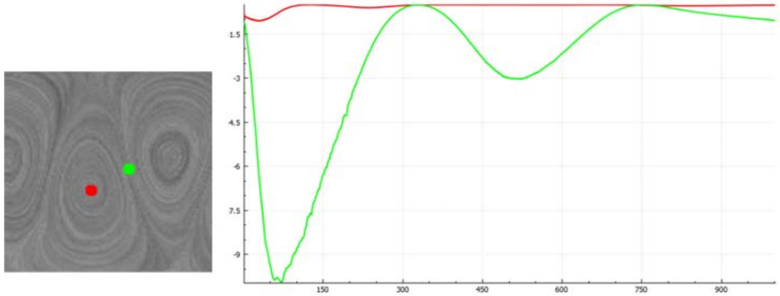
(a) Eulerian accumulation (b) Lagrangian accumulation

Comparison of Eulerian (a) and Lagrangian (b) accumulations using various attributes of the flow behind cylinder data. Note that the Eulerian accumulation highlights the places where the vortices sweep through, while the Lagrangian accumulation emphasizes the oscillating behaviors of the individual vortices.

Conclusion Remarks

- We introduce a Lagrangian accumulation framework for various flow exploration tasks. It is simple to implement and effective in supporting a number of flow exploration tasks to reveal important flow features.

- **Limitations**: when and where do those **<u>local</u>** events occur and for how long?



(a) Two particles released. (b) local attribute profile curves of the pathlines seeded at (a).

Acknowledgment

- Thank all the anonymous reviewers for their valuable comments and suggestions.

- This work was supported in part by NSF IIS 1553329 .

Thank you!