1. Give CFG's for the following languages:
   (a) \( \{ w \in \{a, b\}^* \mid w \text{ is of odd length and the middle symbol of } w \text{ is not the same as the last symbol of } w \} \).
   (b) \( \{ w \in \{a, b, c\}^* \mid w \text{ has equal } a \text{'s and } c \text{'s or } w \text{ has twice as many } a \text{'s as } b \text{'s} \} \).

Run your first design of each CFG on JFLAP with at least 3 strings in the language and at least 3 strings not in the language. Once your design checks out on the 6 strings you selected, run the design on UH-JFLAP to “debug it” further and revise your design if necessary. Turn in the design that checked out on JFLAP with the test strings and the verdicts of JFLAP, and your final design with a list of any problems found using UH-JFLAP, with your solutions.

2. Design two different PDA’s (one direct and one indirect by first constructing a CFG) for each of the following languages:
   (a) \( \{ w \in \{a, b, c\}^* \mid \text{every } c \text{ in } w \text{ is not immediately followed by a } b \text{ and the number of } a \text{'s and } c \text{'s in } w \text{ is equal} \} \).
   (b) \( \{a^m b^n \mid m < n \text{ or } m > 2n, m, n \geq 0 \} \).

Run your first design of each PDA on JFLAP with at least 3 strings in the language and at least 3 strings not in the language. Once your design checks out on the 6 strings you selected, run the design on UH-JFLAP to “debug it” further and revise your design if necessary. Turn in the design that checked out on JFLAP with the test strings and the verdicts of JFLAP, and your final design with a list of any problems found using UH-JFLAP, with your solutions.

3. Prove or disprove the following: there are context-free languages \( L \) and \( L' \) such that both \( L \) and \( L' \) are not regular, but \( LL' \) is regular.

4. Prove carefully and in detail that the closure under kleene star construction for CFL’s given in class is correct.