Time-Dependent (Unsteady) Flow Visualization

A VERY BRIEF INTRODUCTION

Goal: know the difference between steady and unsteady flow; understand the concepts of **pathlines** and streak lines; know some techniques for unsteady flow visualization

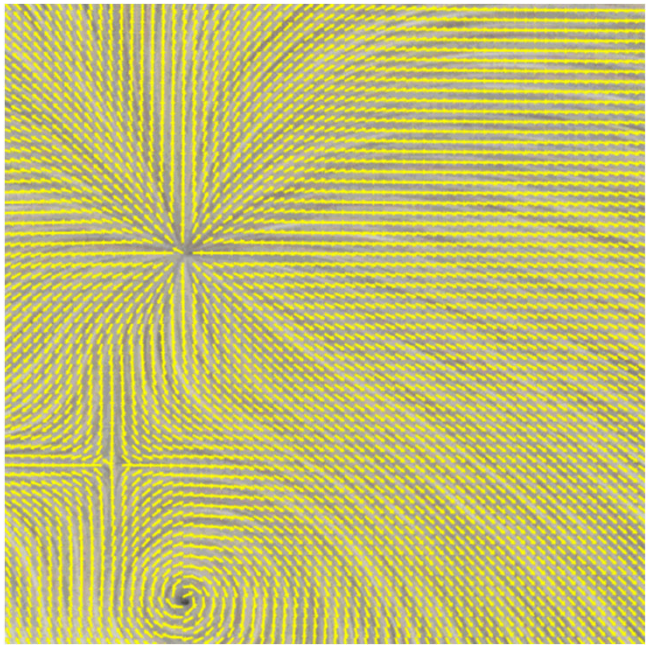
What is Different?

Steady (time-independent) flows:

- flow itself constant over time
- **v**(**x**), e.g., laminar flows
- simpler case for visualization

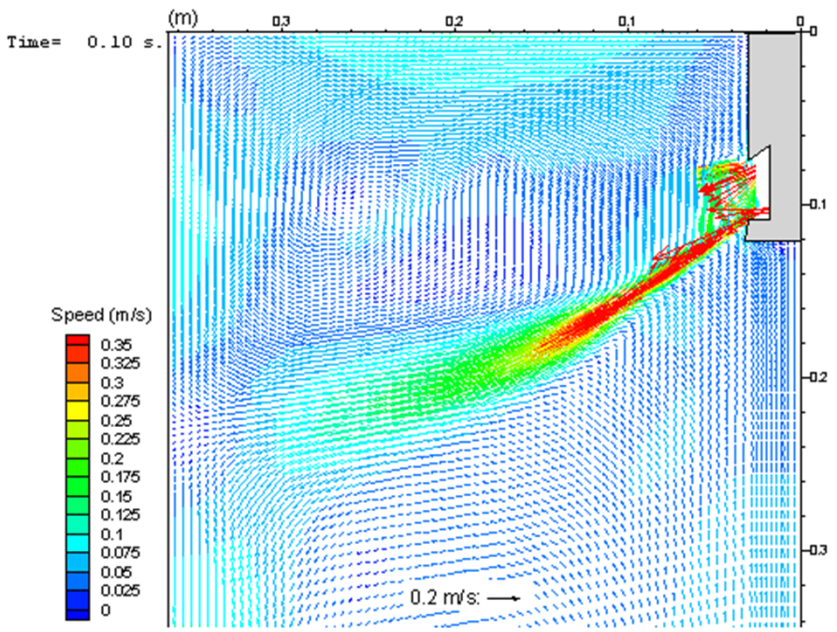
Time-dependent (unsteady) flows:

- flow itself changes over time
- **v**(**x**,***t***), e.g., turbulent flow
- more complex case

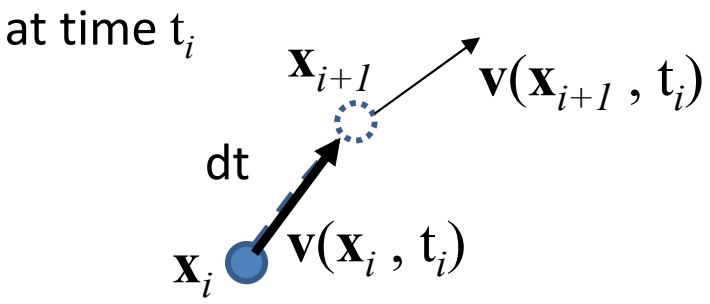


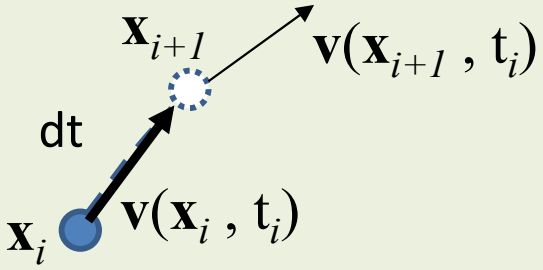
Mathematical Framework

- An unsteady **vector field**
  - is a continuous vector-valued function $\vec{v}(\boldsymbol{x}, \boldsymbol{t})$ on a manifold X
  - can be expressed as a system of ODE $\frac{d\boldsymbol{x}}{dt} = \vec{v}(\boldsymbol{x}, \boldsymbol{t})$
  - is a map $\varphi : R \times X \to X$



Unsteady Vector Fields vs. Steady Vector Fields

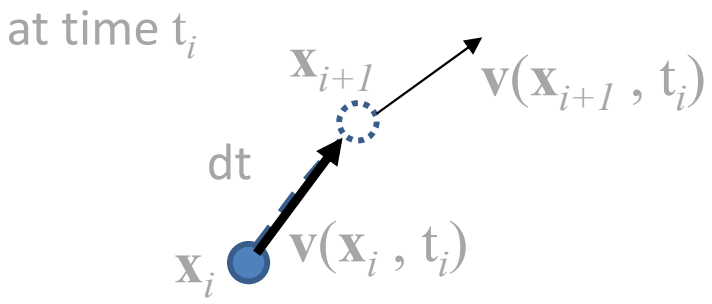


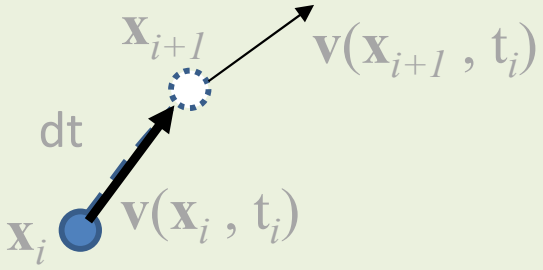


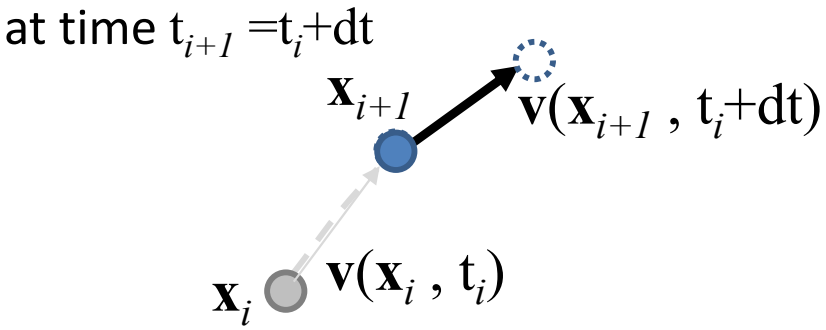
Steady vector field

Unsteady vector field

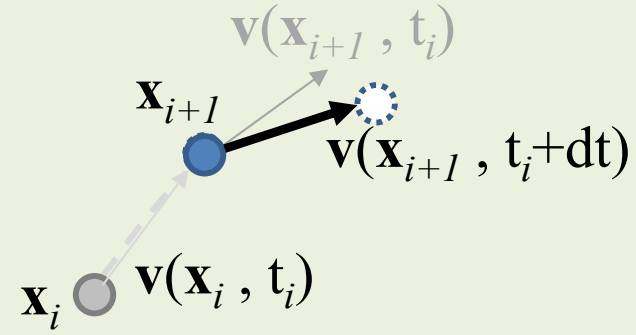
Unsteady Vector Fields vs. Steady Vector Fields







Steady vector field



Unsteady vector field

Unsteady Vector Fields vs. Steady Vector Fields

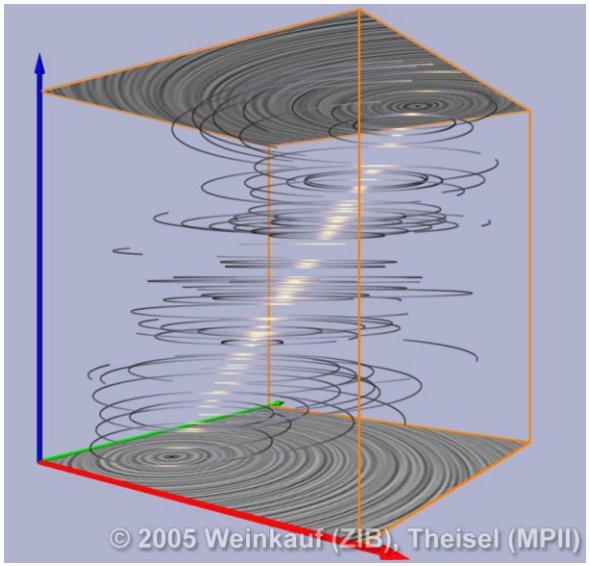
Important feature curves:

- **Streamline**: a curve that is everywhere tangent to the **steady** flow (release 1 massless particle)

$$s(t) = s_0 + \int_{0 \le u \le t} V(s(u))\, du$$

Interpolation in space

streamlines of the vector field at a given time stay at the same time plane!



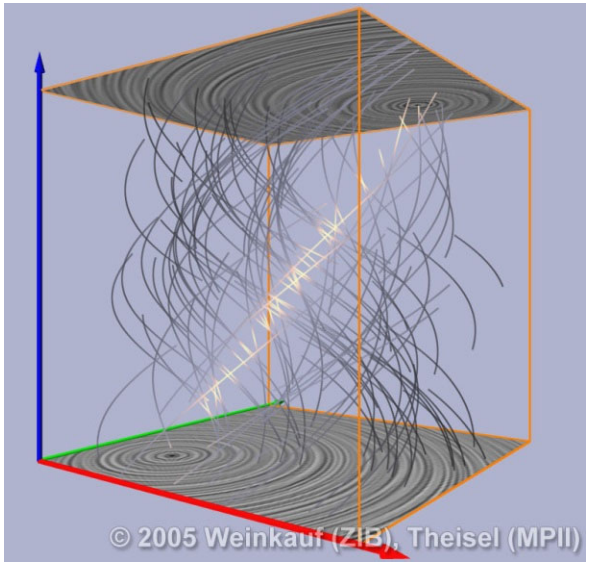
streamlines

- **Pathline**: a curve that is everywhere tangent to an unsteady flow field (release 1 massless particle)

$$s(t) = s_0 + \int_{0 \le u \le t} V(s(u), \mathbf{u})\, du$$

Interpolation in space and time!

Pathlines when shown in the spacetime will traverse through time!



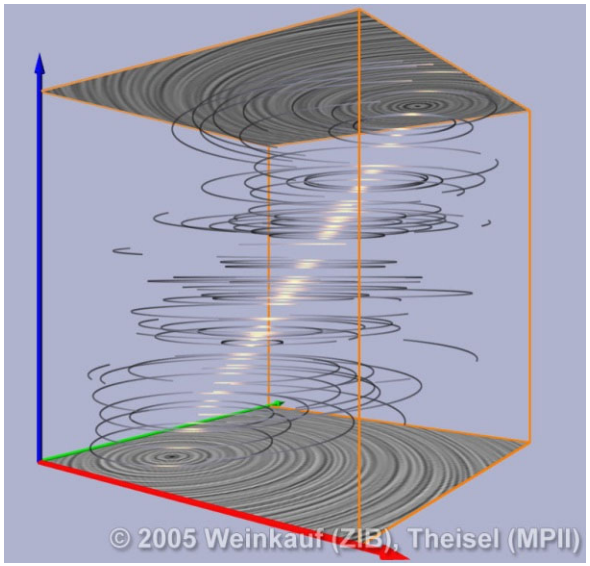
pathlines

Unsteady Vector Fields vs. Steady Vector Fields

Important feature curves:

- **Streamline**:

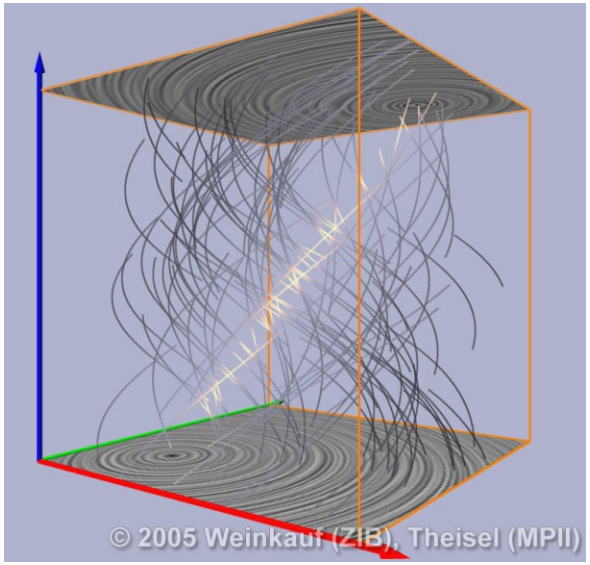
  Streamlines do not intersect!



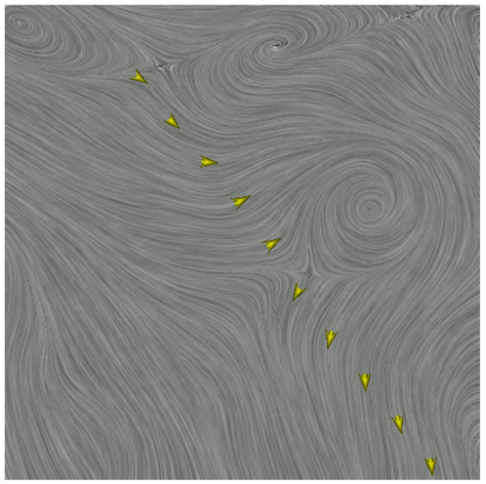
streamlines

- **Pathline**:

  Pathlines may intersect each other or even self intersect when projecting onto the space domain!



pathlines

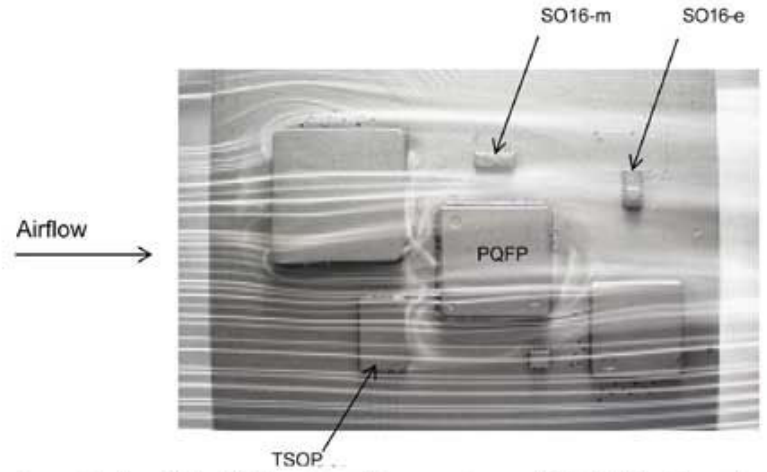


https://www3.nd.edu/~cwang11/2dflowvis.html

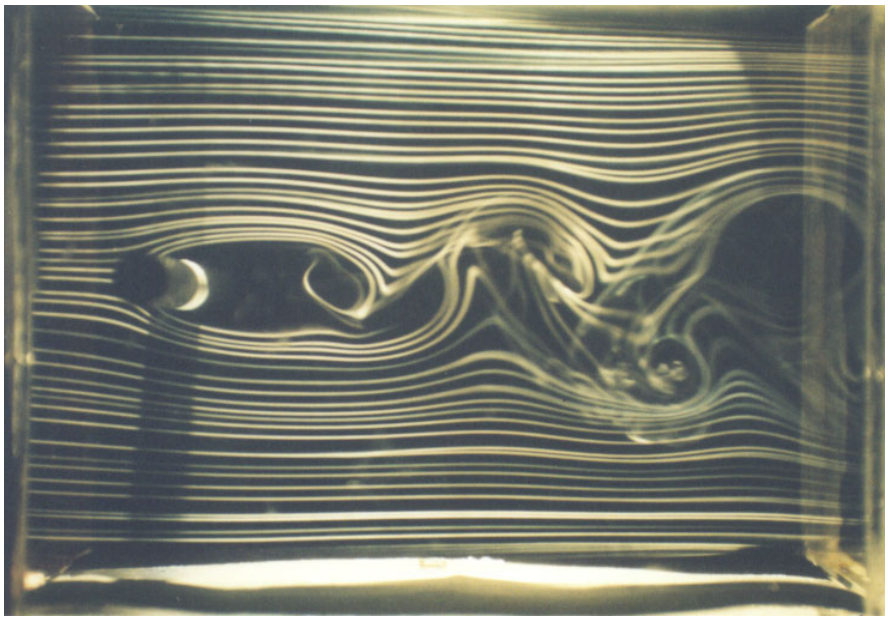
Unsteady Vector Fields

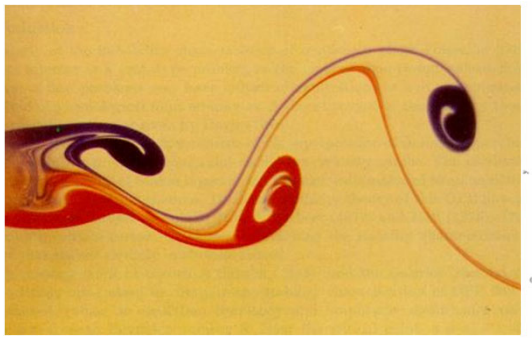
Important feature curves:

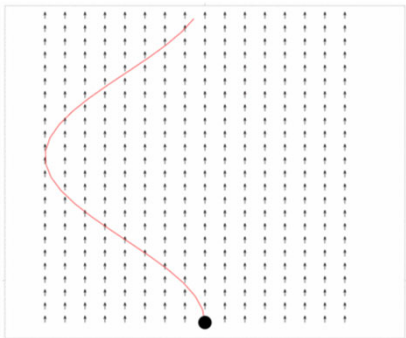
- **Streakline**: a curve traced by the continuous release of particles in unsteady flow from the **same position in space** (release infinitely many massless particles)



Note: Smoke wire set flush with the PCB surface, 25 mm upstream of the PCB leading edge.



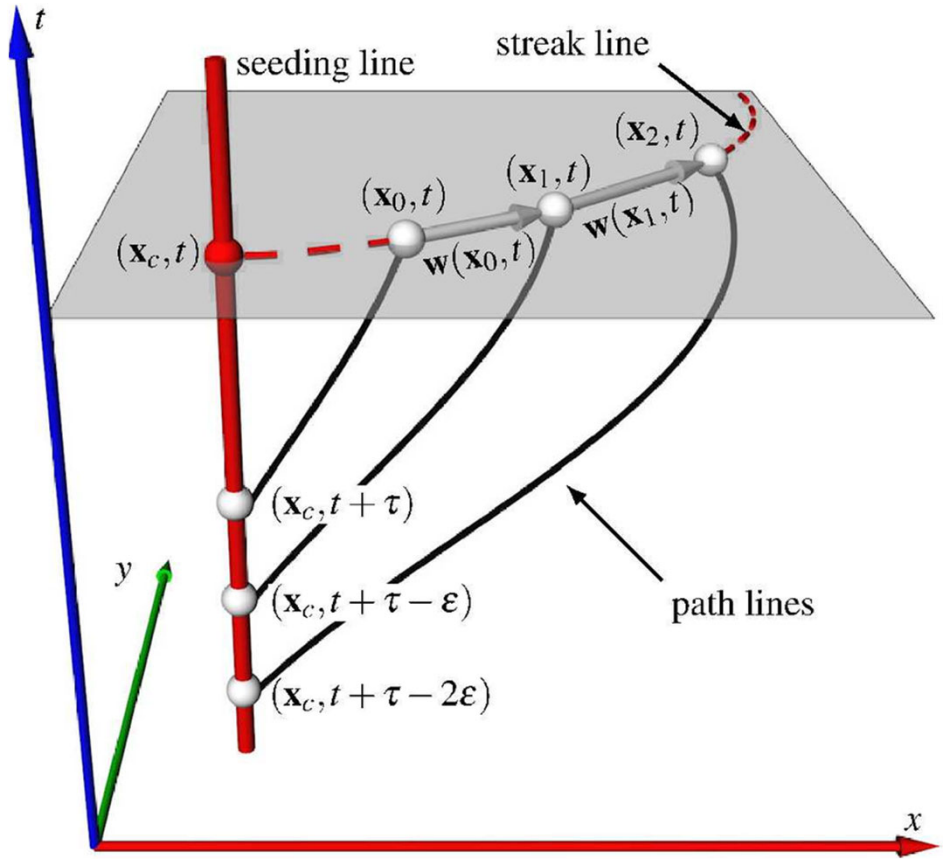




Source: Google images

Streakline computation

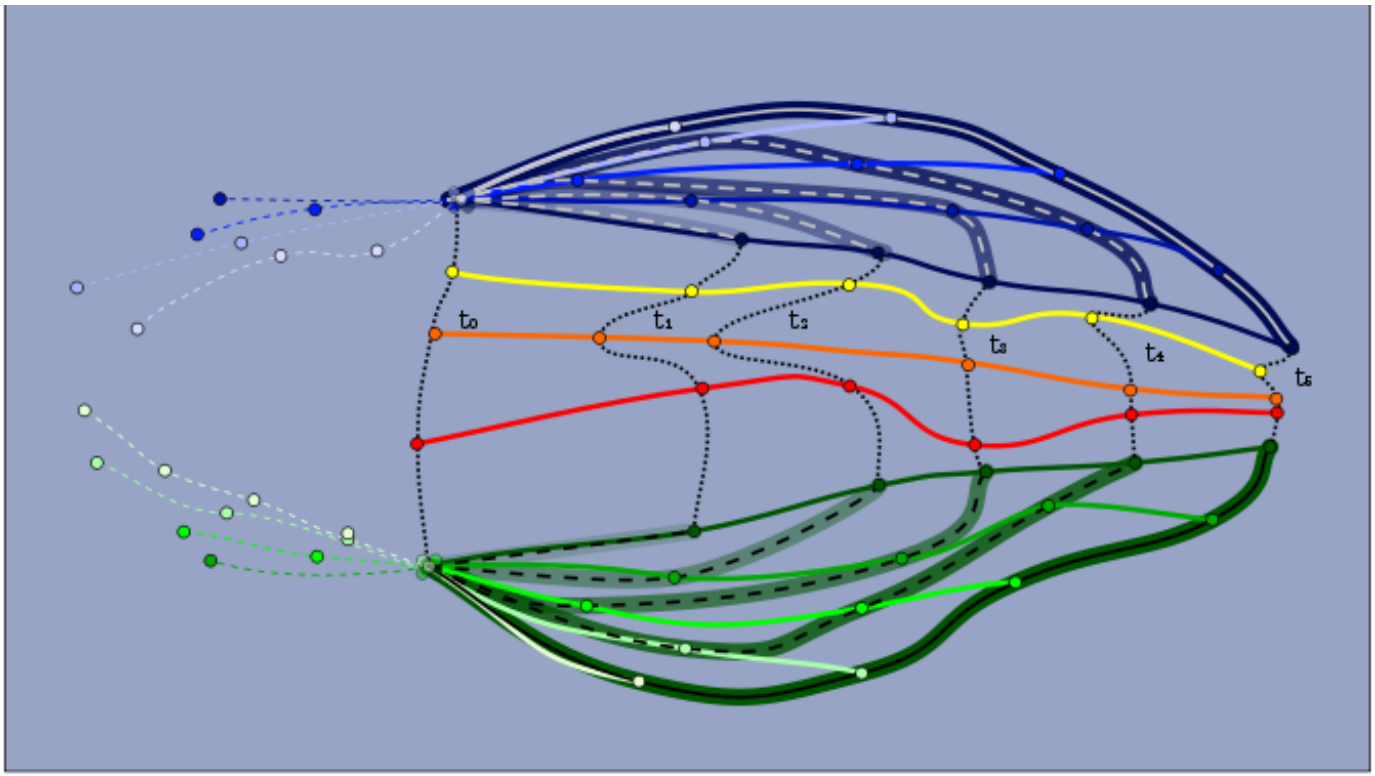
Streakline: a curve traced by the continuous release of particles in unsteady flow from the **same position in space** (release infinitely many massless particles



A 2D illustration of streakline computation [WeinKauf and Theisel, 2010]

Streakline computation

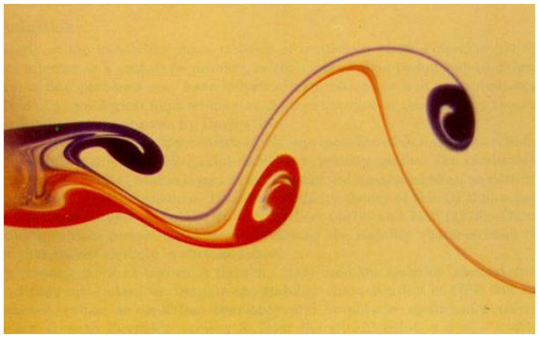
- Not tangent curves to the vector fields
- Union of the current positions of particles released at the same point in space



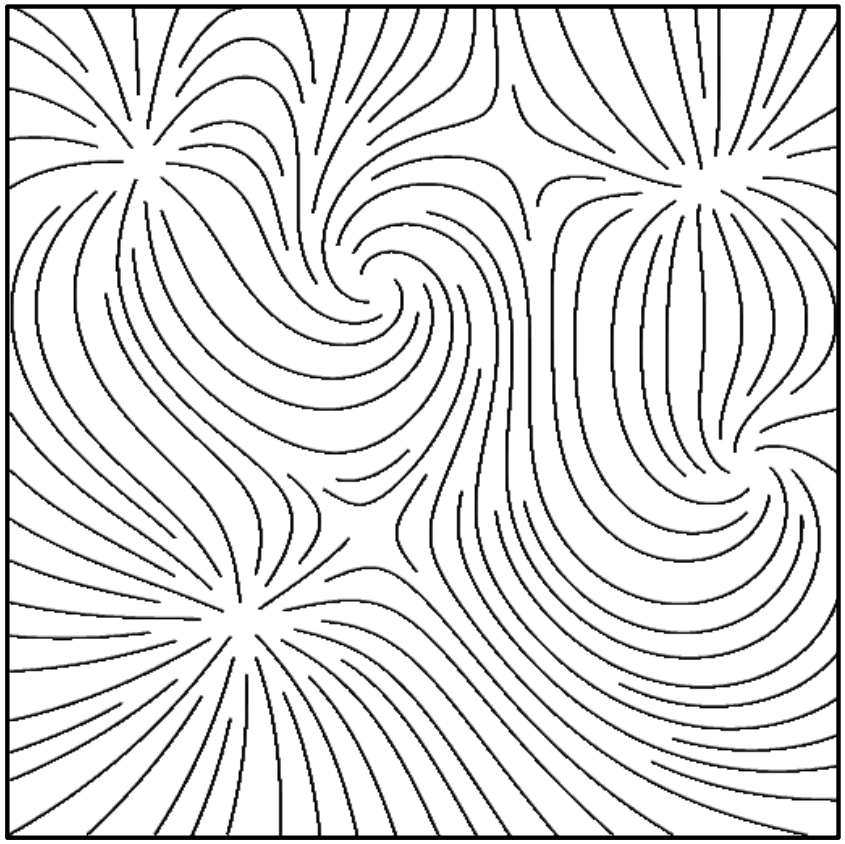
[Source: wikipedia]

Streakline visualization

[Weinkauf et al. 2010]

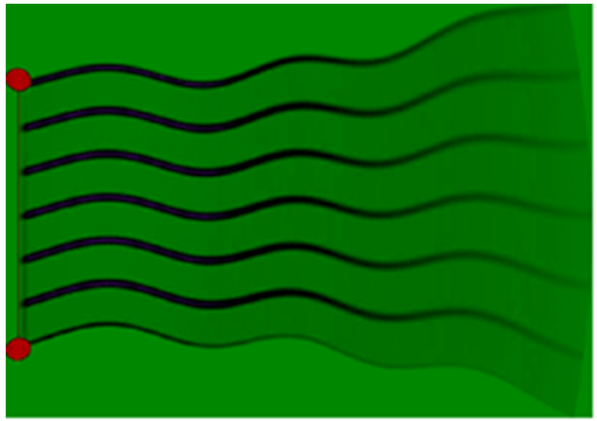


In **steady** flow, streamlines, pathlines and streaklines are **identical** because the vector field is NOT changing over time**.**

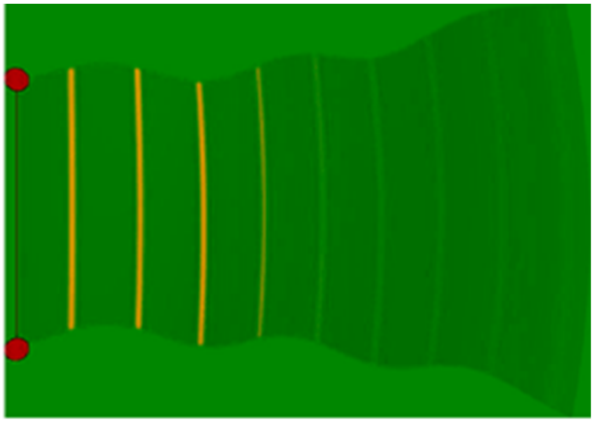


Other feature curve

- **Timelines**
  - Union of the current positions of particles released at the same time in space

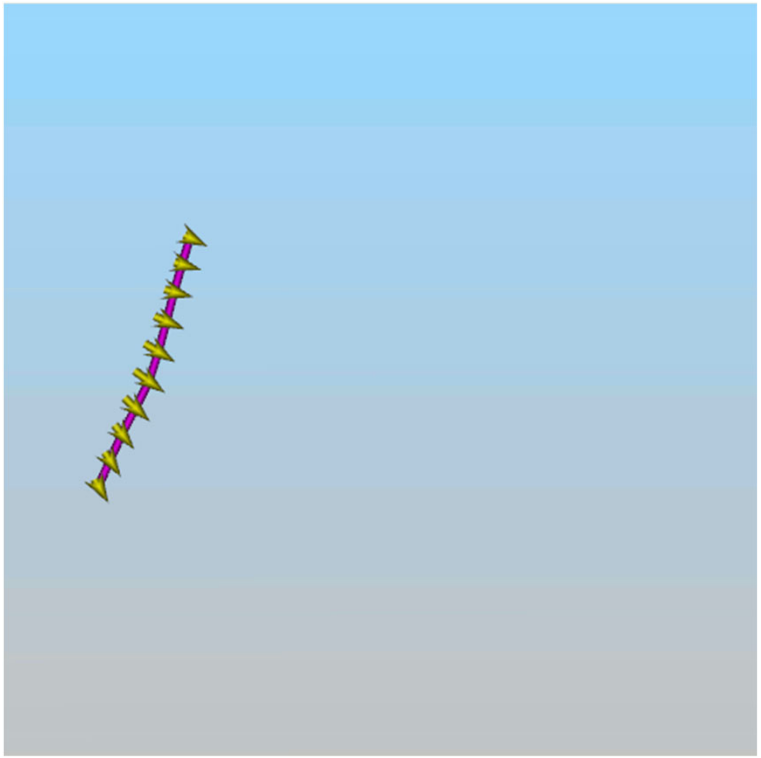


(a) Coloring fixed rows in the array reveals streak lines.



(b) Coloring fixed columns in the array reveals time lines.

Source: doi.ieeecomputersociety.org



https://www3.nd.edu/~cwang11/2dflowvis.html

How to visualization unsteady vector fields?

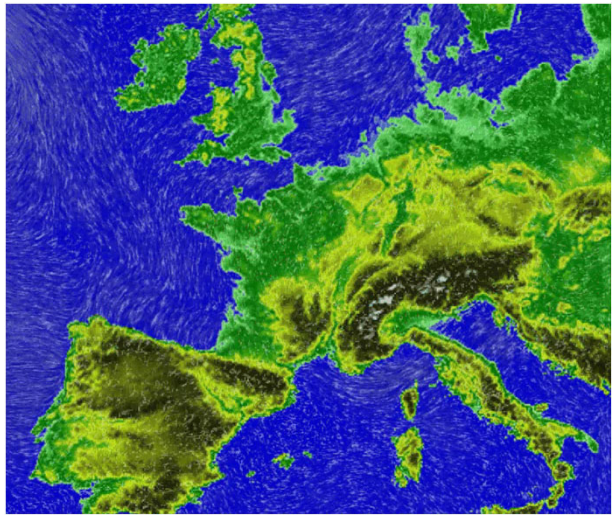
Texture-Based Methods

Unsteady flow LIC (UFLIC): forward scattering + collecting

IBFV: texture advection in forward direction + hardware acceleration

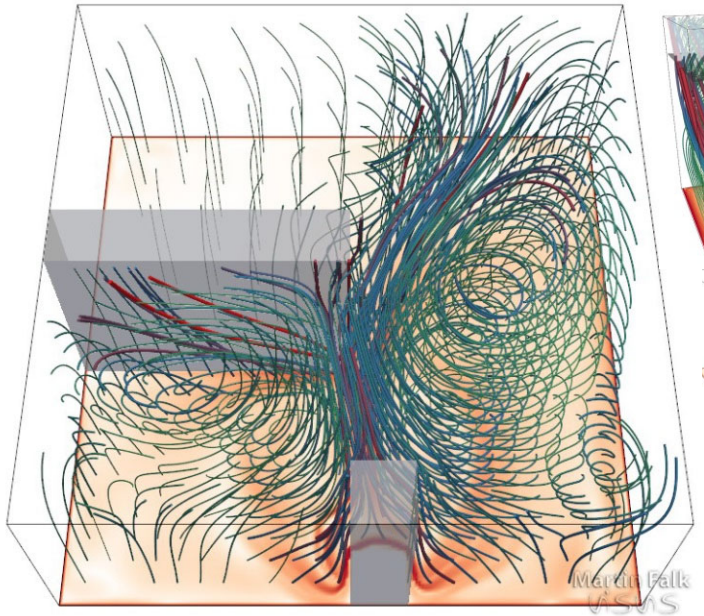
GEOMETRIC-BASED VISUALIZATION

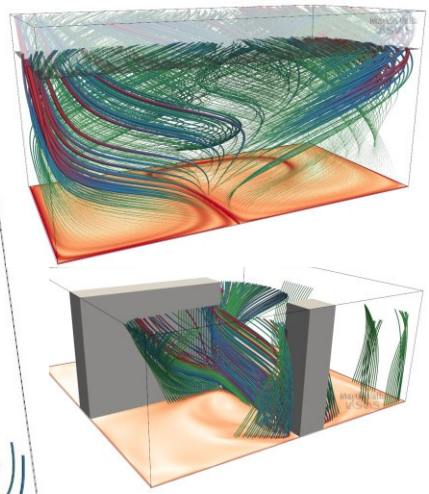
Curve-Based



Streamline-based

[Jobard et al., 2001]





Pathline-based

Source: www.vis.uni-stuttgart.de

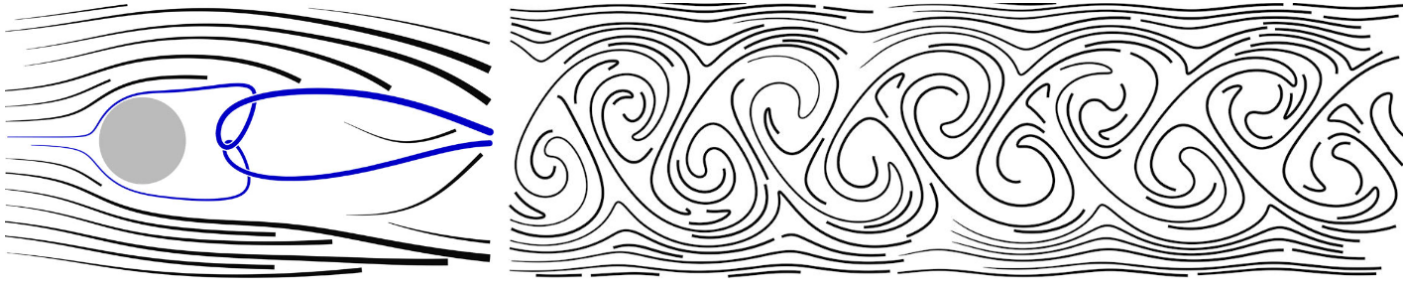
Streakline-based

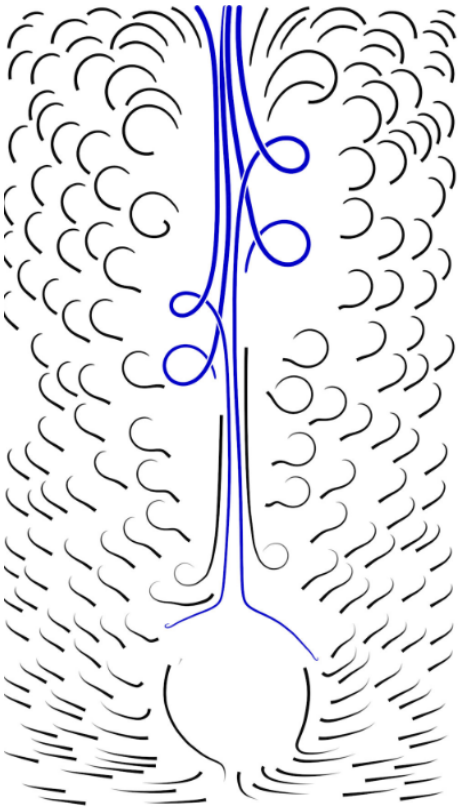
[Weinkauf et al. 2010]

What are the challenges for the curve-based visualization for unsteady vector fields?

What are the challenges for the curve-based visualization for unsteady vector fields?

where to put seeds
how many seeds are needed
termination condition (to avoid or reduce intersections)

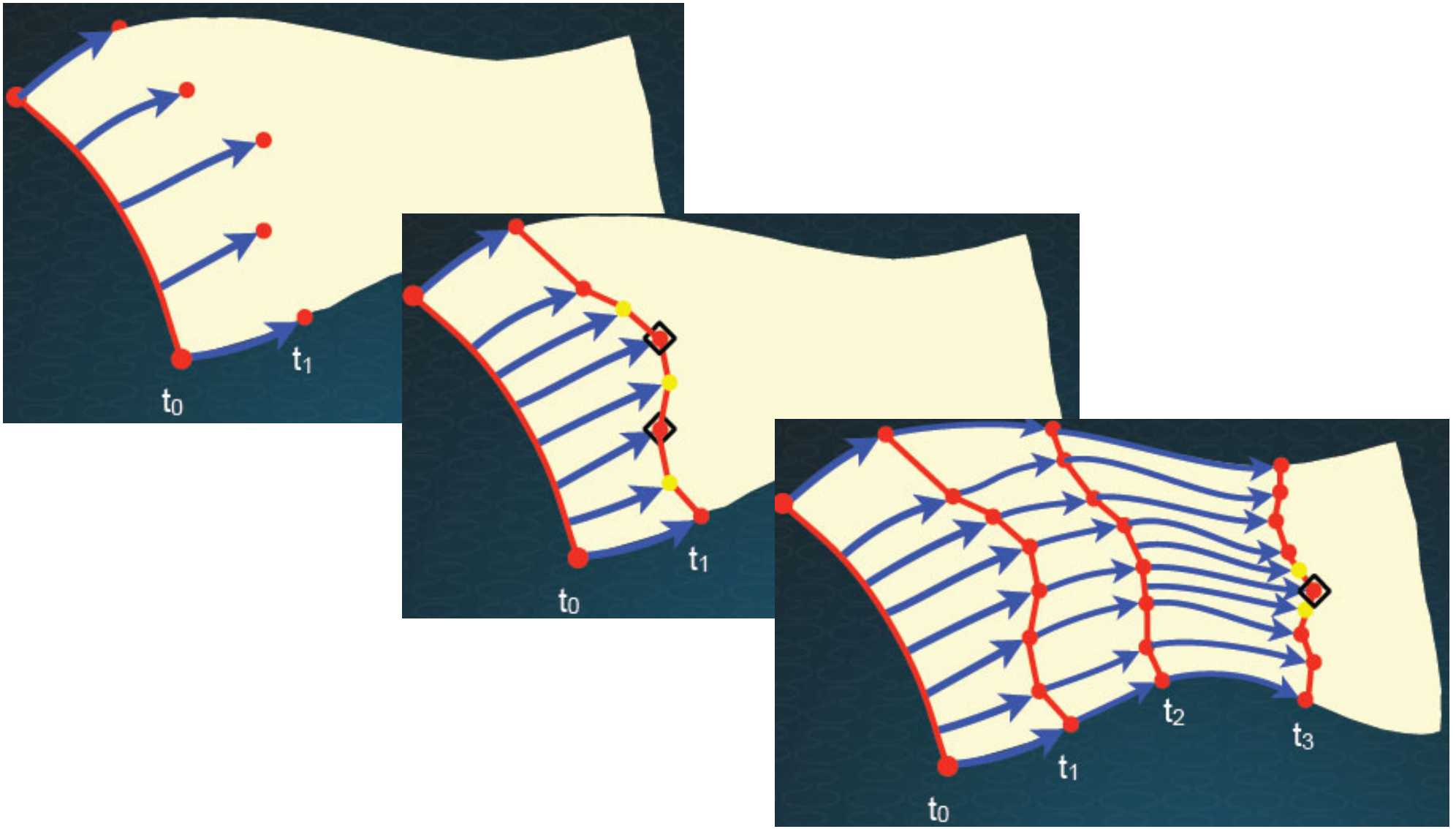




Higher-Dimensional Feature Descriptors

Path surface:

Its computation can use **timeline** advection



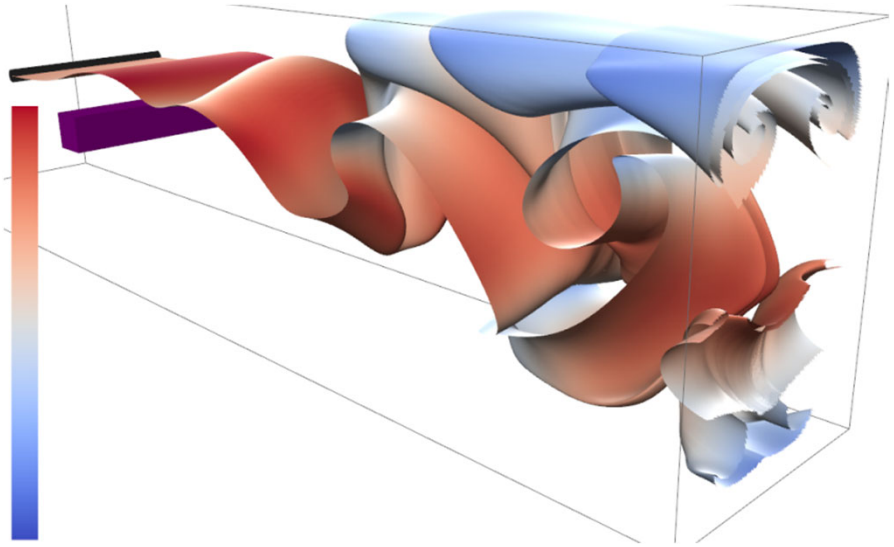
Source: Garth Vis09 Tutorial

Streak Surfaces: Challenges

Streak surfaces are an extension of streak lines (next higher dimension)

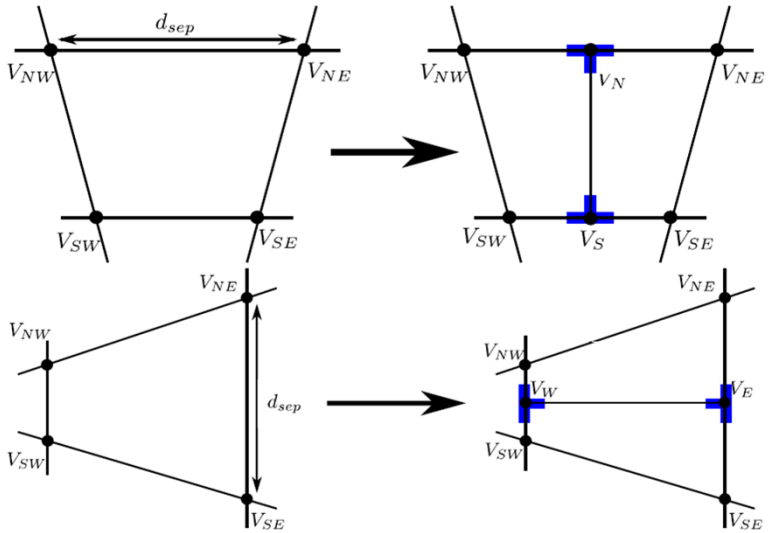
Challenges:

- Computational cost: surface advection is **very** expensive
- Surface completely dynamic: entire surface (all vertices) advect at each time-step
- Mesh quality and maintaining an adequate sampling of the field.
    - Divergence
    - Convergence
    - Shear
- Large size of time-dependent (unsteady) vector field data, out-of-core techniques

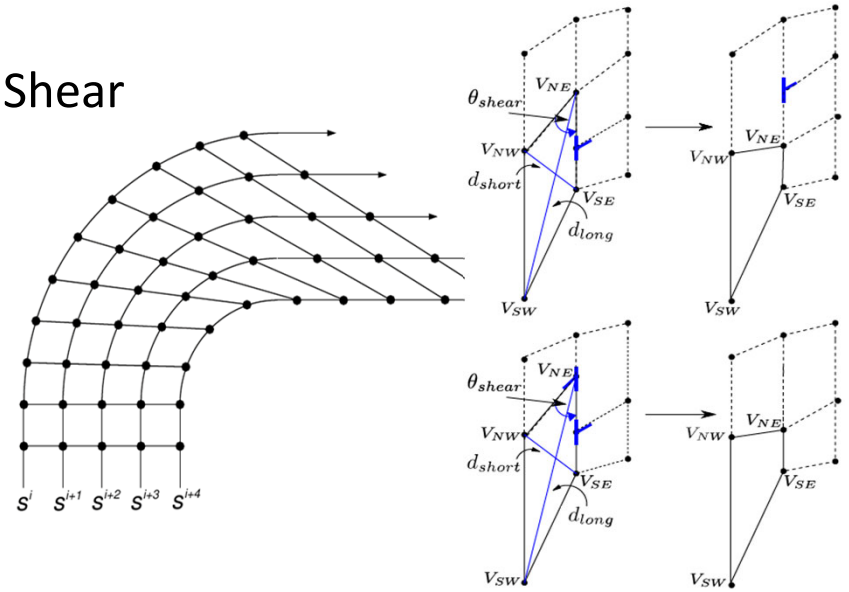


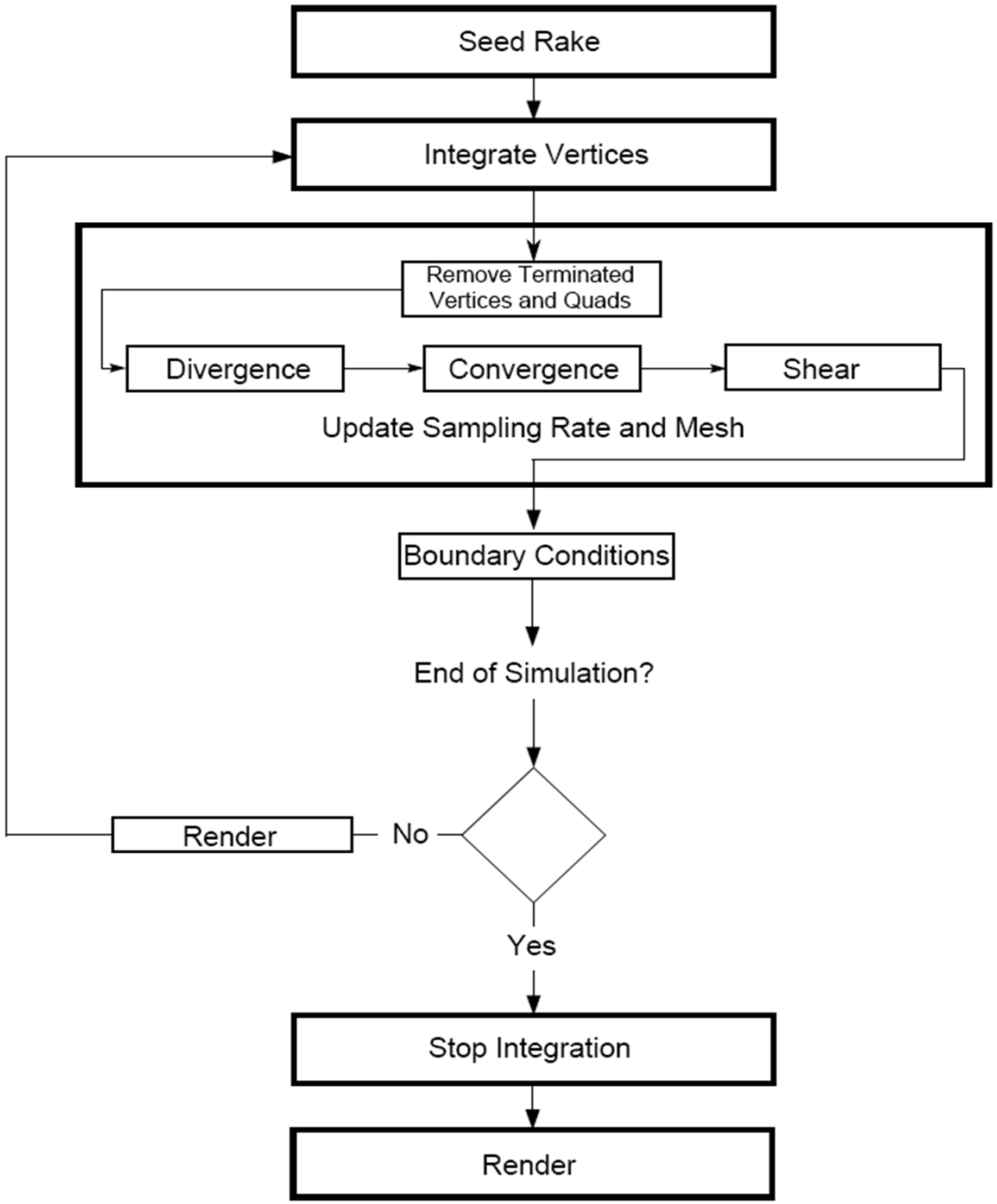
A Streak Surface Computation Pipeline

Divergence: Quad Splitting

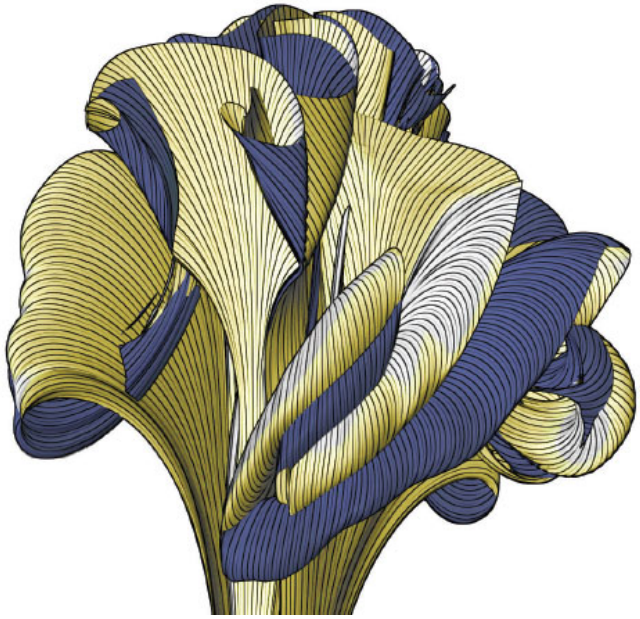


Shear





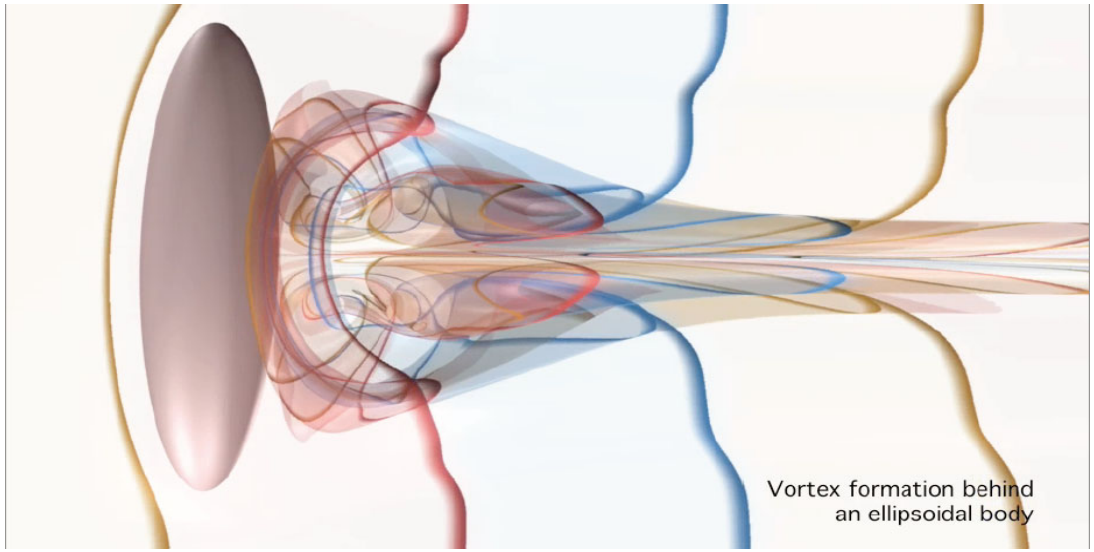
Rendering of the Obtained Surfaces



Illustrative path surfaces: texture + transparency.
Hemmel et al. 2010

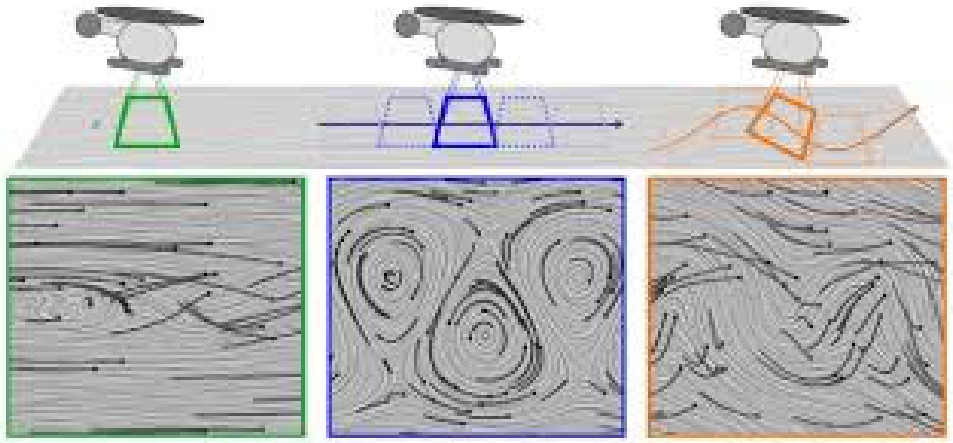
Time Lines on Streak Surfaces

Garth et al. Vis 2008

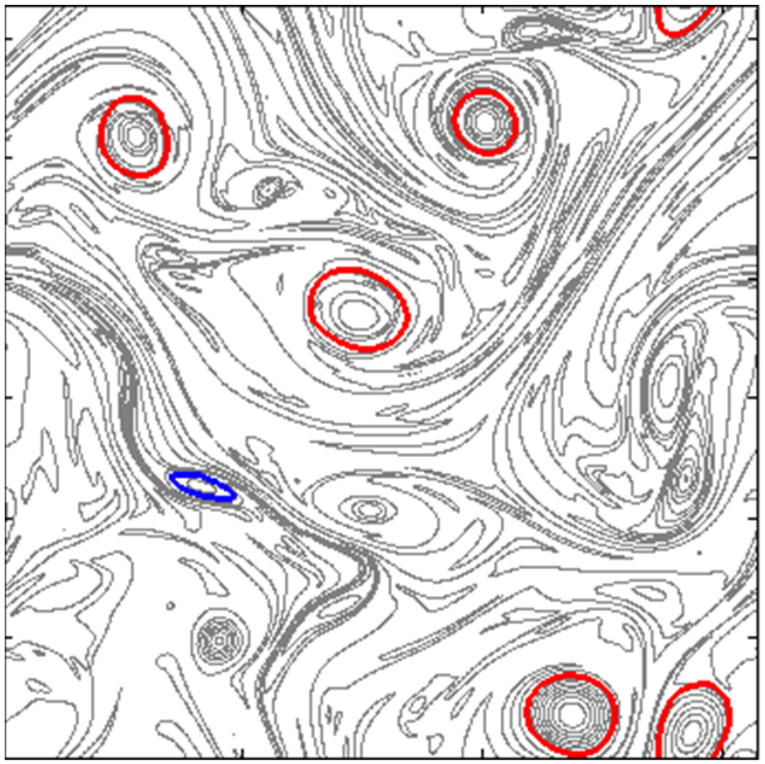


FEATURE-BASED METHOD

Coherent structures (e.g., Eulerian and Lagrangian coherent structures)



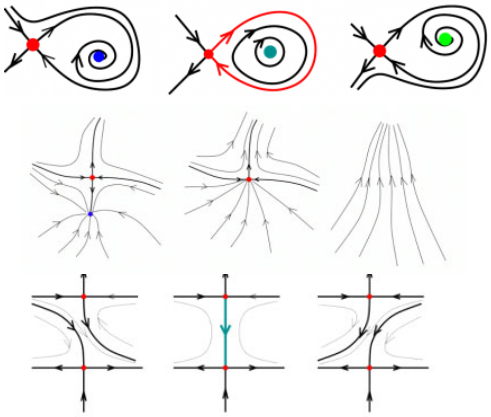
[Guenther et al., SIGGRAPH 2017]

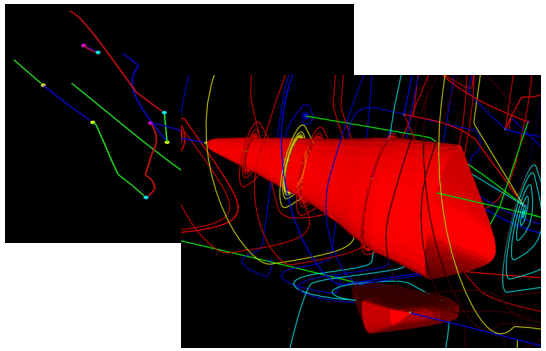


http://georgehaller.com/research/projects.html

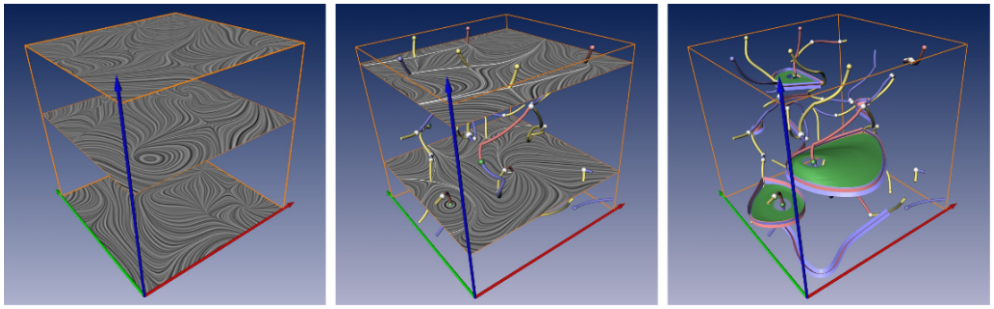
Topology-Inspired Approach for Time-Dependent Vector Fields

- **Track the Evolution of Instantaneous Topology**





[Xavier et al. VisSym01, C&G02, Vis04]



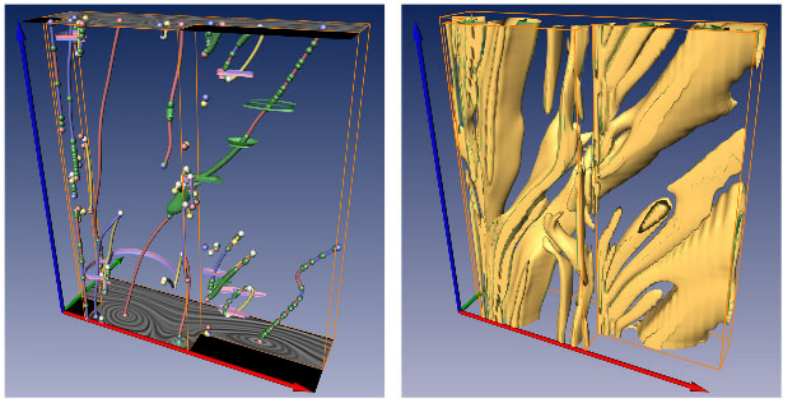
(a) LIC images at 3 different time slices.

(b) Tracking the locations of critical points as stream lines (red/blue/yellow); local bifurcations: Hopf bifurcations (green spheres), fold bifurcations (gray spheres).

(c) Global bifurcations: saddle connections (red/blue flow ribbons), tracked closed stream lines (green surfaces).

[Theisel et al. VisSym2003, Vis04, TVCG05]

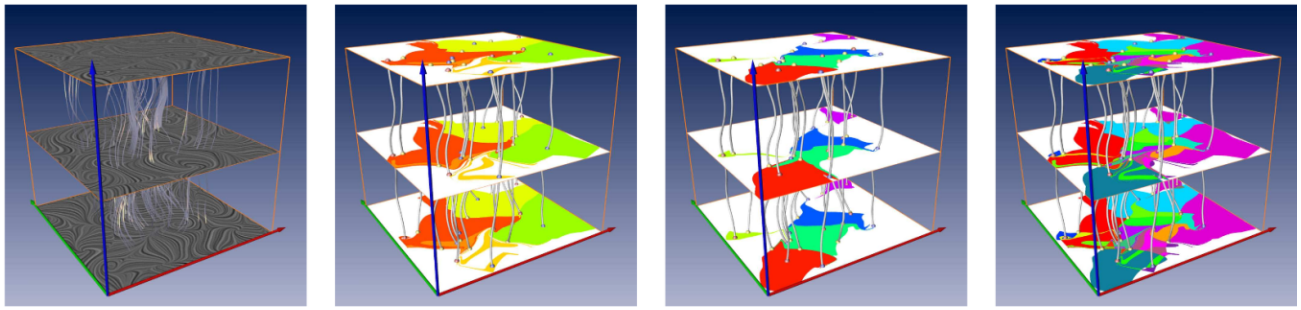
- **Pathline-based**



(b) Stream line oriented topology of the first 100 time steps.

(c) Path line oriented topology of the first 100 time steps.

[Theisel et al. Vis04, TVCG05]



(a) The vector field **p**.

(b) Critical path lines and basins for forward integration.

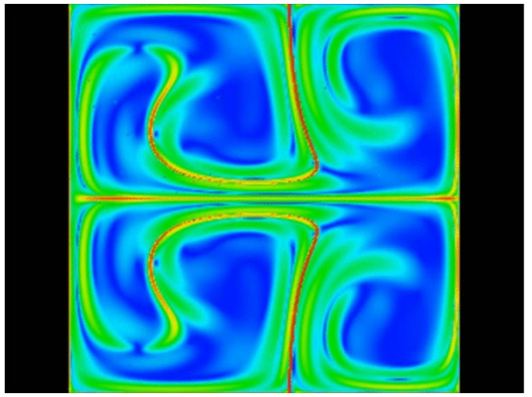
(c) Critical path lines and basins for backward integration.

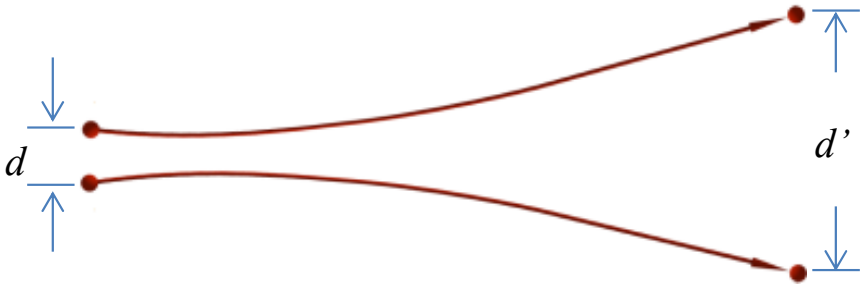
(d) Overlayed basins for forward and backward integration.

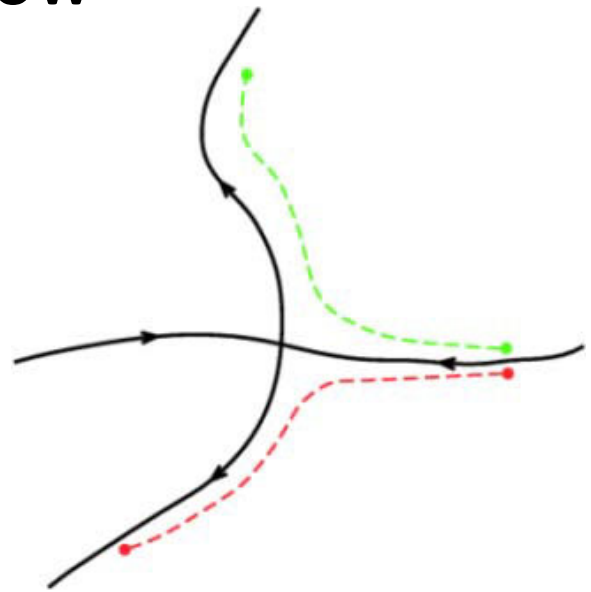
[Shi et al. EuroVis06]

Finite-Time Lyapunov Exponent

- Some observation
  - Observe particle trajectories
  - Measure the **divergence** between trajectories, i.e., how much flow stretch



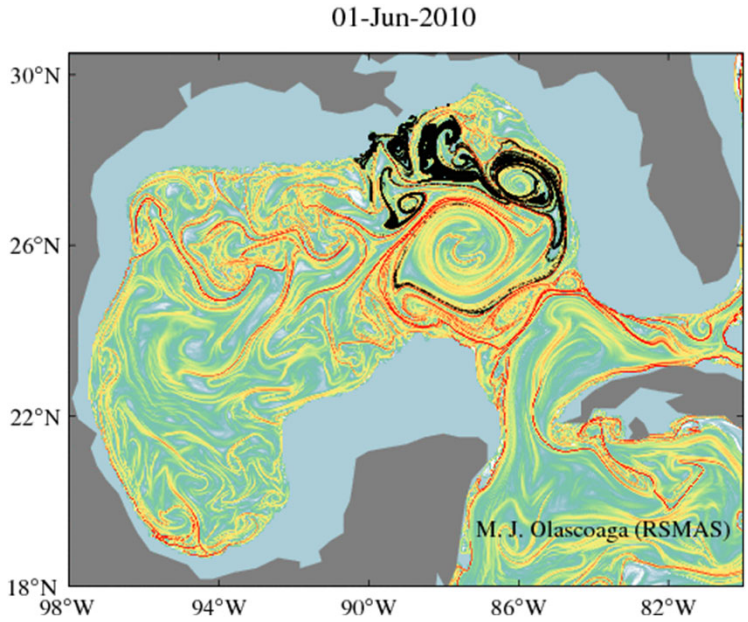




[Shadden]

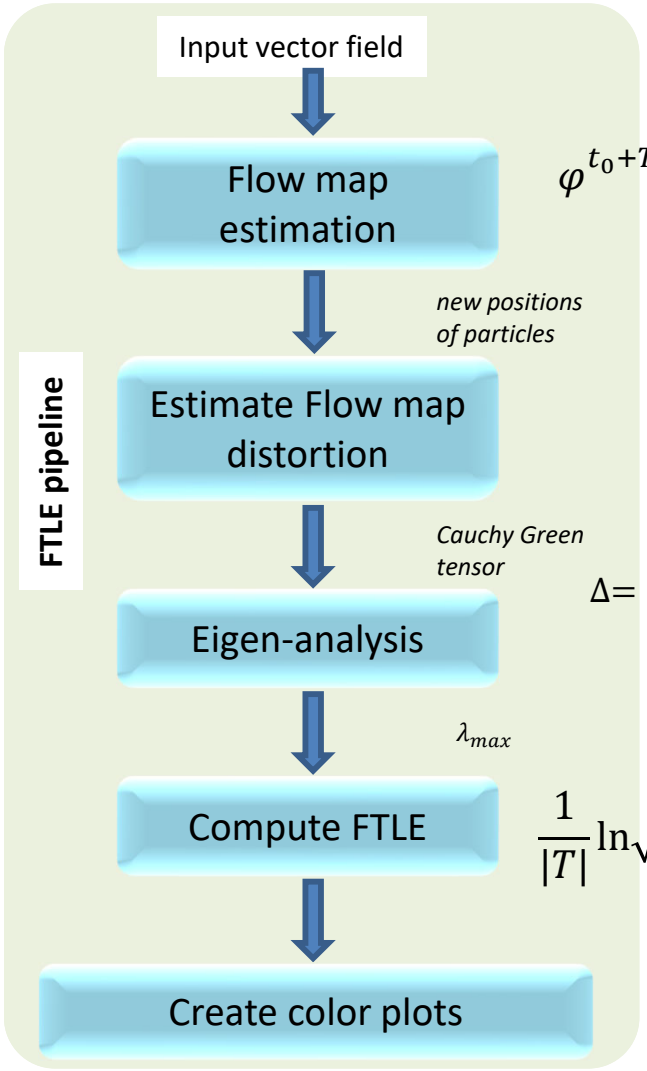
Finite-Time Lyapunov Exponent

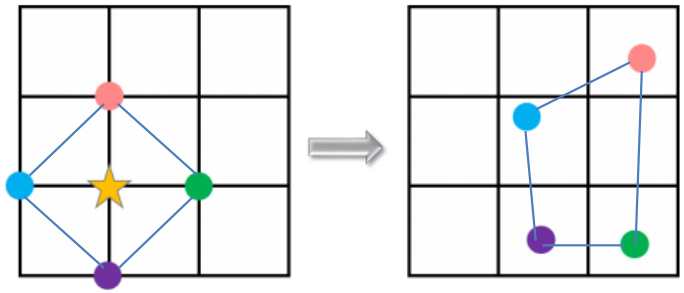
- Description
  - Lyapunov exponents describe rate of separation or stretching of two infinitesimally close points over time in a dynamical system
  - FTLE refers to the largest Lyapunov exponent for only a **limited time** and is measured **locally**
  - *Largest exponent is governing the behavior of the system, smaller ones can be neglected (thresholding)*



Finite-Time Lyapunov Exponent

A computation framework

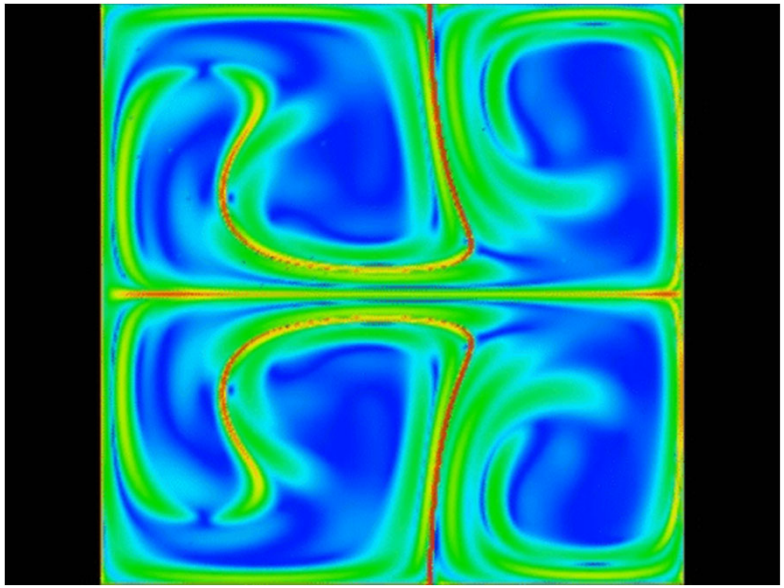




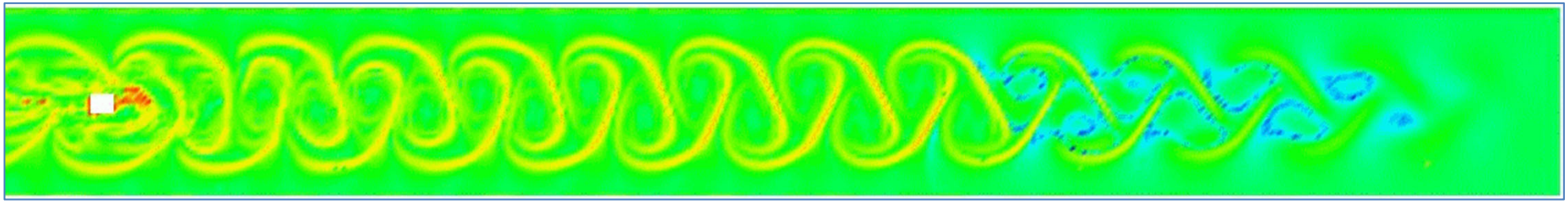
Finite-Time Lyapunov Exponent

- Examples:

unsteady quad-gyre



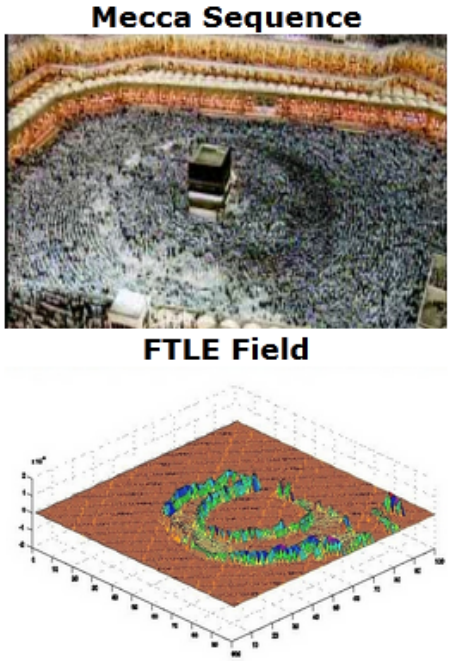
Flow around a cylinder

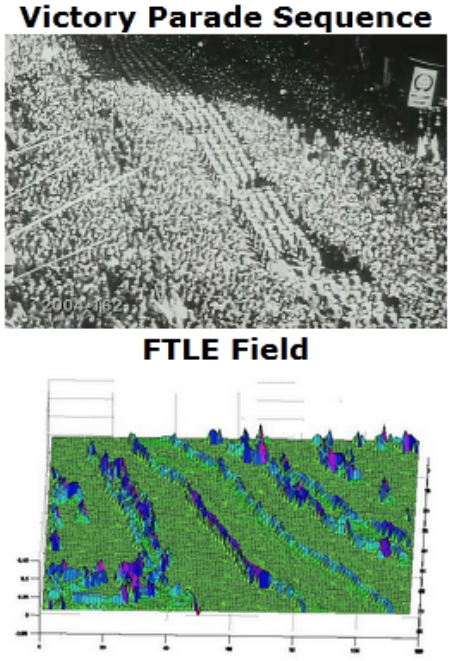


Other examples: http://mmae.iit.edu/shadden/LCS-tutorial/examples.html

Finite-Time Lyapunov Exponent

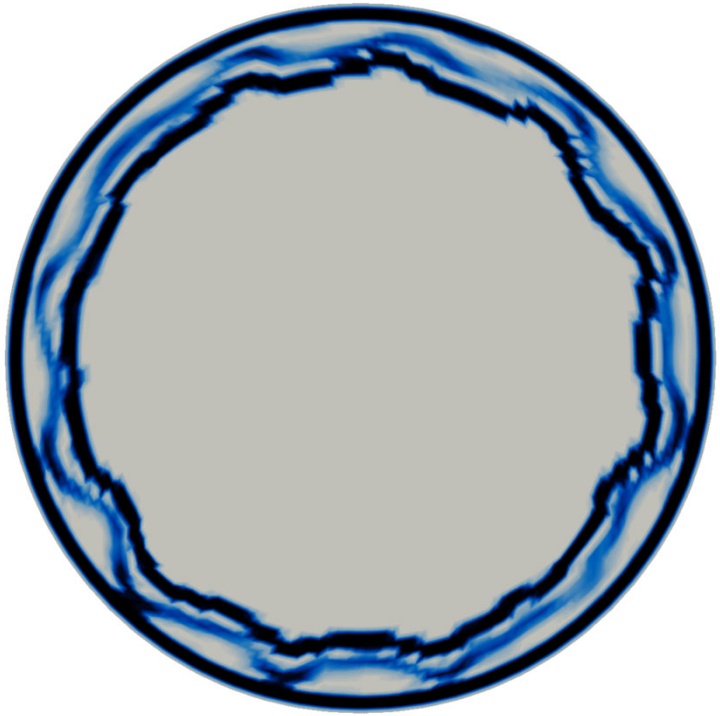
- Further Applications





Detection of crowd flow in videos

[Ali et al.]
http://www.cs.ucf.edu/~sali/Projects/CrowdSegmentation/index.html



Inspecting sound sources in an orifice-jet flow

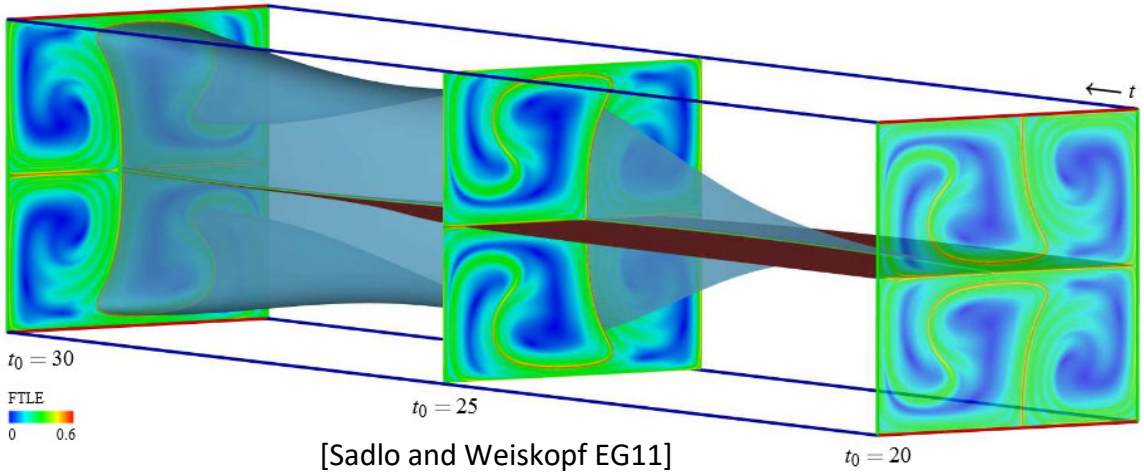
https://www.sciencedirect.com/science/article/abs/pii/S0045793016302626

Finite-Time Lyapunov Exponent

- FTLE ridges are approximately material structures
- Non-zero cross-flux across FTLE ridges
- Accuracy increases with integration time

- **Problem**: data sets often bounded with time

Streak Lines- and Surfaces-Based



[Sadlo and Weiskopf EG11]

[Uffinger et al. TVCG13]

- **What it can do?**

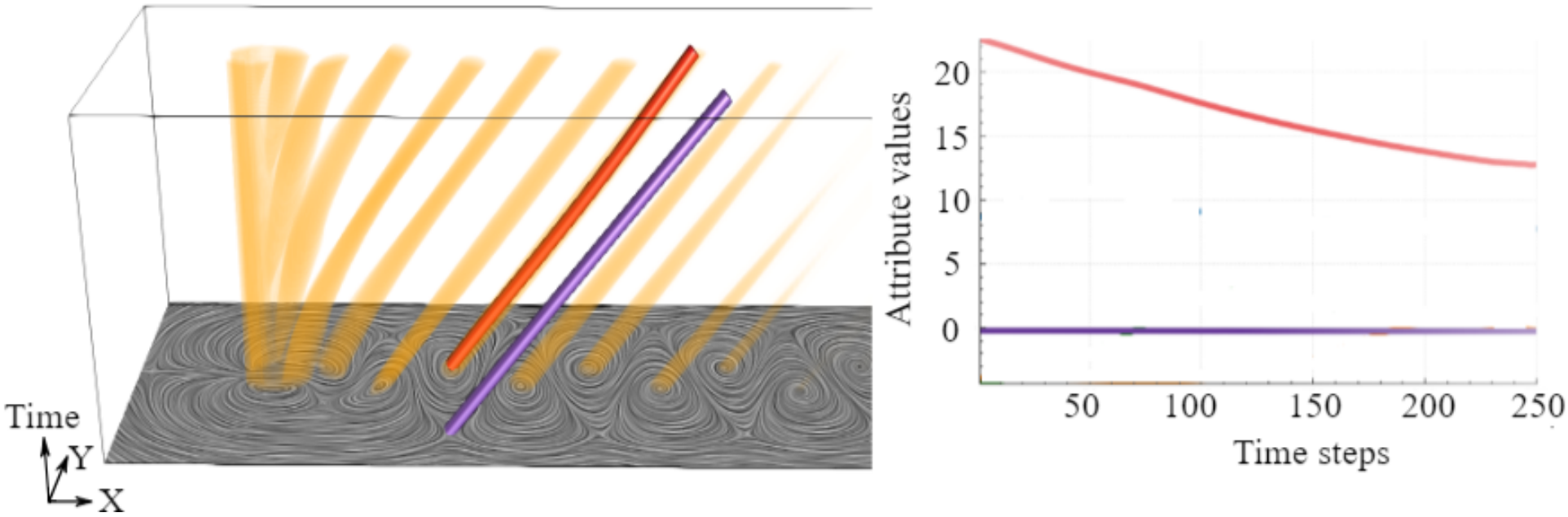
Capture moving saddle-like features!

- **What are still missing?**

Moving sources/sinks!

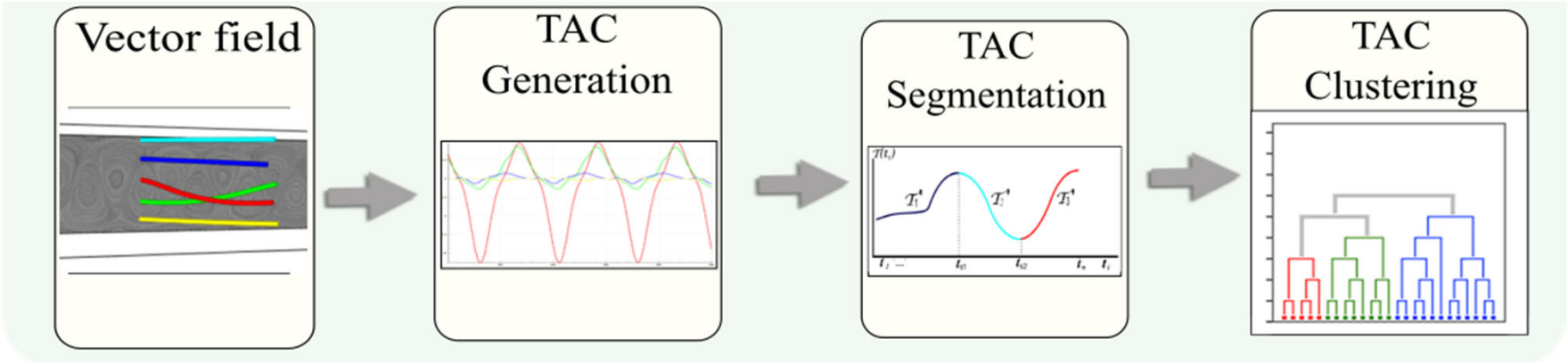
Other recently introduced techniques

Geometric representation sometime cannot faithfully represent the underlying physics of the flow, which is more important to the experts to understand the flow behaviors.



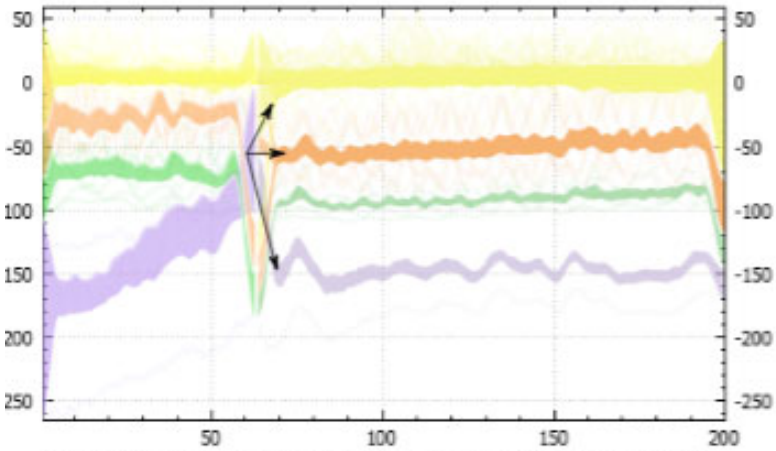
The two pathlines shown in the left image have very similar geometric characteristics, but the attribute values measured along them reveal different characteristics as shown in the right plot.

Time-activity curve (TAC) based processing and clustering

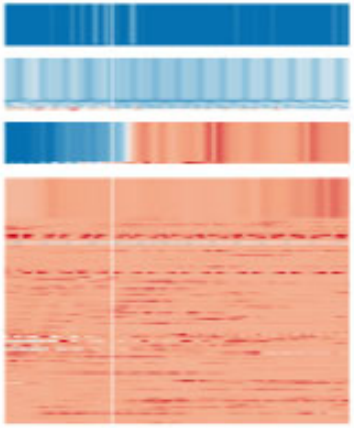


Interactive pathline exploration

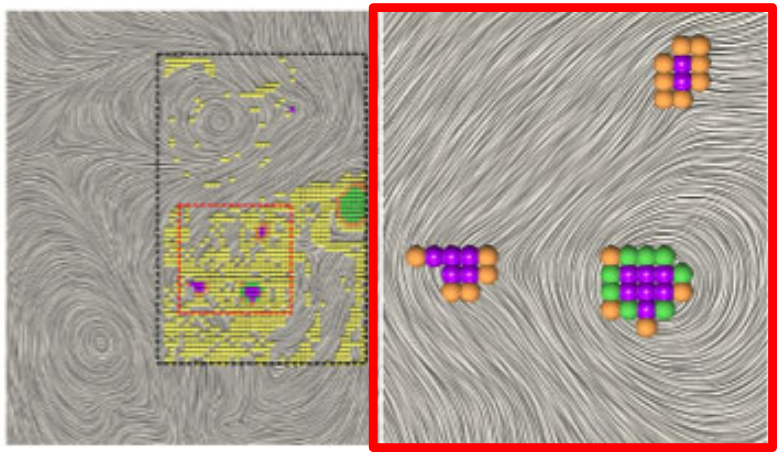
[Duong et al., 2020 CGF]



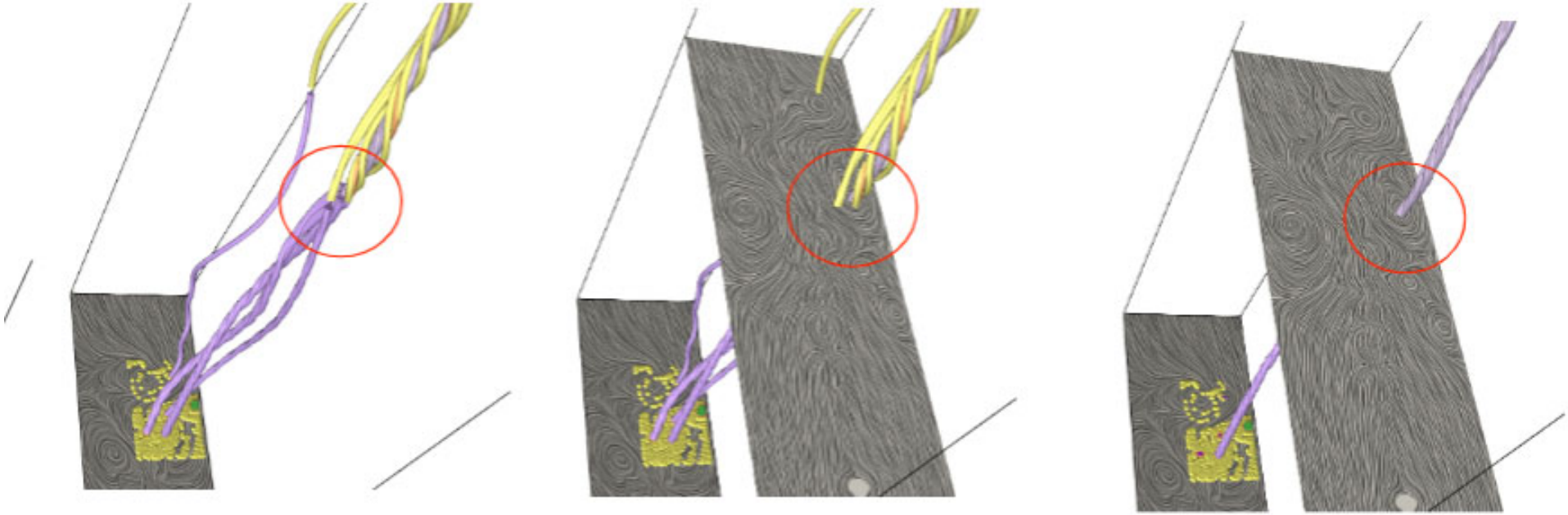
(a1) Splitting behavior of TACs in two time intervals



(a2) Stacked Plot



(a3) Sample points in the region of interest

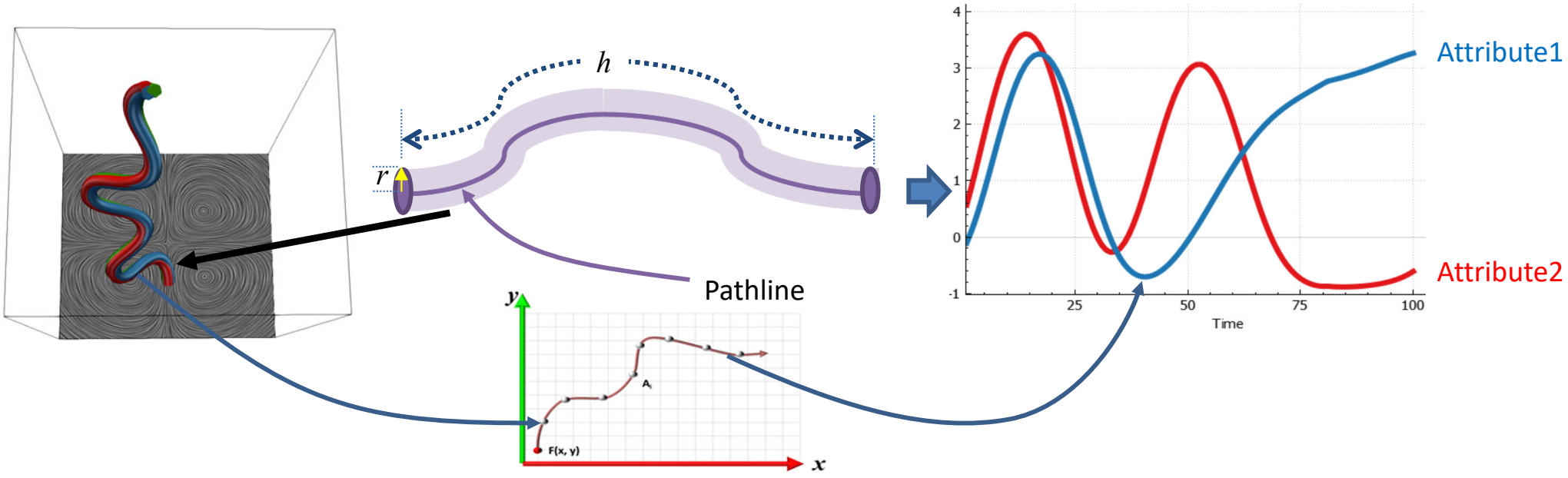


(b1) The vortex merging events are captured by our temporal result

(b2) Pathlines near the core of the newly-formed vortex

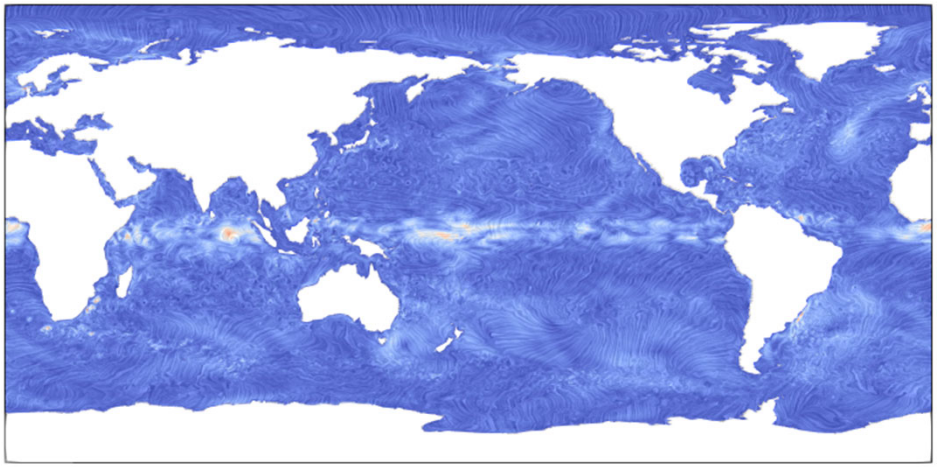
Correlation among different attributes

To measure the (cor-)relation of the temporal behaviors of two attributes of interest along pathlines uniformly seeded in space.

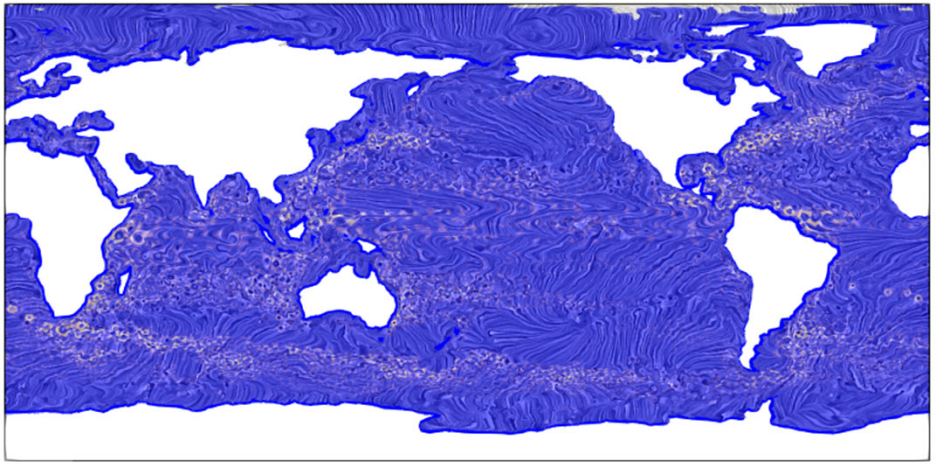


[Berenjkoub et al., 2018 IEEE VIS]

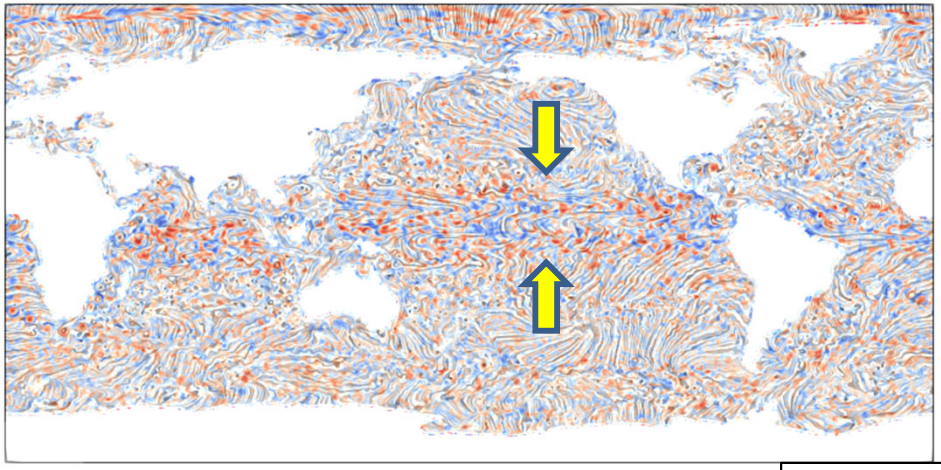
Application: Large Scale Ocean Circulation



Acceleration

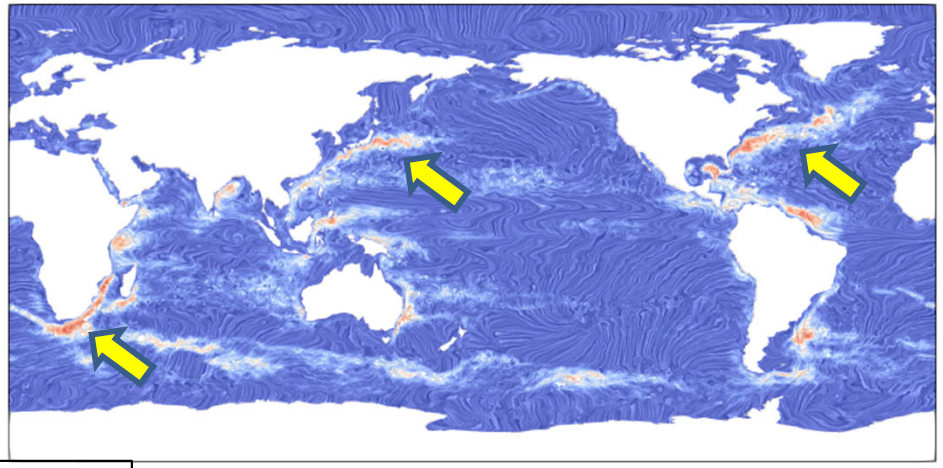


Q



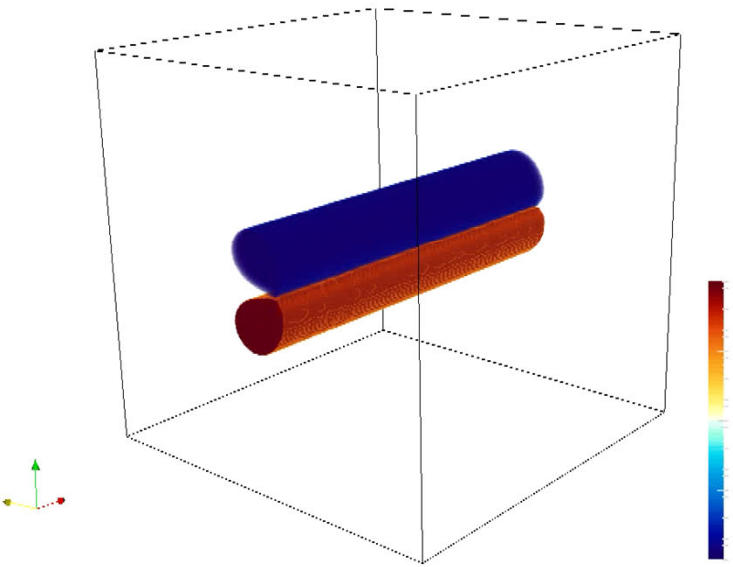
ST_LCC linear correlation

kernel size is 5x50

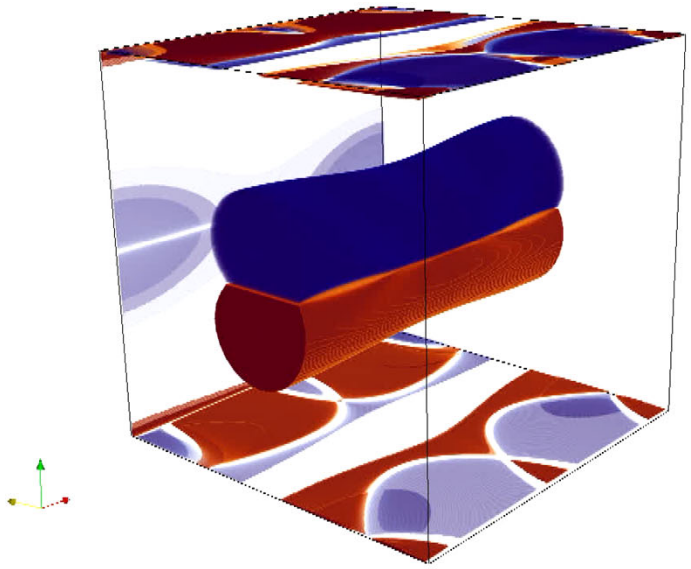


ST_MI dependency

Application: 3D Vortex Tube Data



Elliptical tube result generated with spatial kernel size of 3



Reconnection tube result generated with spatial kernel size of 3

- **Equation for vorticity**

viscous diffusion

$$\frac{D\boldsymbol{\omega}}{Dt} = \boxed{(\boldsymbol{\omega}\cdot\nabla)\boldsymbol{u}} + \nu\nabla^2\boldsymbol{\omega}$$

vortex stretching

dye: a passive scalar with a Schmidt number of unity

- **Equation for dye**

$$\frac{D\mathrm{T}}{Dt} = \nu\nabla^2\mathrm{T}$$

LCC of vorticity and dye

Additional Readings

- Frits H. Post, Benjamin Vrolijk, Helwig Hauser, Robert S. Laramee, and Helmut Doleisch, **The State of the Art in Flow Visualisation: Feature Extraction and Tracking,** in *Computer Graphics Forum (CGF)*, Vol. 22, No. 4, 2003, pages 775-792.

- Helwig Hauser, Robert S. Laramee, and Helmut Doleisch, **Topology-Based Versus Feature-Based Flow Analysis - Challenges and an Application**, in *Topo-In-Vis 2005,* pages 79-90, 2007, Springer-Verlag.

- Armin Pobitzer, Ronald Peikert, Raphael Fuchs, Benjamin Schindler, Alexander Kuhn, Holger Theisel, Kresimir Matkovic, Helwig Hauser. **On the Way Towards Topology-Based Visualization of Unsteady Flow - the State of the Art**, in EuroGraphics 2010.

Acknowledgment

Thanks for the materials from

- Prof. Robert S. Laramee, Swansea University, UK
- Dr. Christoph Garth, University of Kaiserslautern, Germany
- Dr. Holger Theisel, University of Magdeburg, Germany.
- Dr. Xavier Tricoche, Purdue University