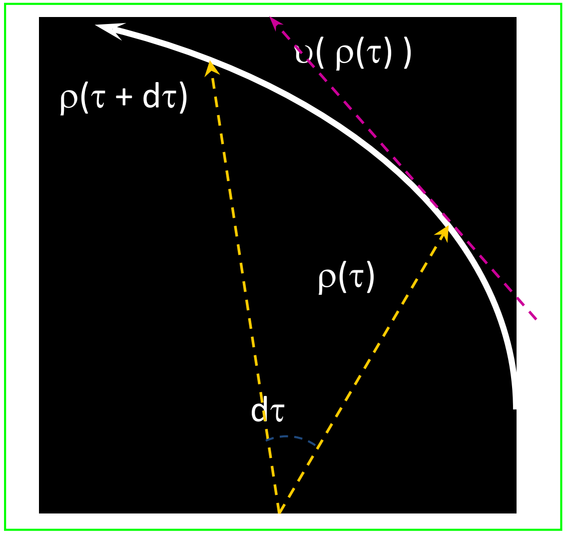
Review of Texture-based Methods

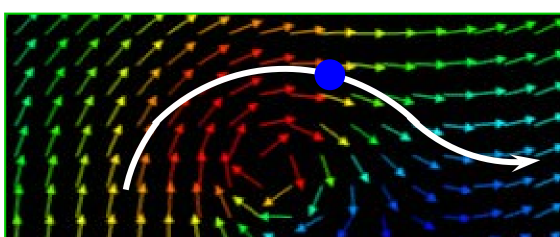
- Employs texture synthesis and image processing techniques to provide global, continuous, dense, and visually pleasing representations *without constructing intermediate geometry*.
- **LIC** family is the most popular texture-based technique
- **IBFV** is an easy but flexible technique
- Both can be extended to (2.5D) surface flow visualization
- IBFV is more computationally efficient than LIC
- Extending to 3D volumetric data visualization is possible but challenging due to the occlusion

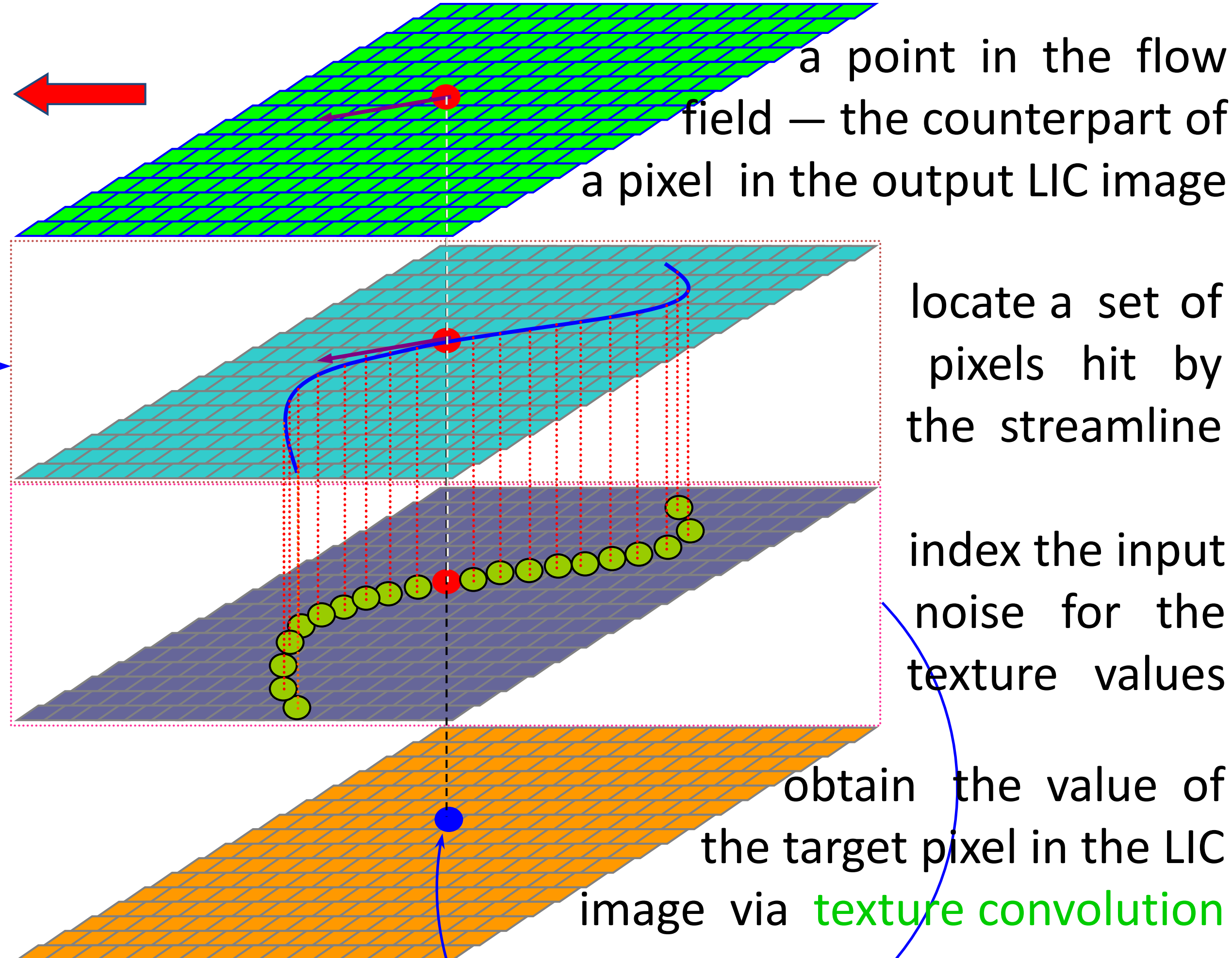
LIC



$$d(\rho(\tau))/d\tau = \upsilon(\rho(\tau))$$

$$\rho(\tau+\Delta\tau) = \rho(\tau) + \int_{\tau}^{\tau+\Delta\tau} \upsilon(\rho(\tau))d\tau$$





$$\frac{\sum (\, texture[i] \times weight[i] \,)}{\sum \; weight[i]}$$

weighting is governed by a low-pass filter

IBFV

http://www.win.tue.nl/~vanwijk/ibfv/

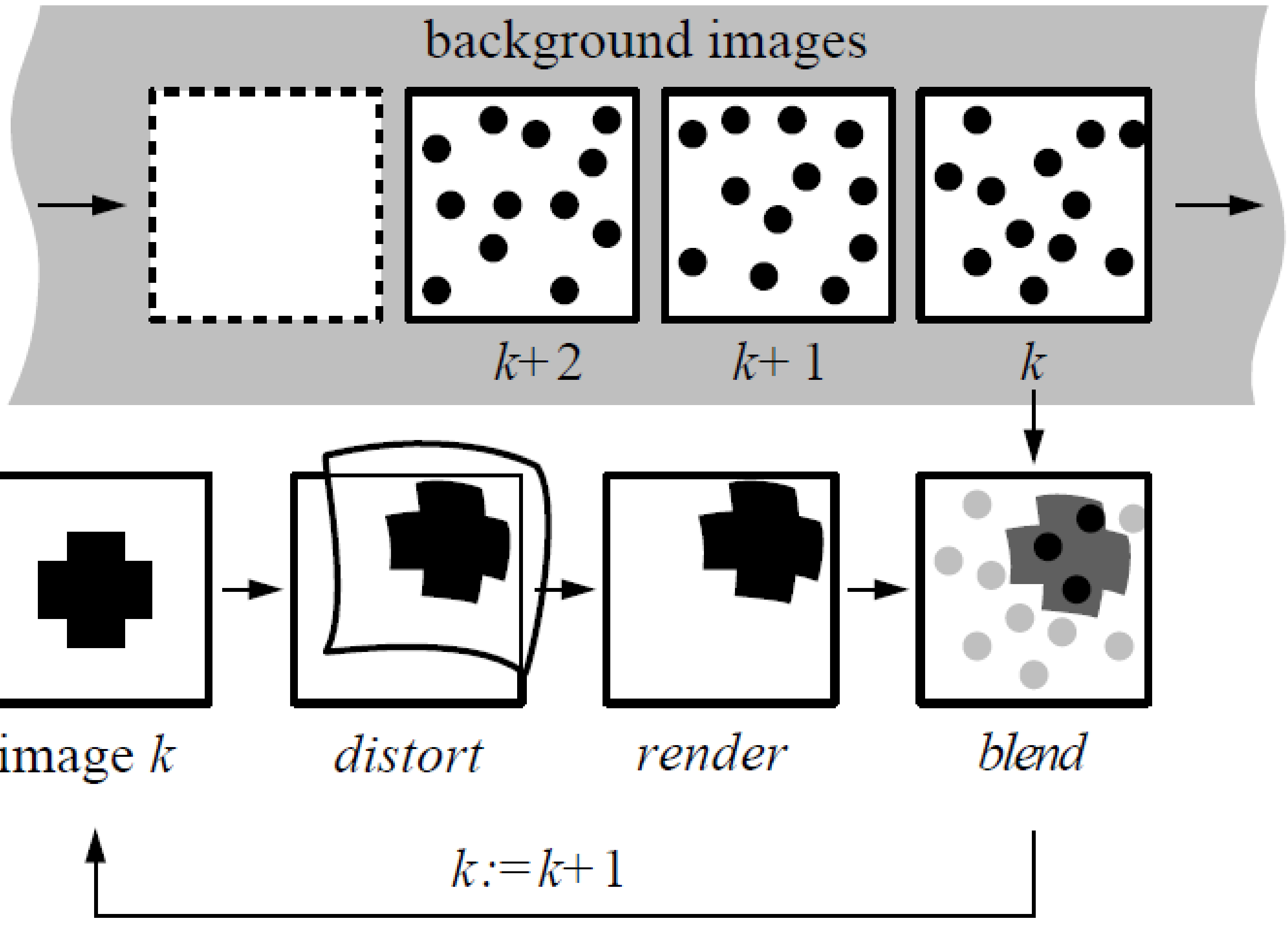
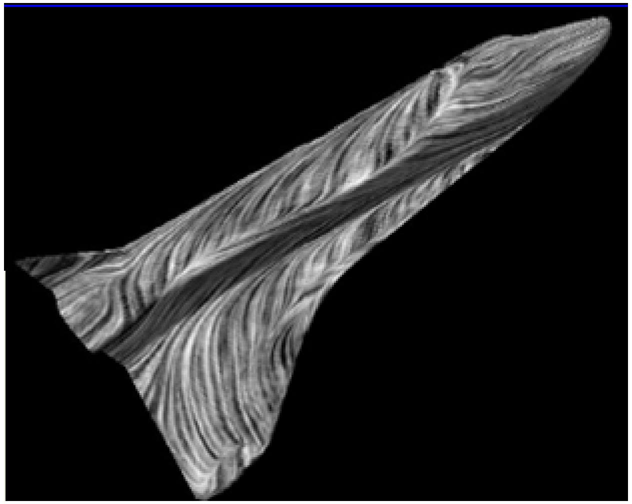
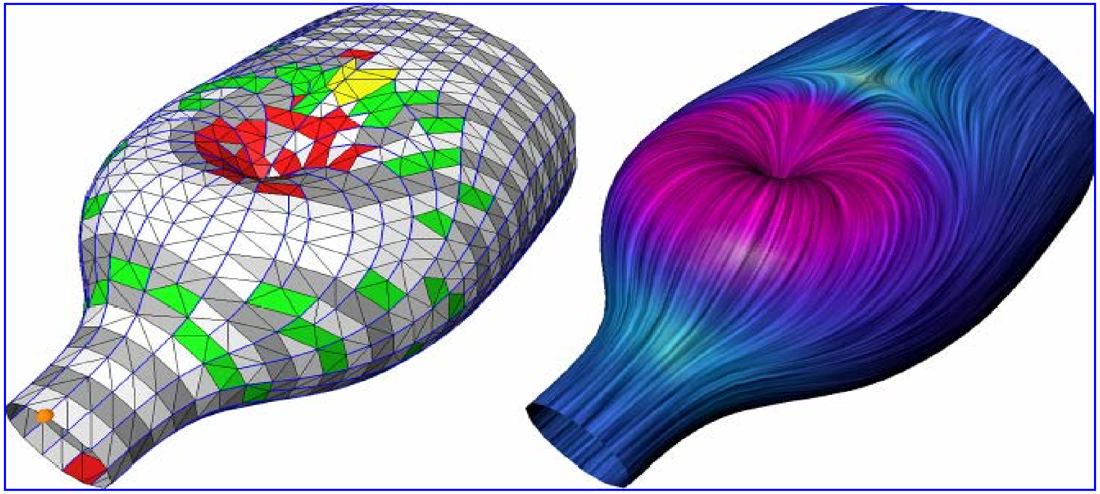


Image source: van Wijk

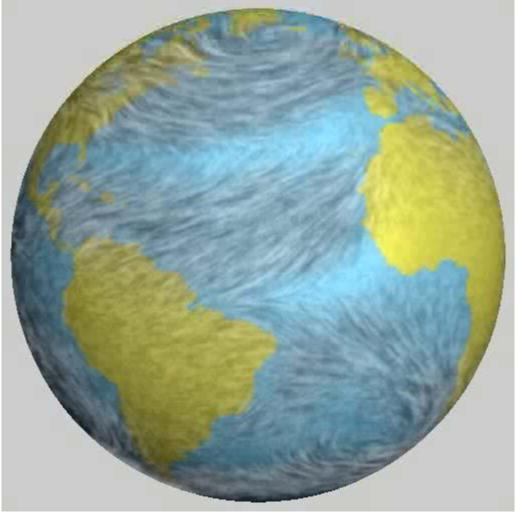
Texture-based Methods on Surfaces



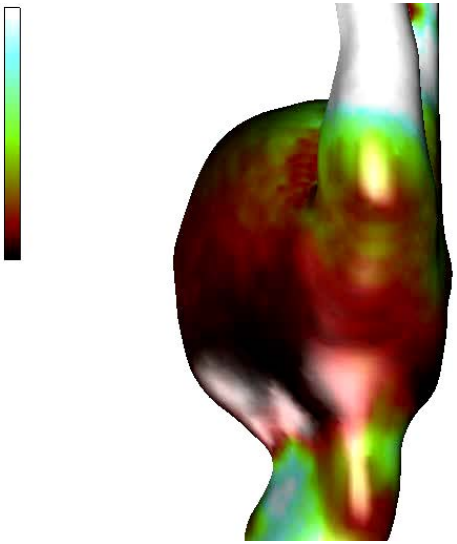
Surface LIC



(Detlev Stalling, ZIB, Germany)

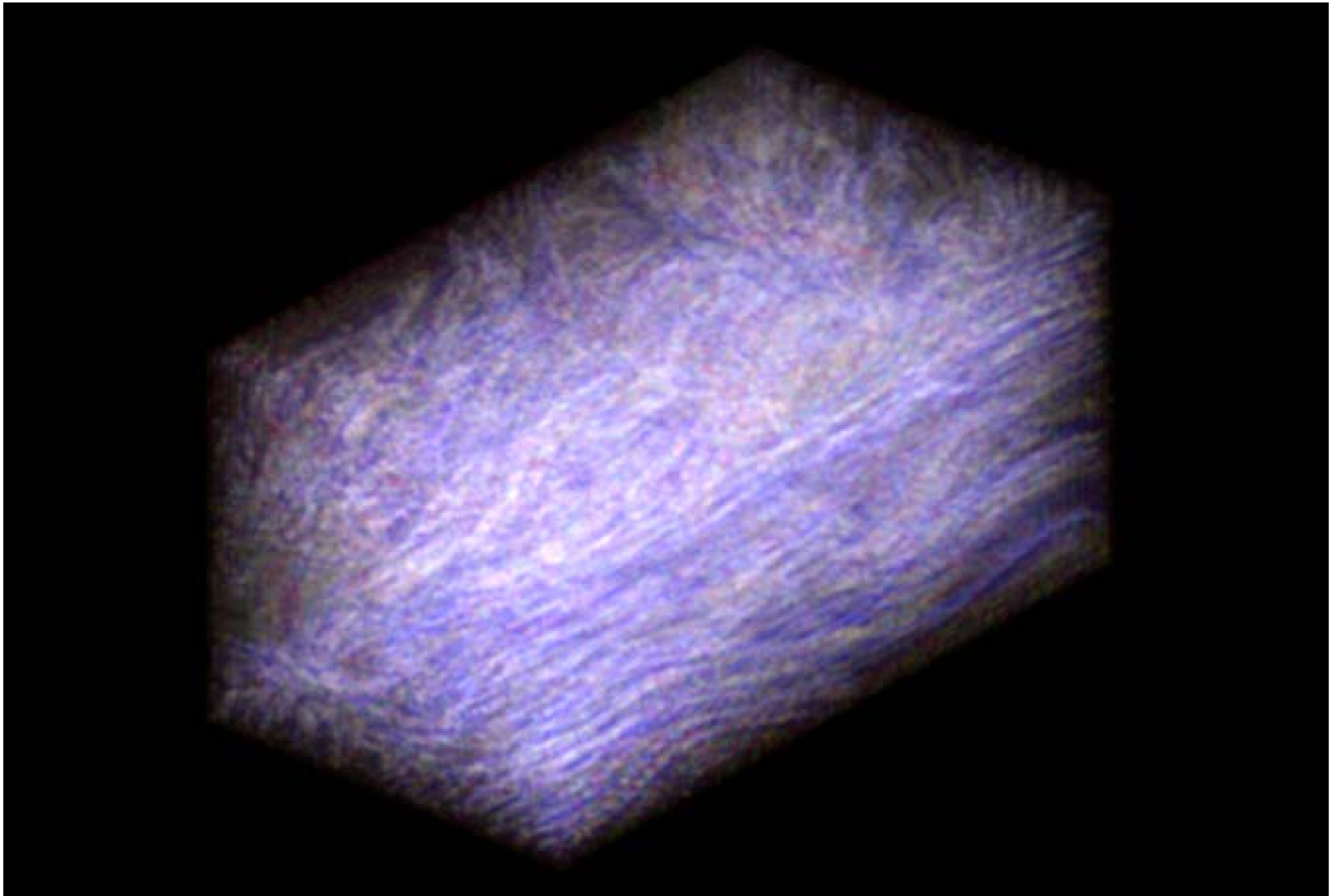


IBFVS



ISA

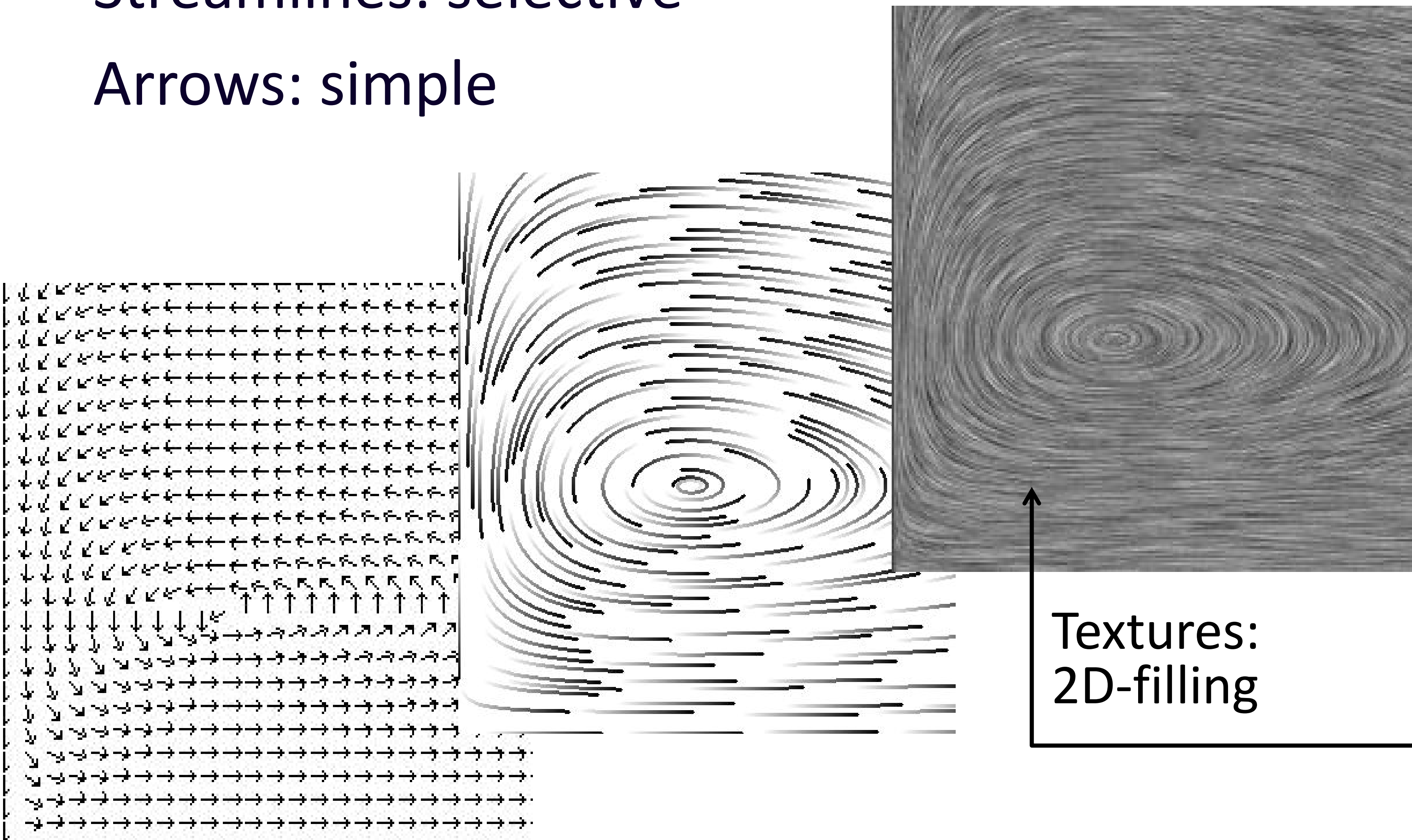
Volumetric Texture

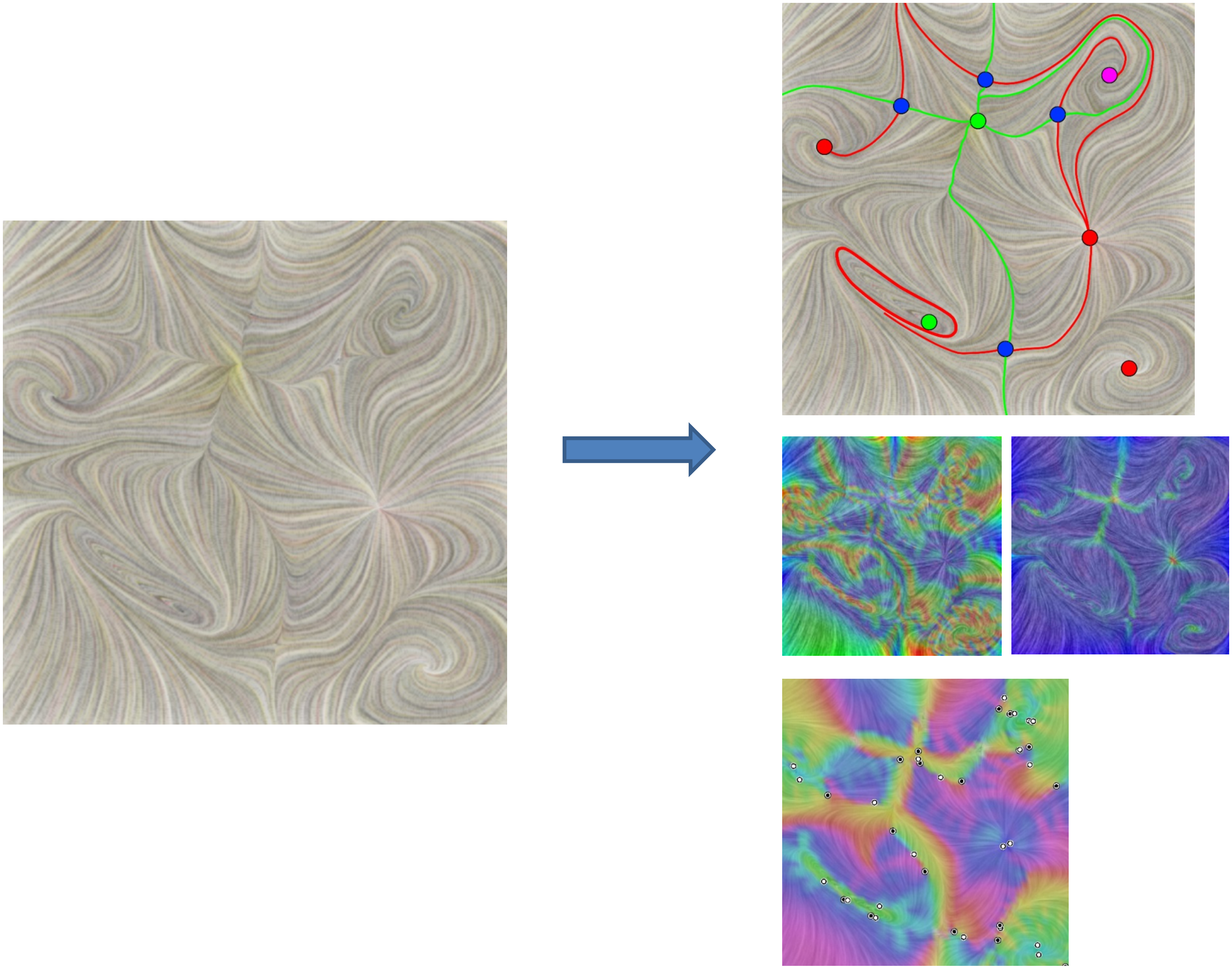


Arrows vs. Streamlines vs. Textures

Streamlines: selective

Arrows: simple





Vector Field Visualization: Feature-based

Goal: how to compute Jacobian of flow; know what features in vector fields are of interest; what are the common physical features; how to use information of Jacobian to extract some of the relevant features.

What features are in flows?

Features are highly application dependent

- Non-topology (physics-based)
- Topology

Physics-based Feature Extraction

Their computation is mostly local

Vector Field Gradient

- Consider a vector field

$$ {}^{d\mathbf{x}}\!/_{dt} = V(\mathbf{x}) = \vec{\boldsymbol{f}}(x, y, z) = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} $$

- Its gradient is

$$ \nabla V = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} & \frac{\partial f_x}{\partial z} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} & \frac{\partial f_y}{\partial z} \\ \frac{\partial f_z}{\partial x} & \frac{\partial f_z}{\partial y} & \frac{\partial f_z}{\partial z} \end{bmatrix} $$

It is also called the **Jacobian** matrix of the vector field.
Many feature detection for flow data relies on Jacobian.

What is the Jacobian of the following vector field?

$$V = \begin{cases} vx = -y \\ vy = \dfrac{1}{2}x \end{cases}$$

What is the Jacobian of the following vector field?

$$V = \begin{cases} vx = -y \\ vy = \frac{1}{2}x \end{cases}$$

$$\nabla V = \begin{bmatrix} 0 & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$$

What is the Jacobian of the following vector field?

$$V = \begin{cases} vx = 10x + 10y + 12340 \\ vy = 12x - 19y - 12000 \end{cases}$$

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$$V = \begin{cases} vx = 10x + 10y + 12340 \\ vy = 12x - 19y - 12000 \end{cases}$$

$$\nabla V = \begin{bmatrix} 10 & 10 \\ 12 & -19 \end{bmatrix}$$

Divergence and Curl

- **Divergence** measures the magnitude of outward flux through a small volume around a point

$$div\, V = \nabla \cdot V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix}$$

- Curl- describes the infinitesimal rotation around a point

$$curl\, V = \nabla \times V = \begin{bmatrix} \frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} & \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} & \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \end{bmatrix}$$

$$\nabla \cdot (\nabla \times V) = 0$$

$$\nabla \times (\nabla \emptyset) = \vec{0}$$

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$$\nabla \cdot (\nabla \times V) = 0$$

$$\nabla \times (\nabla \emptyset) = \vec{0}$$

Both are local computation!

Consider a 2D Steady Vector Fields

- Assume a 2D steady **piecewise linear** vector field

$$d\mathbf{x}/_{dt} = V(\mathbf{x}) = \vec{\boldsymbol{f}}(x, y) = \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} ax + by + c \\ dx + ey + f \end{pmatrix}$$

- Its Jacobian is

$$\nabla V = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix}$$

- Divergence is $a + e$
- Curl is $-(\mathrm{b} - \mathrm{d})$

Given a vector field defined on a discrete mesh, it is important to compute the coefficients **a, b, c, d, e, f** for later analysis.

What are the divergence and curl of the following vector field?

$$V = \begin{cases} vx = -y \\ vy = \frac{1}{2}x \end{cases}$$

$$\nabla V = \begin{bmatrix} 0 & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$$

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$$\nabla V = \begin{bmatrix} 0 & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$$

$$div\, V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} = 0 + 0 = 0$$

$$curl\, V = -\left(\frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x}\right) = -(-1 - \frac{1}{2}) = \frac{3}{2}$$

What are the divergence and curl of the following vector field?

$$V = \begin{cases} vx = x + 2y + z + 0.5 \\ vy = x - 3z - 1 \\ vz = -y + z + 2 \end{cases}$$

What are the divergence and curl of the following vector field?

$$V = \begin{cases} vx = x + 2y + z + 0.5 \\ vy = x - 3z - 1 \\ vz = -y + z + 2 \end{cases}$$

$$\nabla V = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -3 \\ 0 & -1 & 1 \end{bmatrix}$$

What are the divergence and curl of the following vector field?

$$V = \begin{cases} vx = x + 2y + z + 0.5 \\ vy = x - 3z - 1 \\ vz = -y + z + 2 \end{cases}$$

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$$div\, V = \nabla \cdot V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = 1 + 0 + 1 = 2$$

$$curl\, V = \nabla \times V = \begin{bmatrix} \frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} & \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} & \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \end{bmatrix}$$

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$$V = \begin{cases} vx = x + 2y + z + 0.5 \\ vy = x - 3z - 1 \\ vz = -y + z + 2 \end{cases} \qquad \nabla V = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$div\, V = \nabla \cdot V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = 1 + 0 + 1 = 2$$

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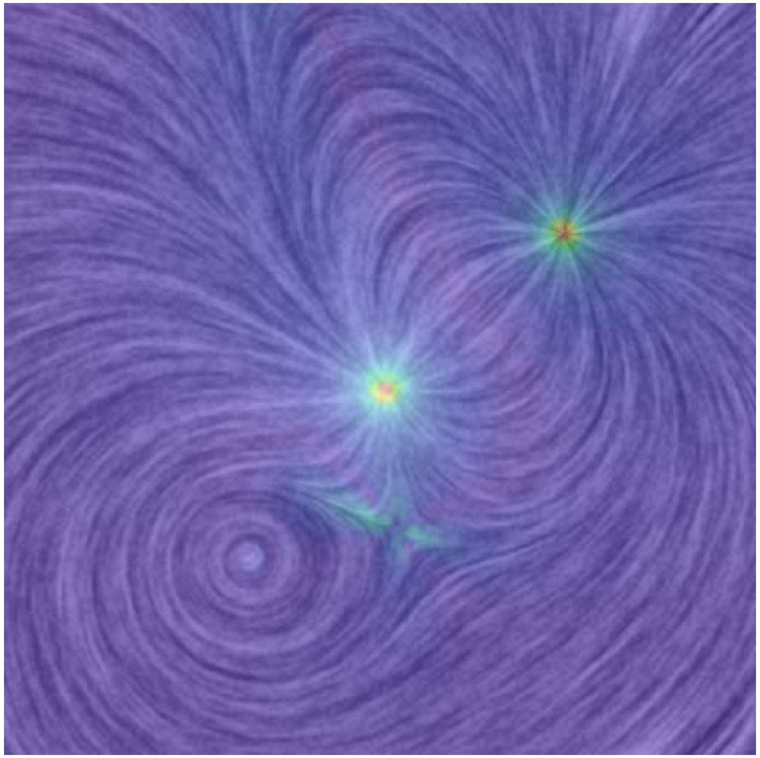
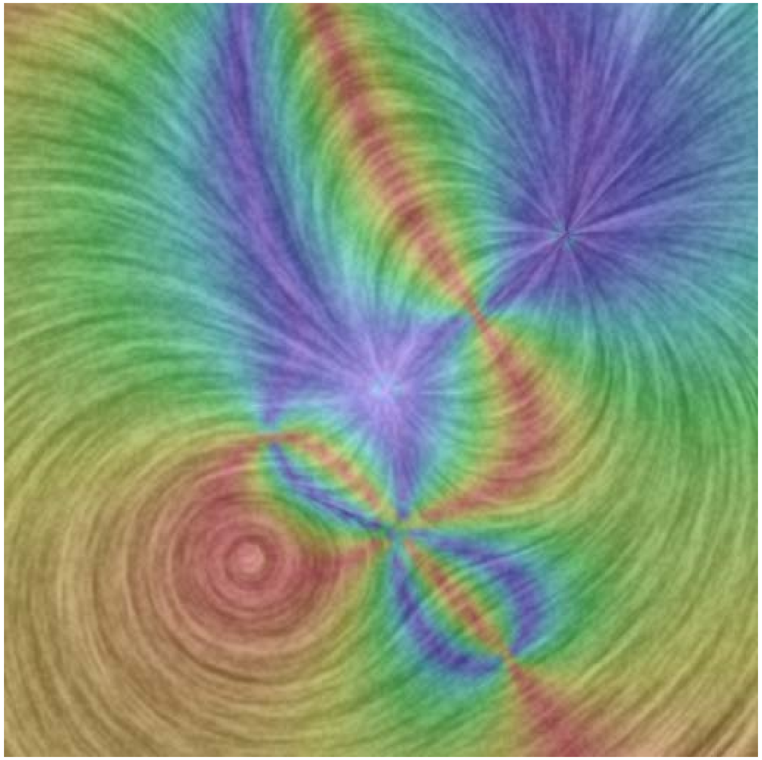
$$= [-1 - (-3), 1 - 0, 1 - 2] = [2, 1, -1]$$

vorticity vector!!

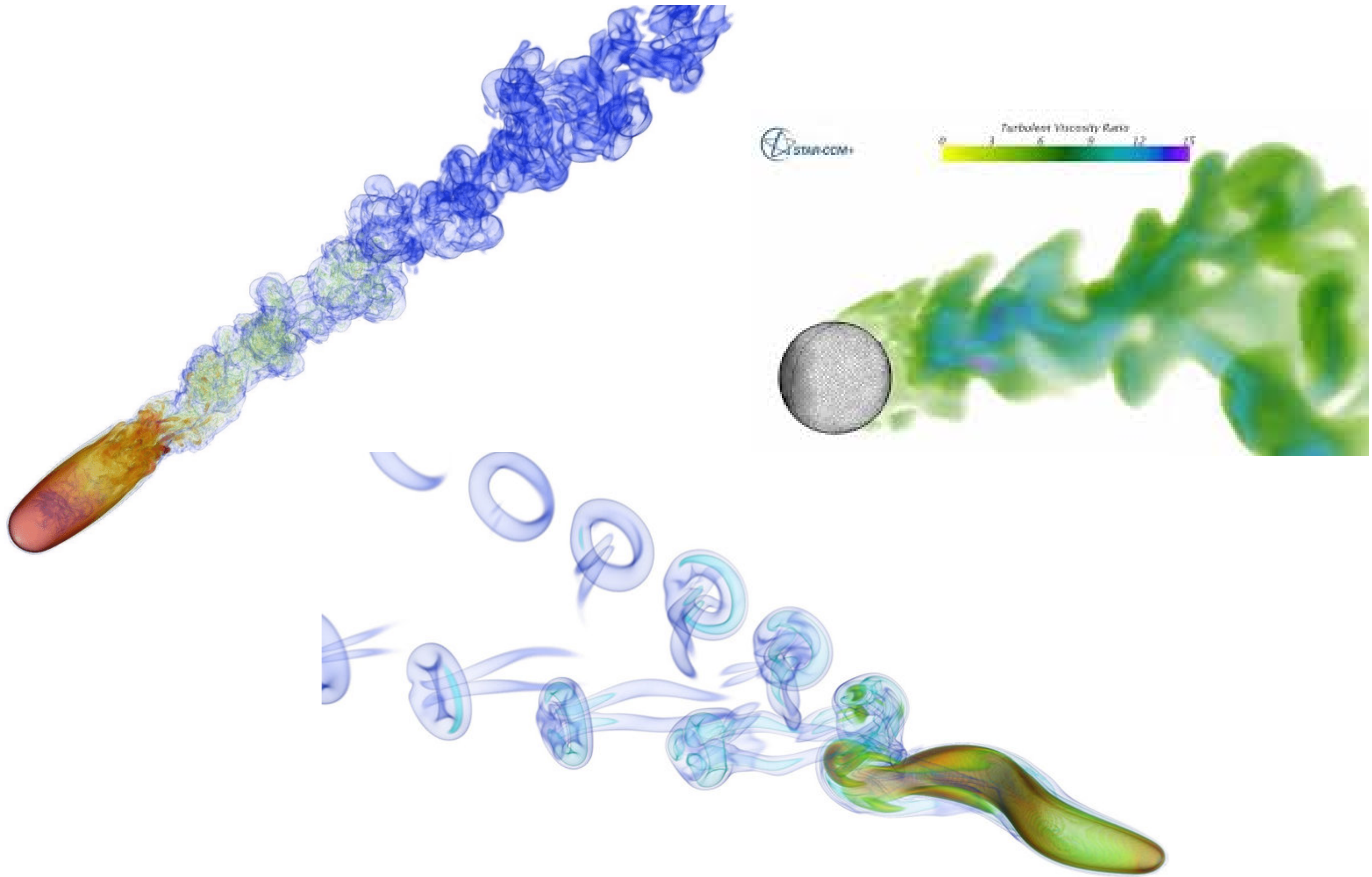
Examples of Divergence and Curl of 2D Vector Fields





Divergence and curl of a vector field
Rainbow color coding is used.

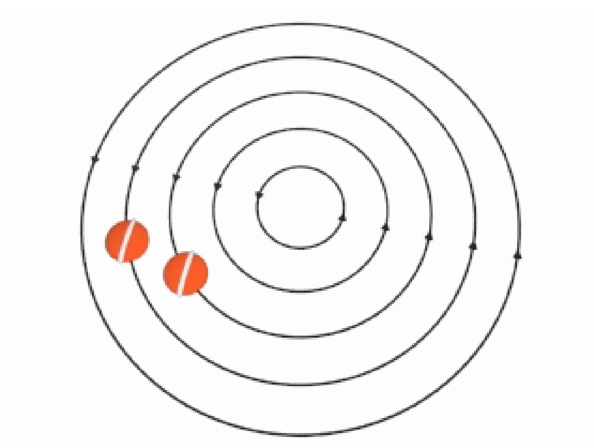
Examples of vector-valued data reduction!

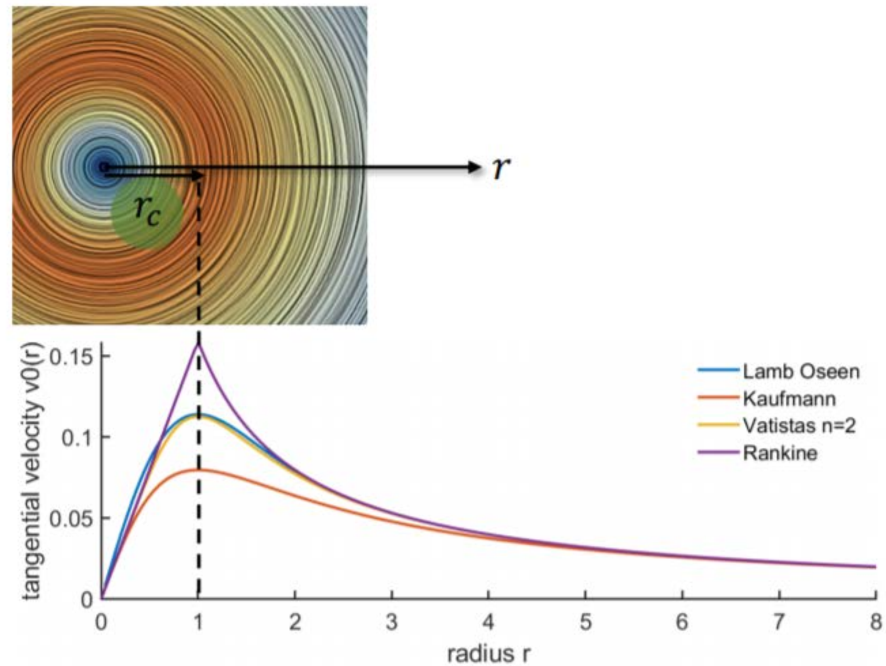


Volume rendering of vorticity in various flows (images from Google images)

Jacobian and its derived physical quantities can be applied to the extraction of certain physics-based features.

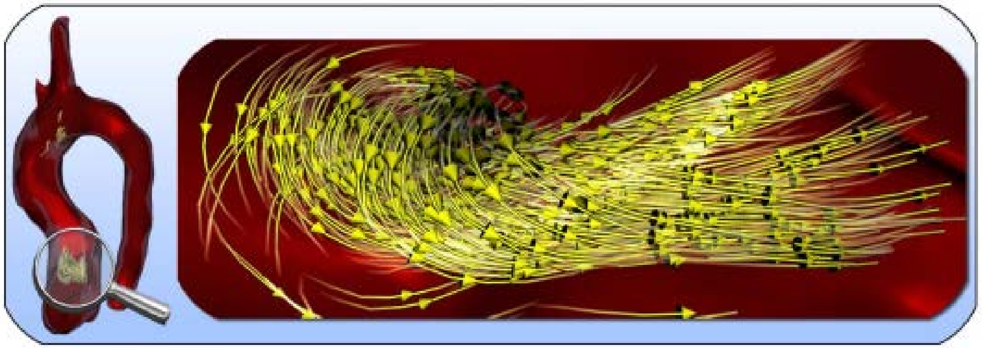
Vortices



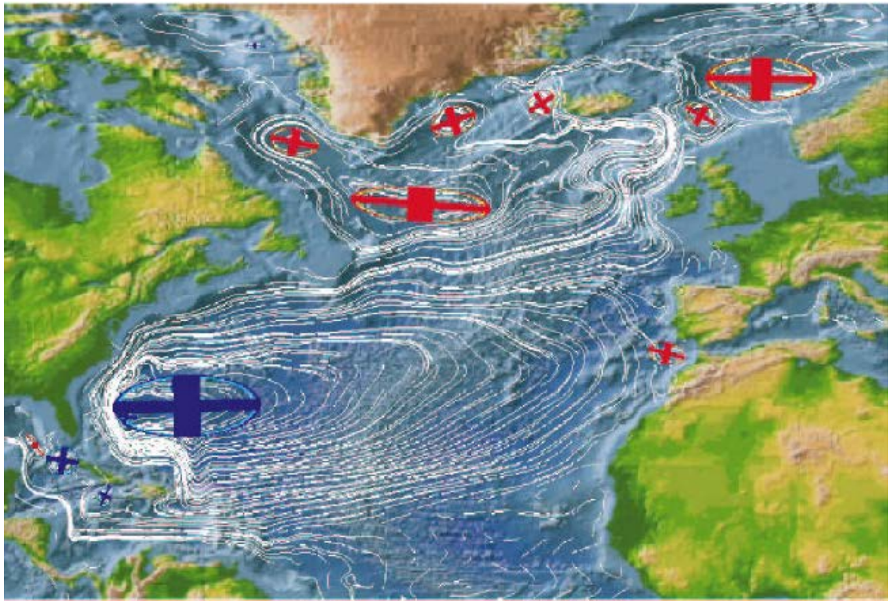


There is NO unified definition of vortices!!!!

Vortices



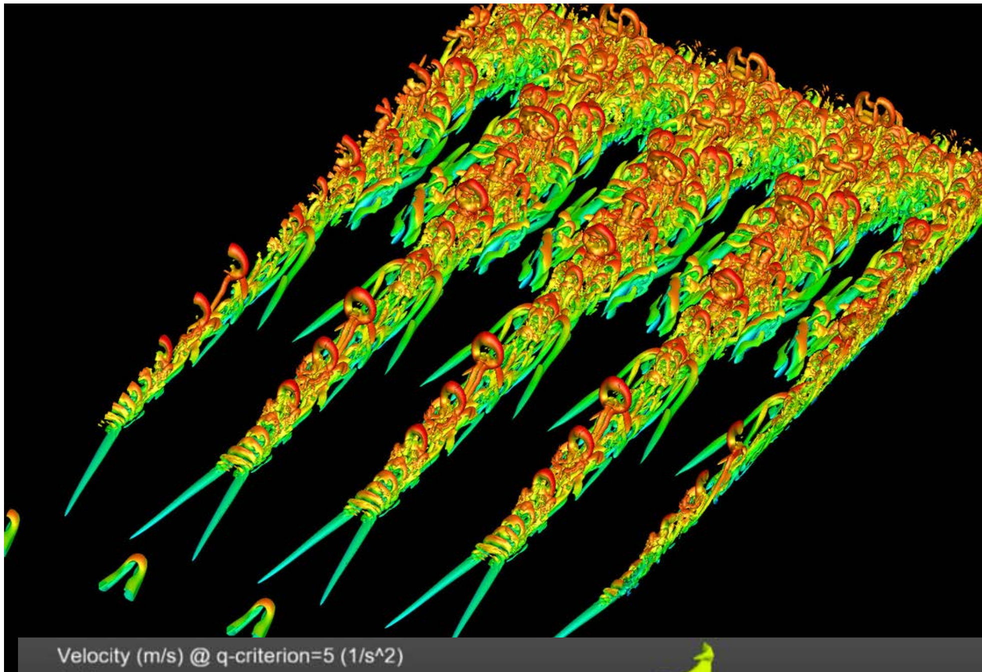
Blood Flow Analysis [Köhler et al. 2013]

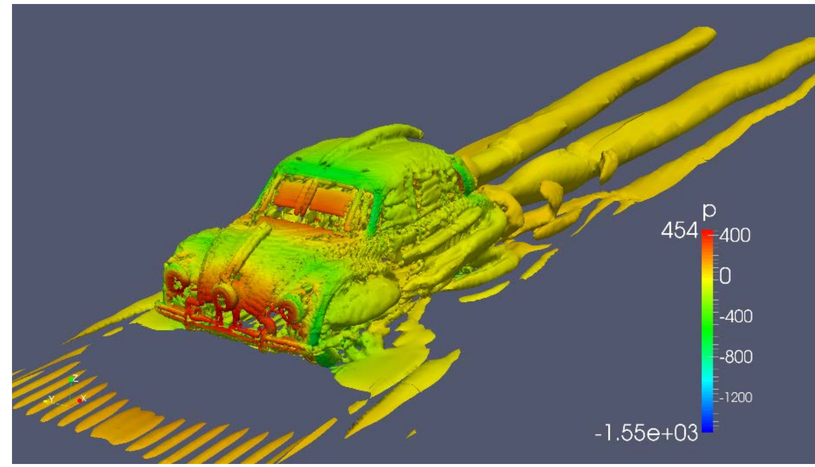


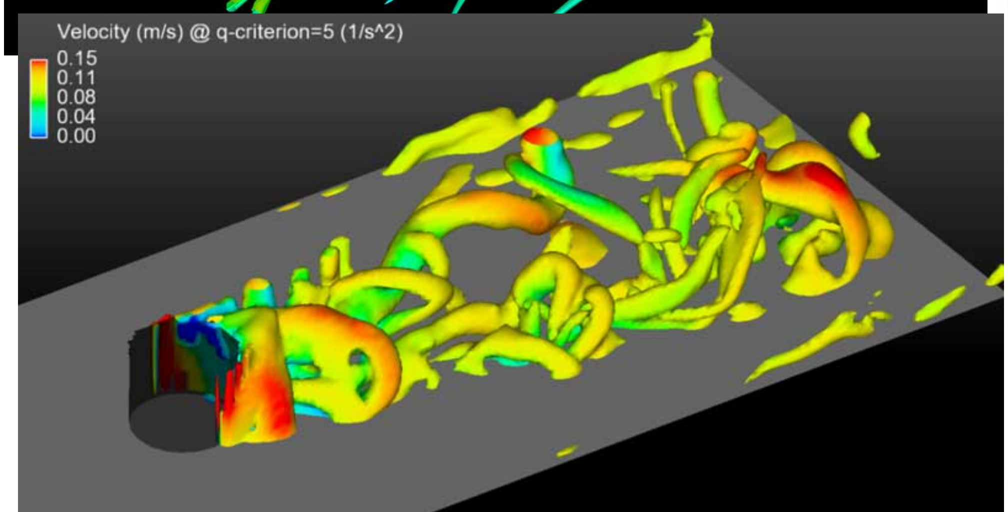
Post et al. STAR report 2003

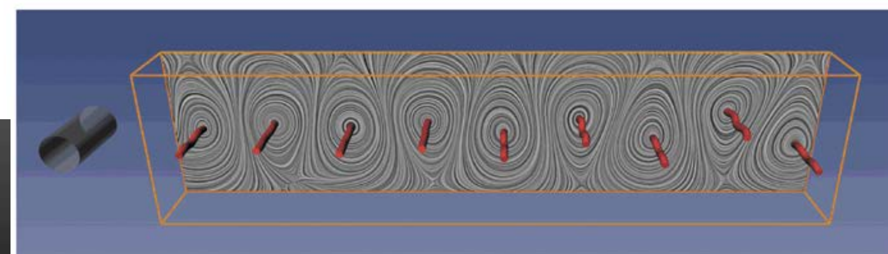
ource:
tp://www.onera.fr/cahierdelabo/english/aerod_ind03.htm

Vortices in turbulent flows

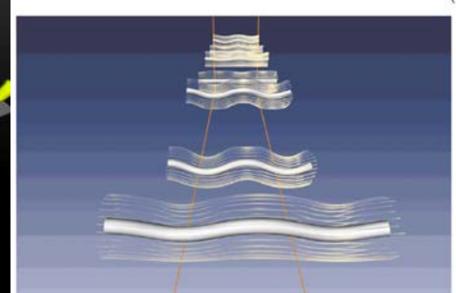




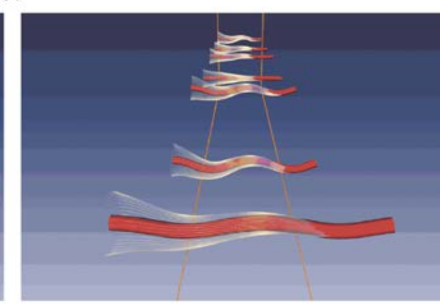




(a)



(b)



(c)

Typical vortex detection techniques

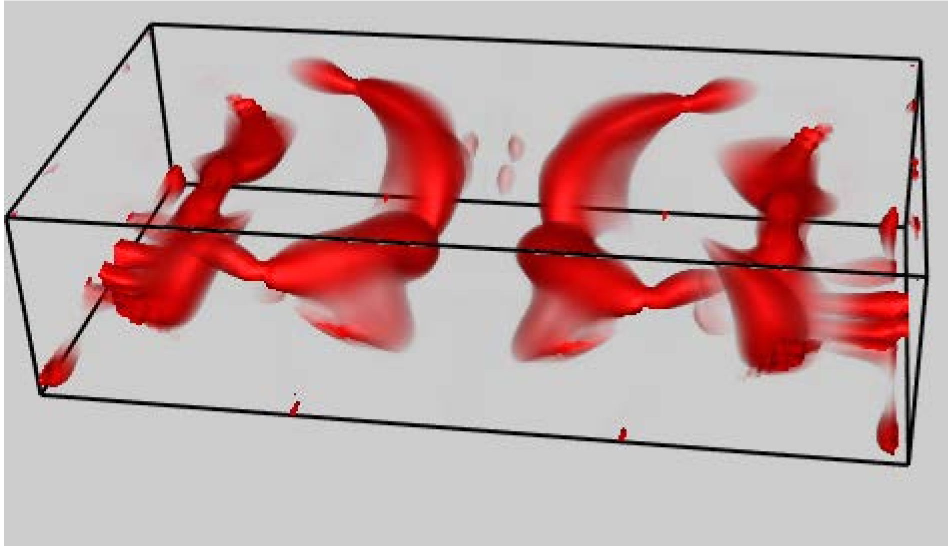
- **Region-based** – using one of the following attributes and some ad-hoc thresholds

$$\boldsymbol{J} = \mathbf{S} + \mathbf{R}, \qquad \mathbf{S} = \frac{1}{2}[\boldsymbol{J} + \boldsymbol{J}^{\mathrm{T}}], \qquad \mathbf{R} = \frac{1}{2}[\boldsymbol{J} - \boldsymbol{J}^{\mathrm{T}}]$$

Vorticity

Q-criterion $\quad Q = \frac{1}{2}\left(\|\mathbf{R}\|^2 - \|\mathbf{S}\|^2\right)$

λ_2 is the second largest eigenvalue of the tensor $\mathbf{S}^2 + \mathbf{R}^2$



Typical vortex detection techniques

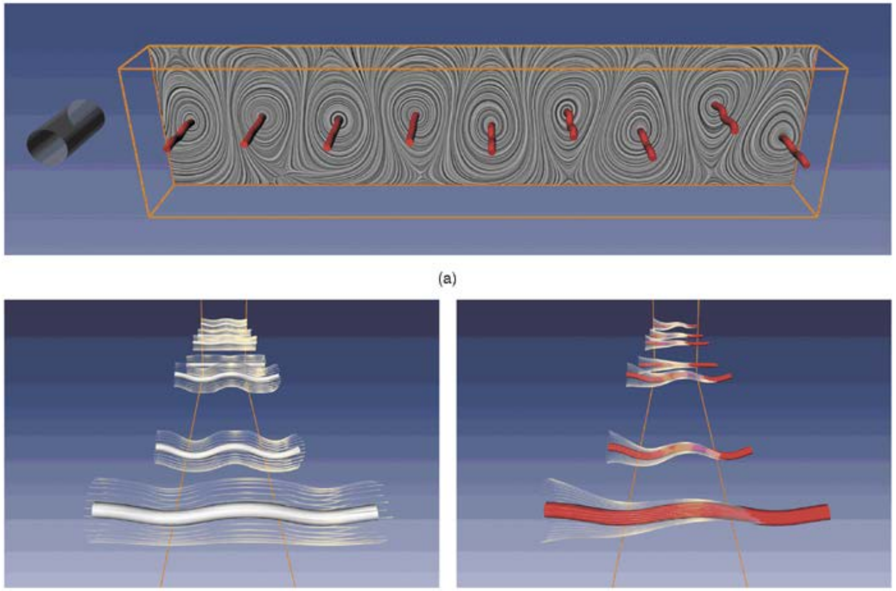
- **Line-based** – detecting vortex cores

PV-parallel vector operator

$$\mathbf{v} || \nabla\mathbf{v} \cdot \mathbf{v}$$

the acceleration is parallel to the velocity

the Jacobian matrix has a complex pair of eigenvalues



(a) (b) (c)

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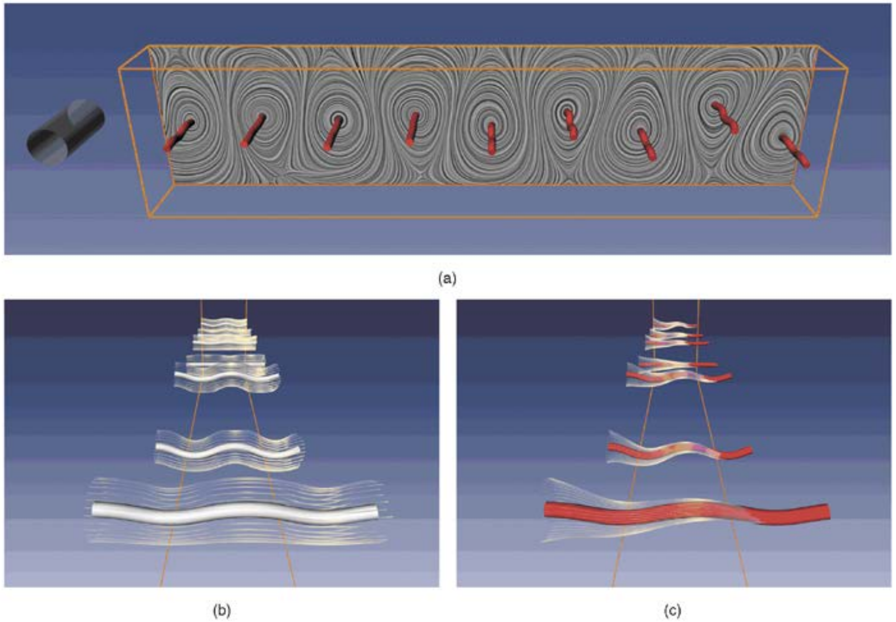
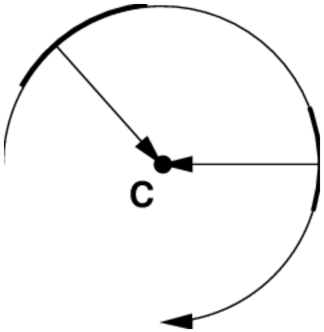
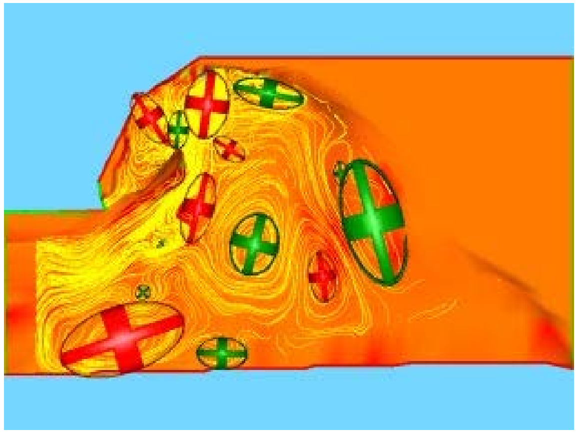


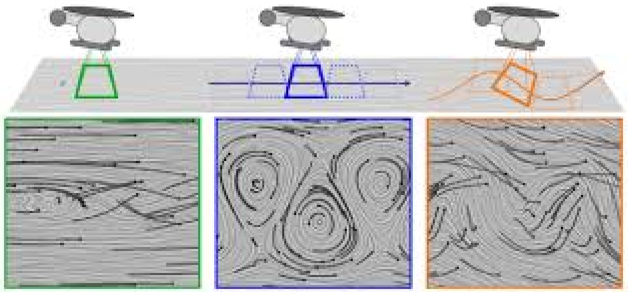
- **Geometric-based** – finding returning streamline using winding angles





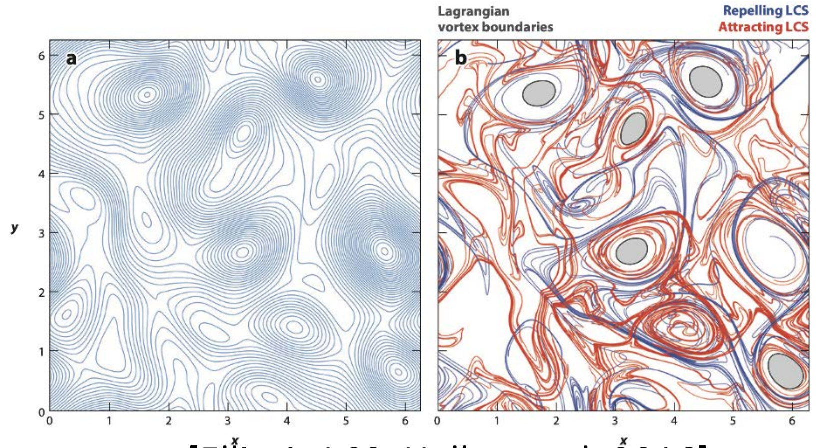
Recent vortex detection techniques

Objectivity-based – extract optimal reference frame

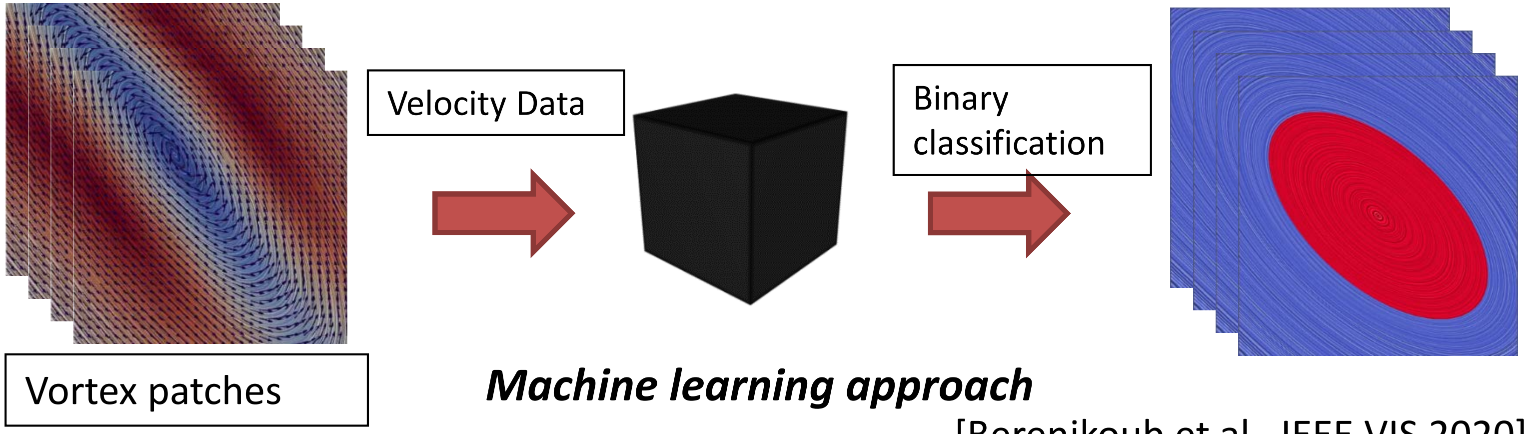


[Guenther et al., SIGGRAPH 2017]

Vortex boundary extraction



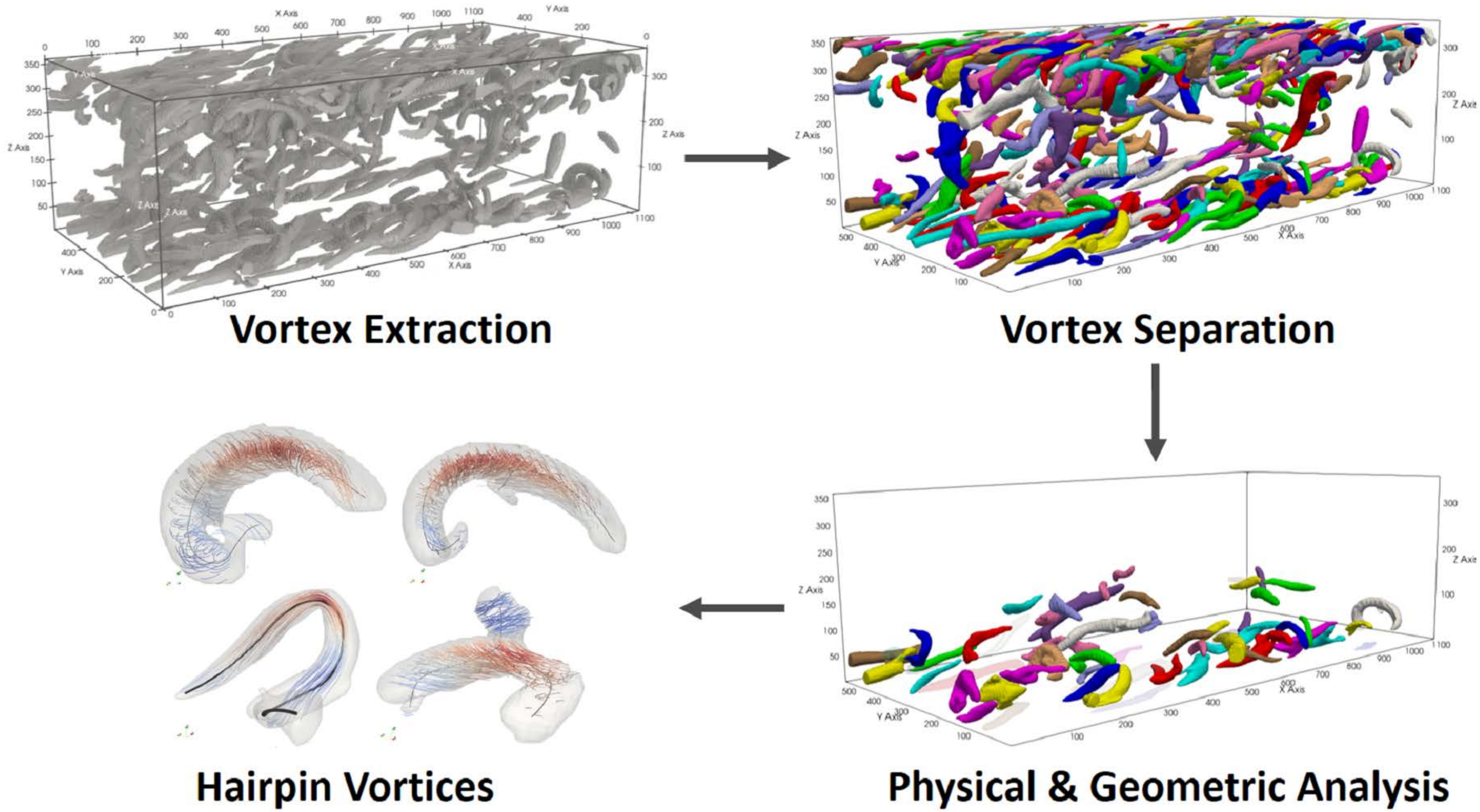
[Elliptic LCS, Haller et al. 2016]

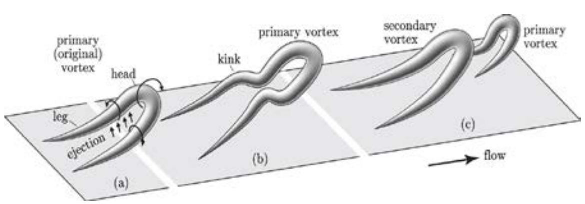


Machine learning approach

[Berenjkoub et al., IEEE VIS 2020]

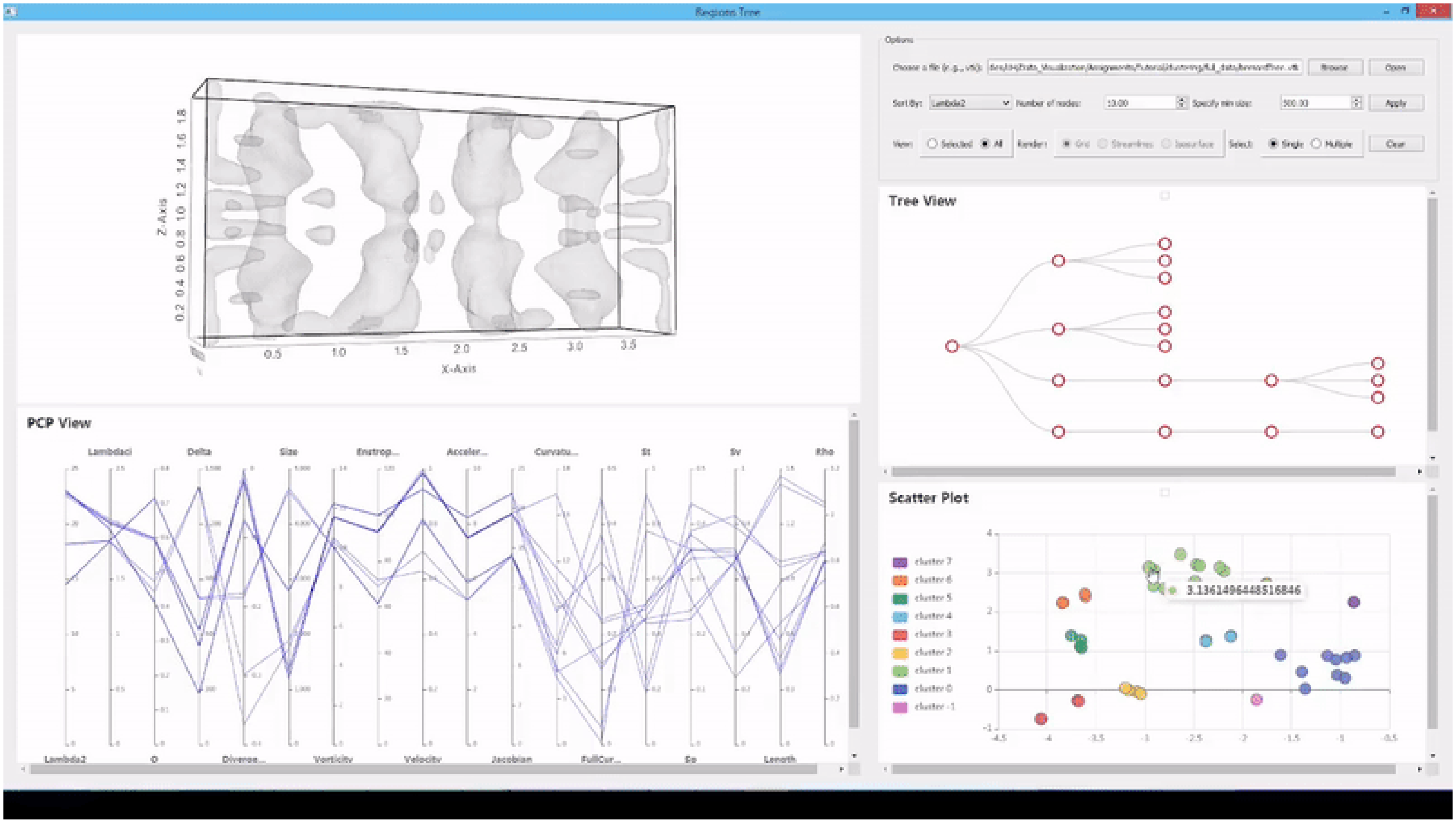
Hairpin vortex extraction and visualization





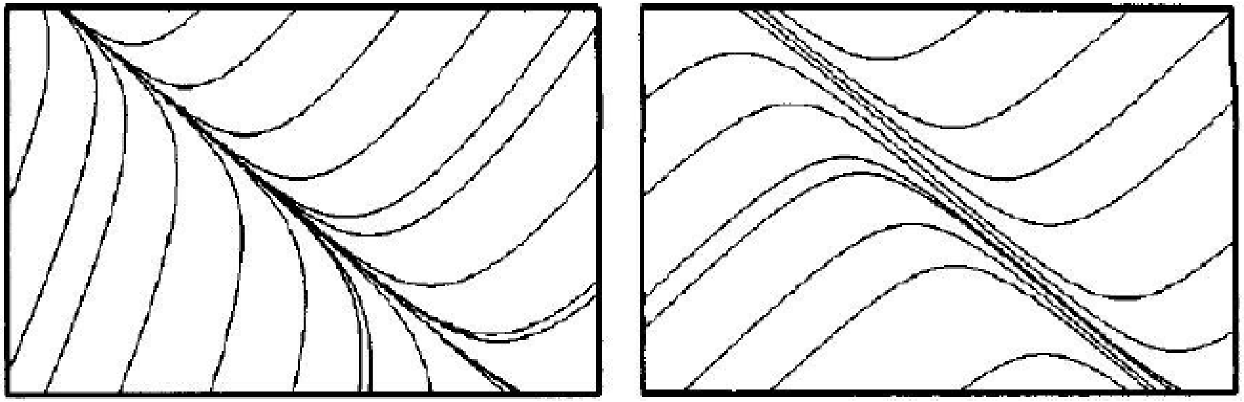
[Zafar et al. IEEE VIS 2023]

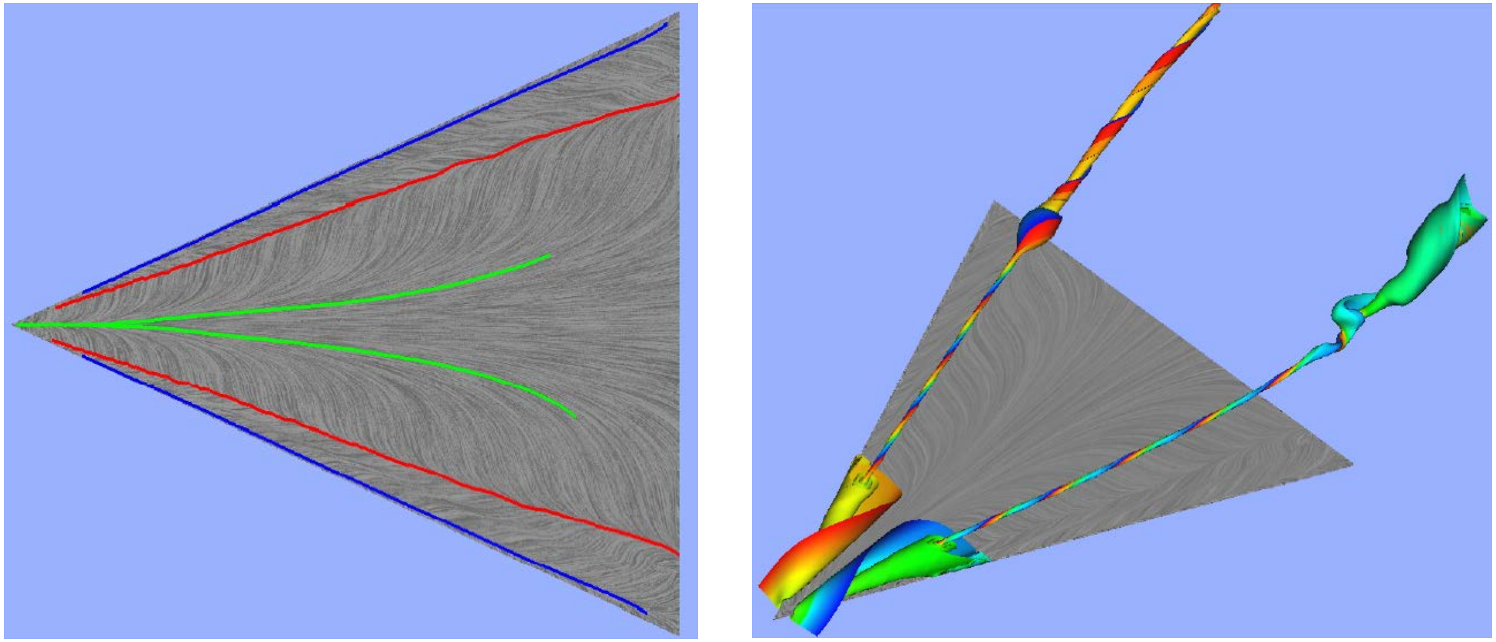
Hairpin vortex extraction and visualization



[Zafar et al. IEEE VIS 2023]

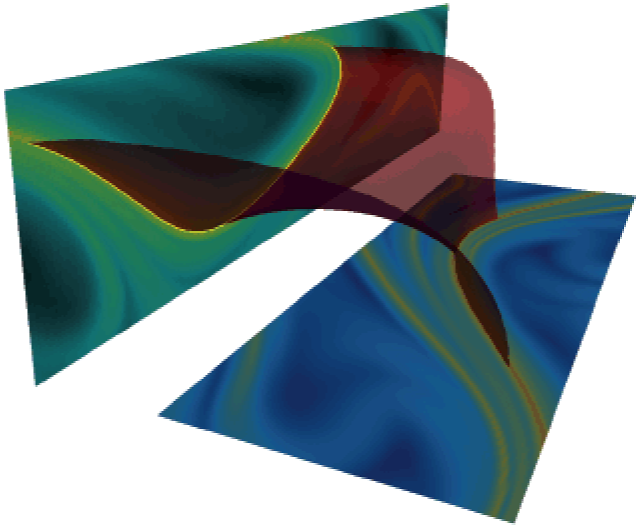
Separation Flows

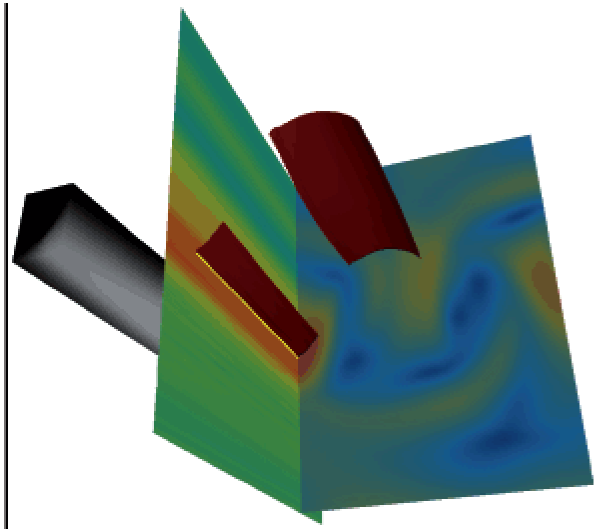


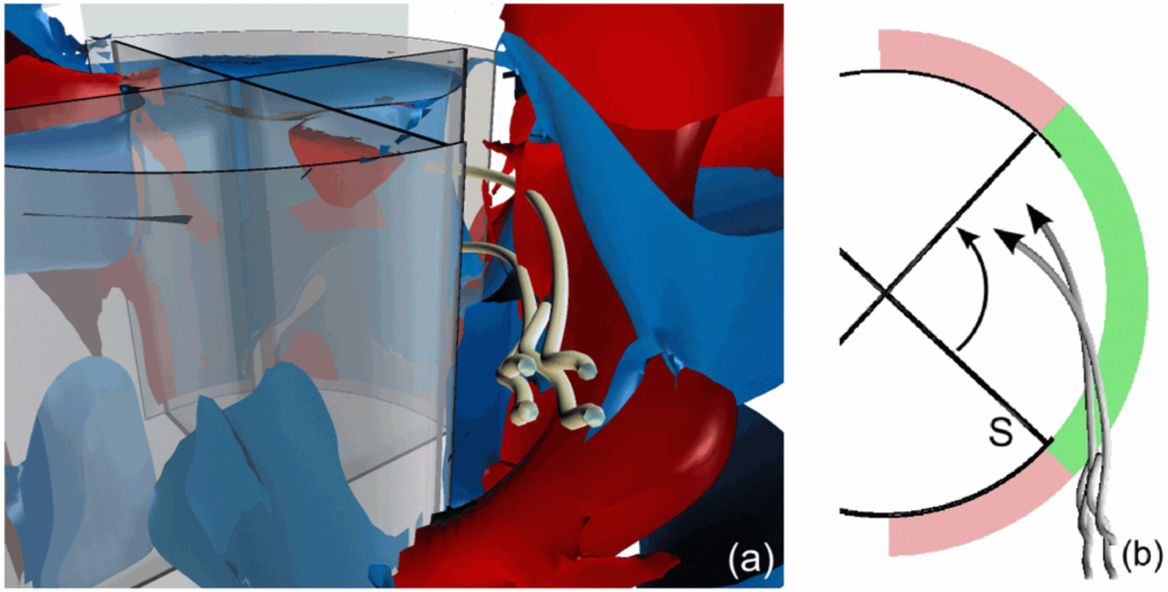


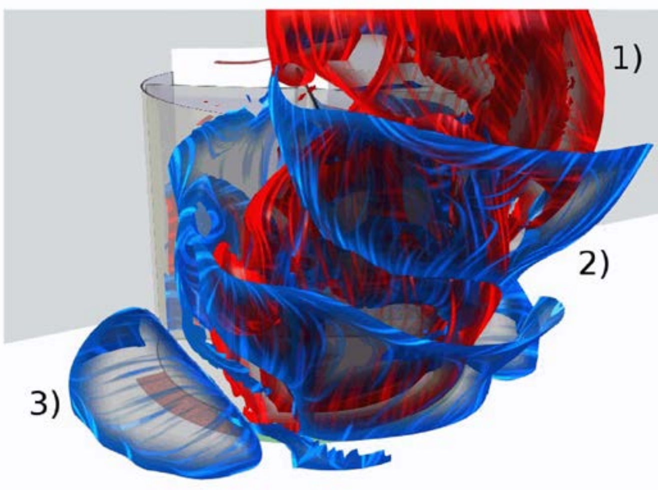
Separation flow on delta wing surface [Tricoche et al. AIAA 2004]

Separation Flows









Separation and attachment line detection is again based on the parallel vector operator (in 2D)

PV-parallel vector operator

$$\mathbf{v} || \nabla \mathbf{v} \cdot \mathbf{v}$$

both eigen-values of the Jacobian matrix are real number, leading to real eigen-vectors

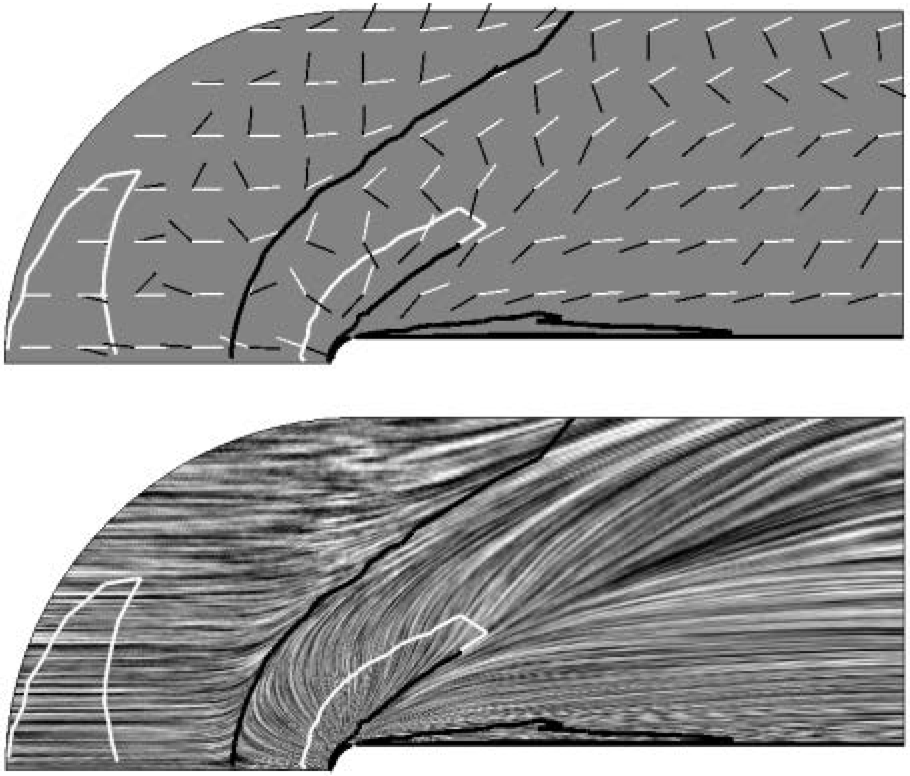
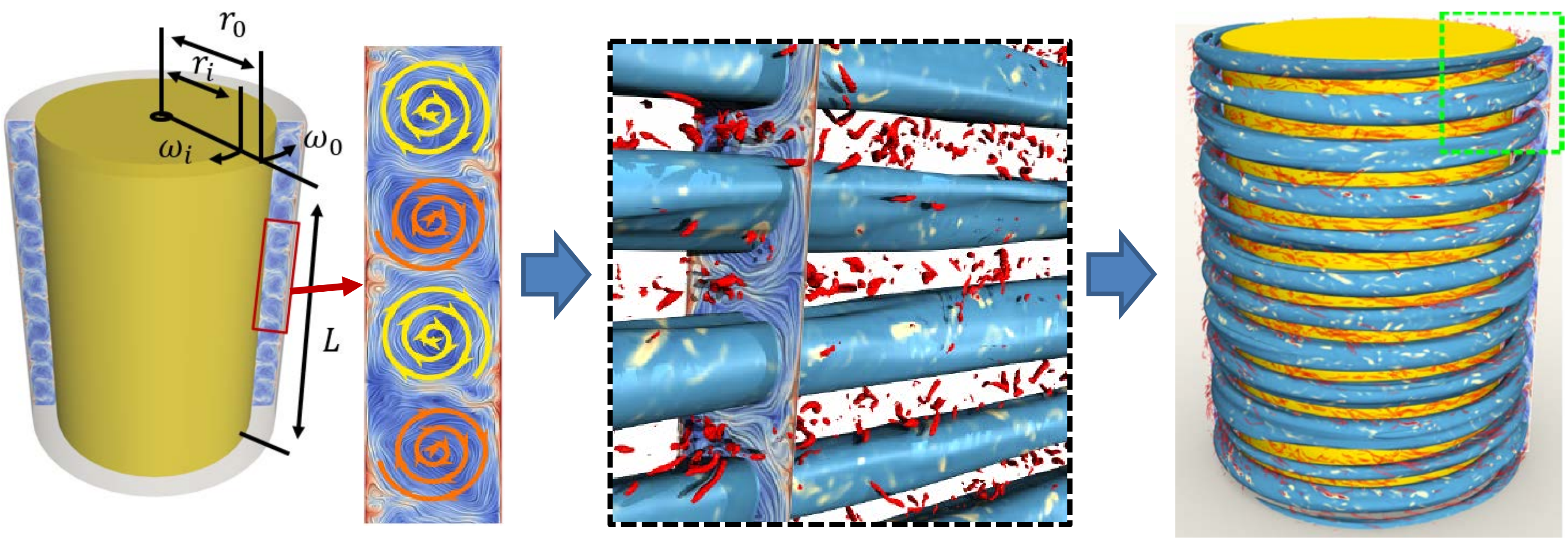


Figure 3: a) Separation and attachment lines on a solid surface of the "bluntfin" data set (black lines). The white arrows indicate the projected velocity **v**, *the black arrows the field* **w** = (∇ **v**) **v**. *b) The same lines compared to an LIC of* **v**.

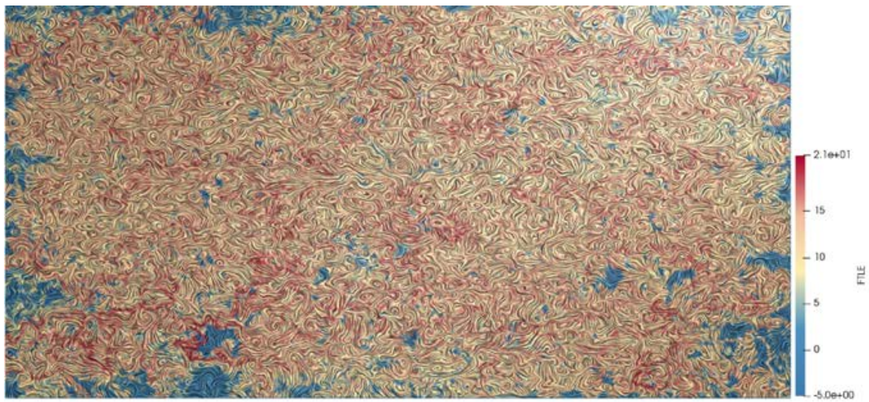
[Peikert and Roth, IEEE VIS 99]

Coherent structures in turbulent flows



Coherent structures in turbulent flows are in different scales and tangled in space and time

Images from [Nguyen et al, IEEE VIS 2020, TVCG2022]

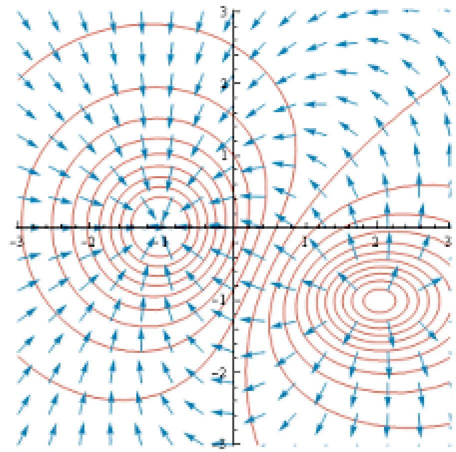


Helmholtz decomposition

$$V = \nabla\varphi + \nabla \times A$$

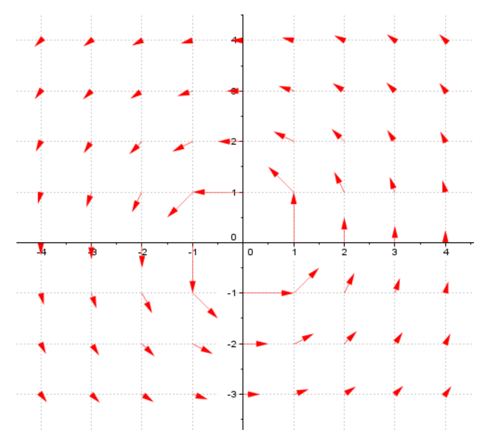
Curl (or rotation) free

Divergence free



$\nabla\varphi$ is the gradient of a scalar field φ

Divergence free (or no-solenoidal field)



Helmholtz decomposition

$$V = \nabla\varphi + \nabla \times A$$

Curl (or rotation) free

Divergence free

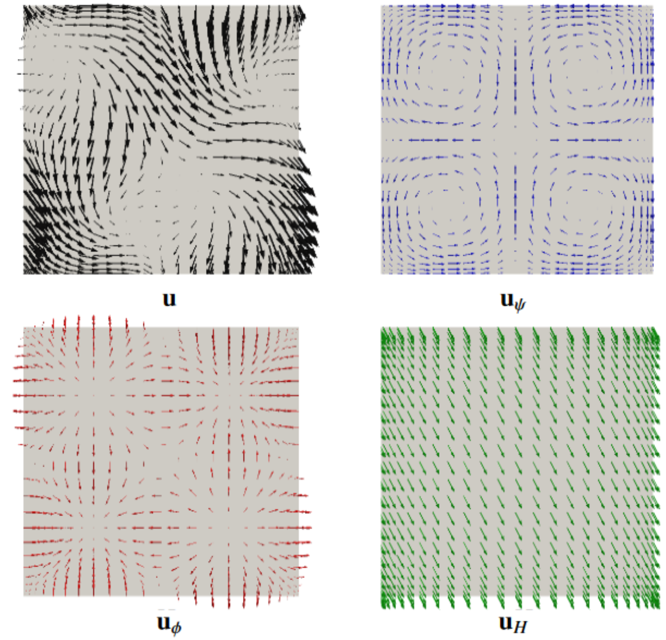
Hodge decomposition

$$V = \nabla\varphi + \nabla \times A + \gamma$$

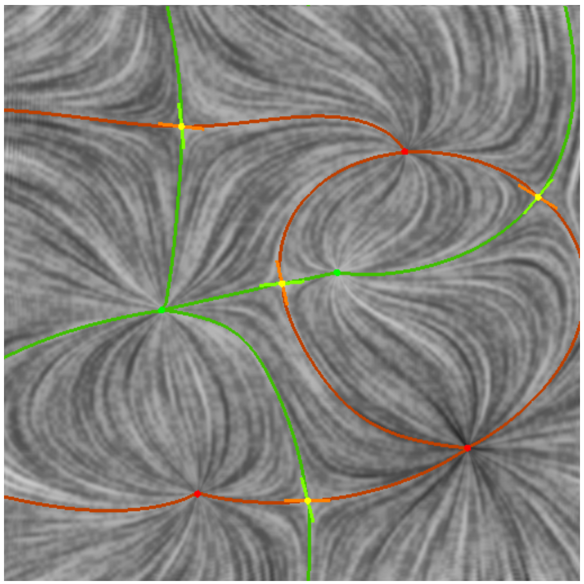
Curl (or rotation) free

Divergence free

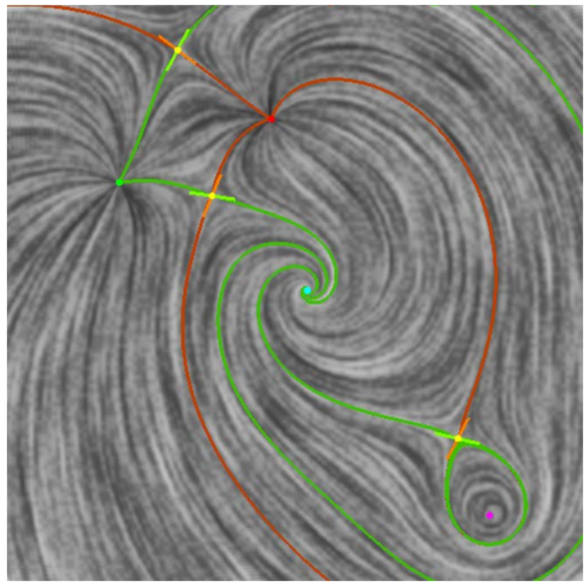
Harmonic



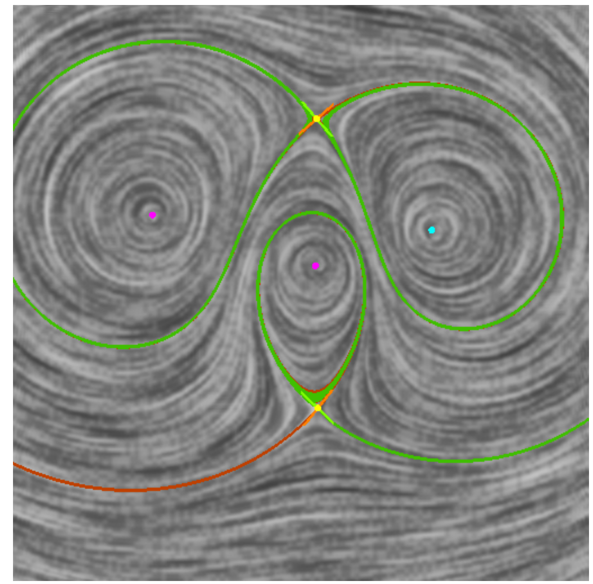
Example: Helmholtz decomposition



curl-free



general



divergence-free

Additional Materials

Günther, Tobias, and Holger Theisel. "The state of the art in vortex extraction." In *Computer Graphics Forum*, vol. 37, no. 6, pp. 149-173. 2018.

Bhatia, Harsh, Gregory Norgard, Valerio Pascucci, and Peer-Timo Bremer. "The Helmholtz-Hodge decomposition—a survey." *IEEE Transactions on visualization and computer graphics* 19, no. 8 (2012): 1386-1404.