Creating Polygon Models for Spatial Clusters

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ABSTRACT

This paper proposes a novel methodology for creating efficient polygon models for spatial datasets. Polygon models are essential in spatial data mining applications for the quantitative analysis of relationships and change between clusters. Moreover they can be utilized for representing and visualizing spatial clusters. However, there is not much research concerning the usage of polygons as cluster models for spatial clusters. There are algorithms, which can generate a representative polygon for a set of points. However, most of the proposed algorithms do not meet the requirements of spatial data mining applications such as removing outliers, dealing with separate regions with varying densities and working with different shapes of cluster, and lack support for automatically selecting proper input parameters. Moreover, quantitative evaluation measures that can guide the generation of polygons are missing. In this paper, we propose a comprehensive analysis framework that takes a spatial cluster as an input and generates polygon model for the cluster as an output. The framework uses pre-processing to remove outliers, detect subregions and novel post-processing algorithms to create a visually appealing, simple, and smooth polygon for each subregion in the cluster. The polygons themselves are generated using Characteristics shapes. The paper proposes two novel polygon fitness functions to automatically select the proper input parameter for Characteristic shapes and other polygon generating algorithms. Moreover, as a by-product, a novel emptiness measure is introduced for quantifying the presence of empty spaces inside polygons. We tested the methodology on several different datasets and verified that each step of the methodology is essential and effective for creating simple and accurate polygon models for spatial datasets. The novel fitness functions proved to be very effective for generating representative polygon models for clusters of differing characteristics.

Categories and Subject Descriptors

I.5.3 [Clustering]: Algorithms H.2.8 [Database Management]: Database Applications – data mining, spatial databases and GIS.

General Terms

Algorithms, Design, Experimentation.

Keywords

Spatial data mining, Polygon Models for Point Sets, Spatial Clustering, Polygon Fitness Function, Polygon Emptiness Measure

1. INTRODUCTION

Polygons serve an important role in the analysis of spatial data. In particular, polygons can be used as a higher order representation for spatial clusters, such as for defining the habitat of a particular type of animal, for describing the location of a military convoy consisting of a set of vehicles, or for defining the boundaries between neighborhoods of a city consisting of sets of buildings. It is attractive to use polygons for this purpose due to the following reasons.

First, it is computationally much cheaper to perform certain calculations on polygons than on sets of objects. For example, polygons have been used to describe the functional regions of a city [17]. A given location can be assigned to one of those functional regions very efficiently by checking in which polygon the location is included.

Second, relationships and changes between spatial clusters can be studied more efficiently and quantitatively by representing each spatial cluster as a polygon. Polygon analysis is particularly useful to mine relationships between multiple related datasets, as it provides a useful tool to analyze discrepancies, progression, change, and emergent events [1]. Polygon analysis is also useful for summarizing new trends in streaming spatial data [3], such as traffic data.

Furthermore, polygons have been studied thoroughly in geometry. Therefore, they are mathematically well understood. There are many libraries and tools in various programming languages for manipulating polygons and other geometric objects (ArcGIS, CGAL, GeoTools etc.). Spatial extensions of popular database systems, such as ORACLE, Microsoft SQL Server, MySQL, and PostgreSQL support polygon search, polygon manipulation, and many other related geometric operations.

However, there is not an established procedure in the literature on how to derive polygonal models from spatial clusters. The objective of the research described in this paper is to find an optimal set of polygons for two dimensional spatial clusters. To achieve this goal, we propose a three-step methodology that consists of:

1) A *Pre-processing* step for removing outliers and detecting subregions in a cluster
2) A *Polygon-generating* step for generating polygons for each subregion in a cluster with the aid of a polygon fitness function

3) A *Post-processing* step which deals with overlapping polygons and smoothens the generated polygons

The input of this process is a spatial cluster containing a set of points and the output is a set of polygons—the model of the cluster. In most cases, the result of this process is just one polygon representing the cluster. However, multiple polygons will be used to represent the cluster if the cluster has well-separated subregions. In general, we want to generate polygon models that represent the cluster as closely as possible, and at the same time we want these polygon models to be simple, consisting of straight, non-intersecting line segments without holes. However, it is not trivial to generate such representative polygons. As shown in Figure 1 (taken from [4]), many different polygons (or a set of polygons as in Figure 1e) can be generated for the same set of points. Therefore, it is desirable to define application specific criteria for evaluating different polygon models. Coming up with such criteria and evaluation measures is the focus of this paper. Depending on the application context, a different one of the 7 shapes in Figure 1 may be desirable.

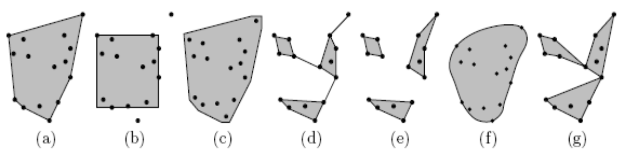


Figure 1. Different shapes generated for the same set of points (taken from [4]).

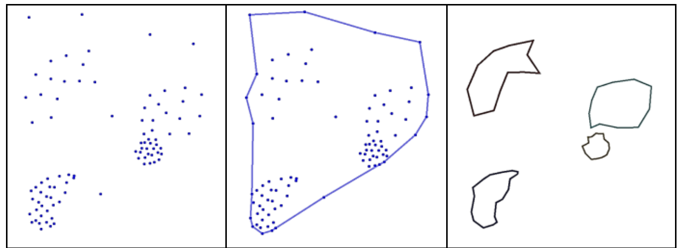
Although, using polygons is not the only option for modeling a spatial cluster (i.e. splines or polylines can also be used); this paper will only consider the usage of polygons as cluster models because of the advantages of using polygon models as discussed earlier. Moreover, in this paper, we will not consider holes inside polygons.

Main contributions of this paper include:

- A comprehensive three-step methodology for generating polygons from spatial clusters is proposed.
- Two novel quantitative polygon fitness functions are introduced to guide the generation of polygons from point clouds, alleviating the parameter selection problem when using existing polygon generation methods. The proposed methodology uses Characteristic Shapes [7] to generate polygon models; however, we wish to emphasize that other polygon generation methods, such as Concave Hull algorithm [9] can be used in conjunction with those fitness functions.
- An automated pre-processing procedure for removing outliers and detecting subregions in a spatial cluster is presented.
- A novel emptiness measure is introduced that quantifies the presence of empty areas in a polygon.
- Novel algorithms for smoothening polygons are proposed.

The rest of the paper is organized as follows. In Section 2, we discuss the challenges that algorithms face which create polygon models for spatial clusters. In Section 3, we compare the existing methods for creating polygon models. Section 4 provides a detailed discussion of our methodology. We present the experimental evaluation in Section 5, and Section 6 concludes the paper.

2. REQUIREMENTS FOR CREATING POLYGON MODELS



a. Spatial Cluster	b. Single Polygon model	c. Multiple polygon models

Figure 2. A spatial cluster with subregions and outliers.

Let's consider the artificial spatial cluster shown in Figure 2 which contains some subregions and outliers. It is obvious that this cluster cannot be well represented with just one polygon; a separate polygon for each subregion needs to be generated as in Figure 2c; otherwise, a polygon with large empty space would be created as in Figure 2b. Using a single polygon cannot represent the density of the points in the cluster. If this polygon is used for statistical purposes for this cluster, incorrect statistics (larger area, less density, etc) will be obtained for the cluster. Thus, a good polygon model needs to allow generating multiple polygons for a cluster. On the other hand, determining the subregions is not a trivial task as varying densities inside a cluster makes this task quite difficult. The region on the right in Figure 2a can be considered as a single region or it can be divided into two subregions. However, most polygon generating algorithms cannot deal with varying densities as we will discuss in Section 3.

Outliers pose another challenge, as most of the existing polygon generating algorithms create models which contain all the points of the spatial cluster. Therefore, not removing them will lead to large empty space in the polygon model, as can be seen in Fig. 2b.

In summary, a methodology which creates polygons for spatial clusters has to be able to create multiple polygons, deal with outliers, varying densities in a cluster, and overlapping polygons[1]. In general, we want to create polygons as shown in Figure 2c. These polygons represent the shape of each subregion in the cluster quite well. Furthermore, the generated polygons are smooth and have small number of edges and cavities. Section 4 will explain how we will accomplish this task.

3. RELATED WORK

Representing a set of points as polygons (or similar geometric shapes), creating the boundary of a set of points, or defining the perceived shape of a dot pattern has been a research area in computational geometry, computer graphics, computer vision, pattern recognition, and geographic information science for many years.

Convex hulls are the simplest way to enclose a set of points in a convex polygon. However, convex hulls may contain large empty areas that are not desirable for good representative polygons. Creating polygon models based on Voronoi diagrams or Delaunay triangulations is another commonly used approach. Alani et al. [6]

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[1] The discussion of this third challenge will be postponed until Section 4.3.

describe a method for generating approximate regional extents for sets of points that are respectively inside and external to a region. However, the proposed method requires defining a set of points outside the given cluster so that cluster boundaries can be obtained. Matt Duckham et al. [7] propose a "*simple, flexible, and efficient algorithm for constructing a possibly non-convex, simple polygon that characterizes the shape of a set of input points in the plane, termed a Characteristic shape*". The algorithm firstly creates the Delaunay triangulation of the point set—which actually is the convex hull of the point set—and then reduces it to a non-convex hull by replacing the longest outside edges of the current polygons by inner edges of the Delaunay triangulation until a termination condition is met. The Alpha shapes algorithm, introduced by Edelsbrunner et al. [5] also uses Delaunay triangulation as the starting step and generates a hull of polylines, enclosing the point set and this hull is not necessarily a closed polygon. Thus, the Alpha shapes algorithm requires post-processing for creating polygons out of the polylines. Alpha shapes can deal with outliers which are points which are not connected to any of the generated polylines. However, the Alpha shapes algorithm does not work well with varying densities in the cluster. Melkemi et al. [10] introduced the A-shapes algorithm with the aim of curing the limits of Alpha shapes in dealing with varying densities. This method applies a different parameter for each region in the dataset whereas Alpha shapes algorithm uses the same parameter for the whole dataset. However, a different set of points outside the original point set needs to be available when this approach is used and this algorithm does not always create simple polygons.

A. Ray Chaudhuri et al. [8] introduce s-shapes and r-shapes; the proposed algorithm firstly generates a staircase like shape called s-shape, which is determined using an s parameter and then reduces it to a smoother shape using the r parameter. The algorithm can cope well with varying densities in the point set. However, there is no easy way to estimate a good r parameter; the authors state that "*to get a perceptually acceptable shape, a suitable value of r should be chosen, and there is no closed form solution to this problem*".

A commercial algorithm, called Concave Hull [9], generates polygons by using a method that is similar to the "gift-wrapping algorithm" [14] used for generating convex hulls. It employs a k-nearest neighbors approach to find the next point in the polygon and creates a simple connected polygon unless the smoothness parameter *k* is too large and the points are not collinear.

A density-based clustering algorithm, DContour [11] is the only algorithm which is known to use density contouring for generating polygonal boundaries of a point set. DContour uses Gaussian Kernel density estimation techniques to derive a density function and then applies contouring algorithms to create polygons from the generated non-parametric density function. However, when using this approach selecting the kernel width parameter for the density estimation approach is non-trivial; moreover, the approach does not work well in presence of varying densities in the dataset.

Table 1. Capabilities of each algorithm

Capability:	**1**	**2**	**3**	**4**	**5**	**6**
Convex Hull	**No**	**No**	**No**	Yes	Yes	Yes
Voronoi	**No**	**No**	**No**	Yes	**No**	Yes
Characteristic shapes	**No**	**No**	**No**	Yes	Yes	Yes
Alpha shapes	Yes	**No**	Yes	**No**	Yes	No
s-shapes	Yes	Yes	Yes	**No**	Yes	**No**
Concave hull	**No**	**No**	**No**	Yes	Yes	Yes*
DContour	Yes	**No**	Yes	**No**	Yes	**No**
A-shapes	Yes	Yes	Yes	Yes	**No**	**No**

1: Can cope with outliers

2: Can cope with varying densities in a cluster

3: Can cope with separate regions

4: Has easy to select parameters or no parameters required

5: Does not require a separate set of points outside the cluster

6: Creates simple polygons

**unless the points are collinear*

Table 1 summarizes the capabilities of each algorithm. Following items are the most common problems with these algorithms:

- Most of the algorithms cannot cope with varying densities
- Many of the algorithms cannot cope with outliers
- Many of the algorithms cannot cope with separate regions in a cluster
- Some algorithms require a different set of points to be declared outside the cluster, which is not always available
- Parameter selection is difficult for some algorithms

In the next section, we will introduce a comprehensive three-step methodology that deals with all six requirements.

4. METHODOLOGY

In this section, we propose a three-step methodology that addresses the shortcomings of the existing polygon generating algorithms, and improve the overall quality of the generated polygons.

4.1 Pre-processing

The pre-processing step deals with the following two tasks:

i. Outliers should be detected and removed before generating clusters, so that large empty areas inside polygons can be avoided.
ii. Well-separated regions inside clusters should be identified. This step will decrease the amount of empty areas that are not relevant to the cluster.

We will consider clustering algorithms for pre-processing since clustering is a natural solution for problems involving (i) and (ii). We reviewed the suitability of existing clustering algorithms for the preprocessing step, based on the following requirements:

- It should be easily automated, that is, its parameters should easily be understood and selected or it should require no parameters at all.
- It should be able to cope with varying densities in the cluster.
- It should be able to detect regions of arbitrary shape.

Considering the above requirements, we can eliminate many of the clustering algorithms such as representative-based clustering algorithms (K-MEANS, PAM), which cannot detect regions of non-convex shapes and are sensitive to outliers. Density-based clustering algorithms such as DBSCAN, CLIQUE, and DENCLUE can find clusters of having non-convex shapes, but cannot cope with varying densities and high dimensional data. Graph-based algorithms like Jarvis Patrick (JP) [12] clustering can cope with varying densities but it cannot cope with some datasets if there is a bridge between clusters as demonstrated by [2].

We reviewed several clustering algorithms for the pre-processing task and selected SNN [2] and AutoClust [13] for this task (more details about the selection process can be found in [16]). In the remainder, of this section we give a little more details about the algorithms and why they were selected.

4.1.1 SNN

SNN (Shared Nearest Neighbors) [2] is a density-based clustering algorithm which assesses the similarity between two points using the number of nearest neighbors that they share. SNN clusters data as DBSCAN does, except that the number of shared k neighbors is used to access the similarity instead of the Euclidean distance. This allows the algorithm to deal with varying densities. Similar to DBSCAN, SNN is able to find clusters of different sizes, shapes, and can cope with noise in the dataset. Moreover, SNN copes better with high-dimensional data.

4.1.2 AutoClust

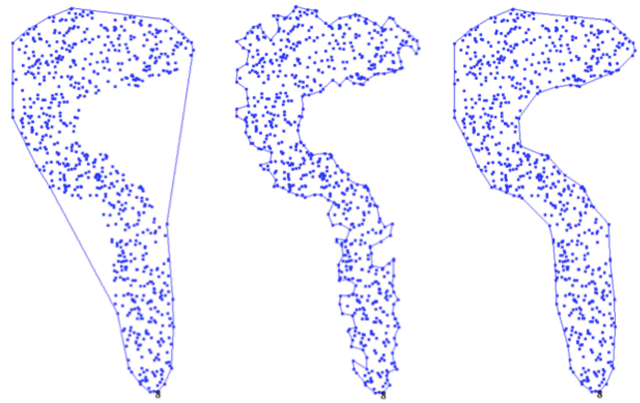
AutoClust [13] is a graph-based clustering algorithm, which makes use of Delaunay triangulation to cluster two-dimensional datasets. AutoClust does not require any parameters. Parameter values are revealed from the proximity structures of the Delaunay triangulation. Multiple bridges between clusters are detected and removed by using a 3-step algorithm that classifies edges in each step according to the statistical features of edges. AutoClust can identify clusters of differing densities even in presence of outliers and bridges between the clusters.

Authors argue that border points between clusters have a larger standard deviation of edge lengths in Delaunay triangulation compared to the points inside the cluster since border points have both long edges connecting them to the points in a different cluster and short edges connecting them to the points inside the cluster. By using the standard deviation of edge lengths connected to points, border points and inner cluster points are identified.

We propose using AutoClust when there is no previous knowledge of the dataset, as it has no parameters. On the other hand, SNN can be tuned according to the analysis task (amount of noise, nearest neighbor size can be set) when previous knowledge of the dataset is available as it has parameters to control the clustering process. Fortunately, the parameters are easy to understand and we were able to propose parameter selection procedure that works well with most spatial clusters, allowing to automate the pre-processing step.

4.2 Generating Polygons

The pre-processing step removes outliers and partitions clusters into well-separated subregions; therefore, the next step of the methodology, the *polygon generation step* is concerned with fitting a single polygon to a single point cloud. Figure 3 depicts three polygons that were created for the same cluster.



(a) A smooth polygon with large empty areas | (b) A tight polygon with too many edges and sharp angles | (c) A smooth polygon without large empty areas and with less cavities

Figure 3. Three different polygons generated for a cluster

The generated polygon in Figure 3a covers the largest area, and has the smallest perimeter, the least number of edges and the smoothest shape. However, it is obviously not a good model for the cluster because it includes large empty areas that are not relevant to the cluster. On the other hand, the polygon in Figure 3b has the largest perimeter, the most number of edges and covers the smallest area. Yet, it is also not a good representation for the cluster due to its ruggedness, as the polygon is quite complex. Additionally, this polygon has a potential overfitting problem as it is quite complex and therefore more sensitive to noise. Although this polygon does not have large empty areas, it has too many cavities which result in too many edges and sharp angles. If we use this polygon as the model for the cluster, having so many edges will make the model less efficient in terms of storage and processing costs. If we use the polygon model for clustering new samples, a new sample inside the cavities may not be assigned to this cluster although the sample is in the middle of many of the samples belonging to the cluster.

The polygon in Figure 2c, on the other hand, balances the two objectives as it does not include large empty areas and has a low degree of ruggedness. In the following, a polygon generation framework will be introduced which fits a polygon P to a set of spatial objects D minimizing the two objectives, we introduced earlier; namely, generating smooth polygons that have *a low emptiness with respect to D* and a low complexity. Additionally, we require that all objects in D are inside the polygon P. More formally, we define the problem of fitting a polygon P to a set of spatial objects as follows:

Let D be a set of spatial objects. Our goal is to find a polygon P that minimizes the following fitness function:

$$\phi(P,D) = Emptiness(P,D) + C * Complexity(P) \quad (1)$$

subject to the following constraint:

$$\forall o \in D: inside(o,P) \quad (2)$$

where C is a parameter which assesses the relative importance of polygon complexity with respect to polygon emptiness; e.g. if we assign a large value of C, smooth polygons will be preferred.

Emptiness(P,D) is a quantitative emptiness measure that assesses the degree to which P contains empty regions with respect to D.
Complexity(P) measures the complexity of polygon P.

In the following, we first introduce our novel emptiness measure. We then describe a polygon complexity measure that has been defined by some other work [15] which will be reused in our work; finally, we introduce different methods that fit polygons P to D, solving the optimization problem described in Eq. (1).

4.2.1 Measuring the Emptiness of P with respect to D

We have a surface, and we like to measure the emptiness in a surface with respect to spatial objects embedded into it. We call a subspace of the surface empty, if the density of the objects that are inside the subspace is low (typically, below a user-defined threshold). In general, we measure emptiness by computing the sum of the volumes/areas of the empty subspaces inside the surface over the volume/area of the surface itself.

In this particular work, we are interested in measuring the emptiness of polygonal surfaces with respect to a set of spatial objects D; moreover, we assume that all objects in D are located inside the polygon P. Several approaches can be envisioned to approach this problem. One approach is to design a non-parametric density function based on the objects belonging to D, and then, relying on a contouring algorithm, we can identify all contiguous regions inside P that are empty; that is, whose density is below a threshold θ; next, we can measure emptiness by computing the sum of the areas occupied by empty regions over the area of the polygons P; more formally:

$$\phi(P,D)=(\Sigma_{r\subseteq P:\ contiguous(r)\ \wedge\ empty(r)}\ area(r))/area(P)$$

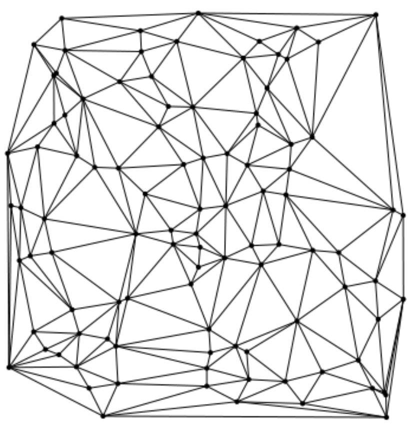


Figure 4: Delaunay Triangulation of a Point Cloud

However, in this paper we use a different approach, which is based on Delaunay triangulations; in general, as can be seen in Fig. 4, areas with very low density can be identified by large triangles in the Delaunay triangulation; that is, triangles whose area is above a certain size θ; for example, if we use the average triangle size in DT(D) as the threshold, the area of the large triangle on the upper right would be identified as an empty area. We introduce an emptiness measure which assesses the emptiness of a polygon P with respect to a point cloud D. Let:

P be a polygon whose emptiness has to be assessed,
D a set of points in 2D that P is supposed to model,
DT(D) the set of triangles of the Delaunay triangulation of D,
θ be the triangle area threshold,
$P_{CONV}=(\cup_{t\in DT(D)}\ t)$ be the outer polygon of the DT(D); P_{CONV} acts as the surface into which the objects of D are embedded and it is also the convex hull of D.
Our definition of emptiness of a polygon P with respect to a point cloud D is as follows:

$$Emptiness(P,D) := (\Sigma_{t\in DT(D)\wedge area(t)>\theta\wedge inside(t,P)}\ area(t)-\theta))/area(P_{CONV}) \quad (3)$$

When assessing emptiness of P with respect to D, we go through the triangles inside P and add the differences between θ and the area they cover, but only if the size of their area is above θ, and divide this sum by the area of the convex hull of D; be aware that p_{CONV} is not the area P covers, but a usually larger polygon which is the union of all triangles of Delaunay triangulation which serves as the surface into which the object of D are embedded in. It should be noted that when measuring emptiness triangles that are not part of P trivially do not contribute to emptiness.

4.2.2 Measuring Polygon Complexity

We assess the complexity of polygons using the polygon complexity measure which was introduced by Brinkhoff et al. [15]; it defines the complexity of a polygon P as follows:

$$Complexity(P) := 0.8 * ampl(P) * freq(P) + 0.2 * conv(P) \quad (4)$$

where *ampl*(P) is amplitude of vibration defined as:

$$ampl(P) := 1 - (\ boundary(convexhull(P)) / boundary(P)\ )$$

and *freq*(P) is the frequency of vibration of a polygon p:

$$freq(P) := 16 * (notches_{norm}(P)-0.5)^4 - 8 * (notches_{norm}(P)-0.5)^2 + 1$$

where $notches_{norm}(P) := notches(P) / (vertices(P)-3)$

and a notch is defined as a vertex with an interior angle greater than 180 degrees. Lastly, convexity of a polygon P is defined as:

$$conv(P) := 1 - (\ area(P) / area(convexhull(P))\ )$$

According to this definition, polygons with too many notches, having significantly smaller areas and larger perimeters compared to their convex hulls are considered complex polygons. Most importantly, it is a suitable measure to assess the ruggedness of a polygon model generated.

Figure 5 depicts the change of emptiness and complexity of the polygons generated using the Characteristic shapes algorithm for the cluster in Figure 3 using different chi parameters. Emptiness was computed by setting triangle area threshold θ to 1.25 times the average triangle area in DT(D).

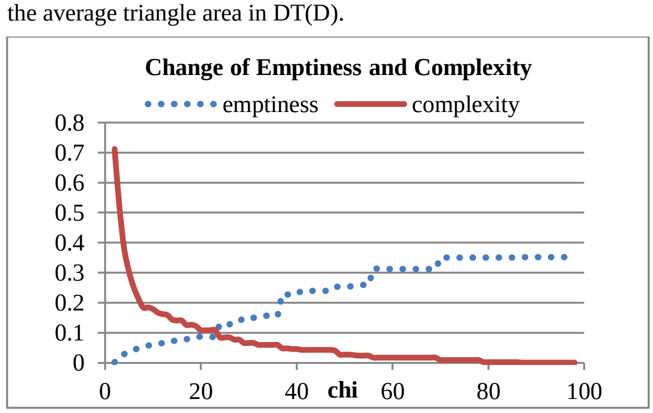


Figure 5. Change of emptiness and complexity of polygons created for the same cluster using different parameters

As seen in Figure 5, emptiness and complexity of the polygons are inversely proportional. Larger chi parameters create polygons which are emptier but less complex and vice versa. We employ objective fitness function defined in (1) to find a balance between the two measures.

4.2.3 A Second Polygon Fitness Function

In this section, we introduce an alternative fitness function which balances the area and the perimeter of the generated polygon. Figure 6 shows the area and perimeter for different polygons created for the same cluster using the Characteristic shapes algorithm while changing the chi parameter of the algorithm.

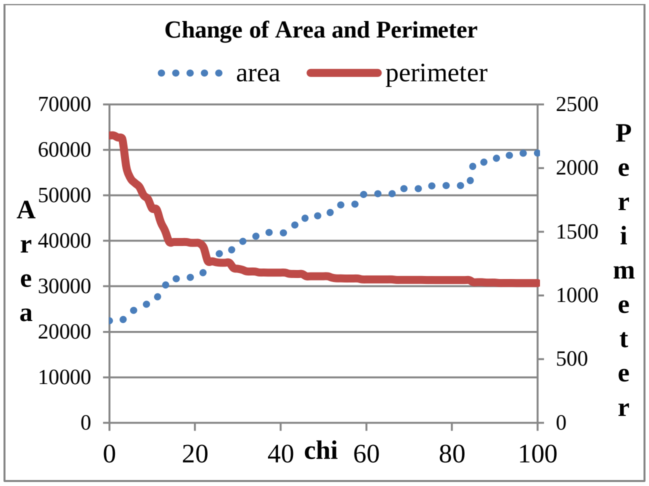


Figure 6. Change of area and perimeter of polygons created for the same cluster using different parameters.

As depicted in the Figure 6, increasing the chi parameter creates polygons with larger areas and smaller perimeters. In order to find a balance between area and perimeter, we postulated that a good fitness function should be in the following form:

$$\psi(P) = area(P)^{\alpha} \; x \; perimeter(P)^{\beta}$$

The polygon minimizing $\psi(P)$ will be considered as the fittest polygon. We have tested many different polygon fitness functions on a large benchmark of spatial clusters in this research; analyzing the results of this study, setting α=1 and β=2 proved to be the most effective, leading to:

$$\varphi(P) = area(P) \; x \; perimeter(P)^2 \qquad (5)$$

We claim that the fitness function φ provides a good default setting also for balancing polygon emptiness and complexity. However, if application requires producing very smooth polygons or very tight polygons, we recommend using the fitness function ϕ with very large C values or very small C values, respectively.

4.2.4 Generating Polygons

At the moment, we use Characteristic shapes to generate polygons in conjunction with the two fitness functions in our methodology as this produces decent polygon models. The algorithm itself has a normalized parameter chi which has to be set to an integer value between 1 and 100. In order to find the value of chi which minimizes the employed fitness function (either $\varphi(P)$ or ϕ(P,D)) we exhaustively test all 100 chi values, and return the most fit polygon as the result. Runtime complexity of the Characteristics shape algorithm is O(n**log(n)*). Thus, our methodology is guaranteed to generate the fittest polygon in O(n**log(n)*) as we exhaustively test 100 parameter values.

4.3 Post-processing

In this section, we will discuss how to tune the generated polygons to make them smoother. While polygon generation step of the proposed methodology generates polygons that are representative of the clusters, it is possible that:

1) some of the polygons generated for a spatial cluster might overlap.
2) polygons may have edges that do add very little value to the model, such as:
   a. successive edges which are almost-parallel to each other
   b. very short edges

In order to decrease the complexity of the generated polygons, we will post process the polygons in the following order:

1. Merge overlapping polygons in a cluster
2. Merge successive edges which are almost parallel and remove very short edges

4.3.1 Merging Polygons

Polygons are merged by creating a new polygon which is the union of all overlapping polygons. Figure 7 shows a polygon merge operation:

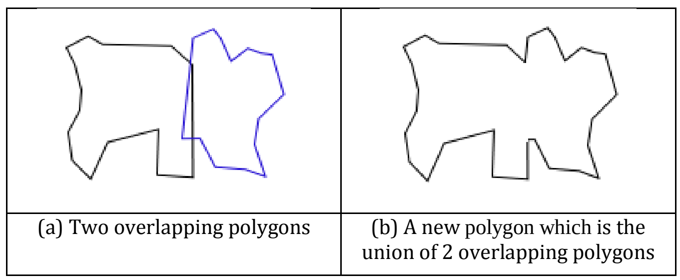


Figure 7: Merging polygons

In Figure 7a, two polygons generated for the two subregions are shown. These polygons overlap and therefore they were merged obtaining a larger polygon (Figure 7b).

4.3.2 Merging almost-parallel successive edges

We define almost-parallel edges as two successive edges in a polygon which have less than a threshold degree of deviation from one edge to another. Based on experiments, we found that setting this threshold value to 15 degrees gives the best results. Figure 5 shows three edges in a polygon where AB and BC are almost-parallel edges. The edge BC is deviating less than 15 degrees from edge AB, thus it can be merged with AB without changing the generated shape significantly. While merging two edges, the connecting point of the two edges is removed (point B) from the polygon and remaining end points of the two edges (points A and C) are connected.

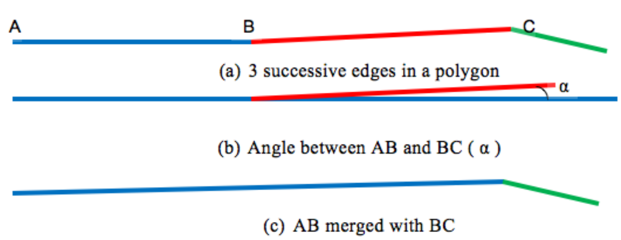


Figure 8: Merging almost-parallel edges

Figure 9 describes the steps of the algorithm for merging two almost-parallel edges:

```
foreach edge e(i) connecting points i and  i+1 in the polygon
    Calculate how much edge e(i+1) deviates from e(i)
        If deviation is less than Θ degrees
        Merge these edges by connecting points i an i+2
        Increase i by 1 to skip next point
```

Figure 9. Pseudocode for merging two almost-parallel edges

As shown in Figure 9, once a point is removed, we increase the counter *i* to skip the next edge in the polygon to avoid topological errors.

4.3.3 Removing short edges

We define short edges to have a length that is less than a threshold edge length. By default, we set this threshold value to 20% of the average edge length. Figure 10 shows three edges in a polygon and how to remove edges that are very short compared to other edges:

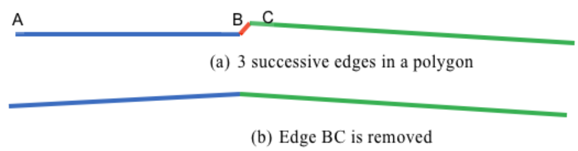


Figure 10. Removing short edges

Edge BC is too short; therefore it was removed from the polygon. Removing the short edge does not change the shape significantly. This process is similar to merging two almost-parallel edges, the connecting point of the short edge and the previous edge (point B) is removed and remaining end points of the two edges (points A and C) are connected.

5. EXPERIMENTAL EVALUATION

To evaluate our methodology, we created an artificial cluster which contains some outliers that consists of 4 separated regions which vary in density. The spatial cluster is depicted in Figure 11.

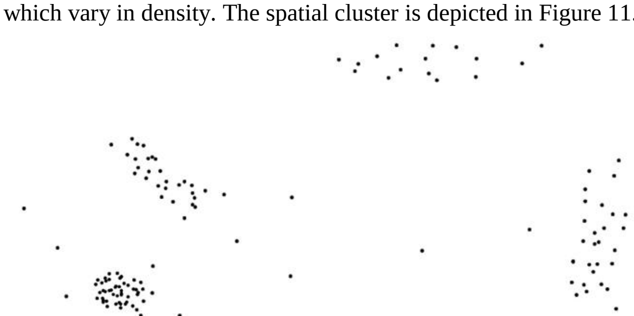


Figure 11. Artificial cluster used in the first experiment.

Figure 12 depicts a polygon that has been created using the Characteristic shapes algorithm without using any pre-processing. As can be seen, the generated polygon contain large empty spaces, and the polygons obtained for other chi values were equally bad, as Characteristic shapes creates a single polygon that contains all the points of the spatial cluster.

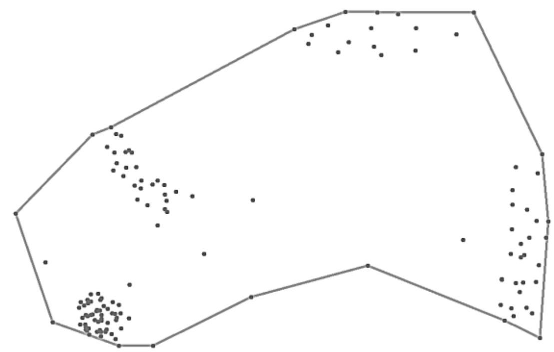


Figure 12. Polygon created by the Characteristic shapes algorithm without pre-processing

5.1 Step 1: Pre-processing

When we ran SSN and AutoClust on the cluster, both algorithms were able to detect subregions and eliminate outliers successfully. Figure 13 depicts the subregions detected by both algorithms.

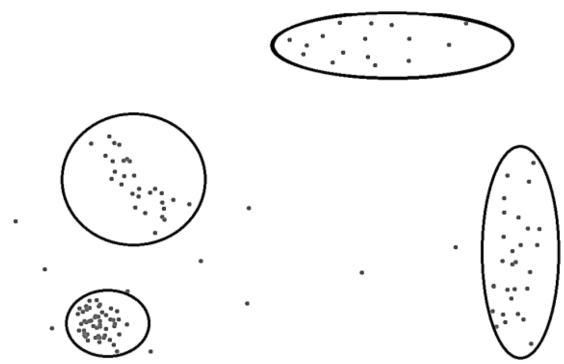


Figure 13. Pre-processing result for the cluster in Figure 11

We experimentally found that when using SNN setting the *nearest neighbors size* parameter to 8 gave best results with most datasets. SNN was able to deal with clusters of varying densities, shapes and outliers successfully with the default parameter setting.

When we used AutoClust algorithm, we obtained the same result; AutoClust does not have any parameters, yet it works very well on clusters of varying densities, and shapes in presence of noise.

5.2 Step 2: Polygon Generation

After detecting the 4 subregions, we generated a polygon for each subregion in the cluster using the Characteristic shapes algorithm in conjunction with fitness function φ to select the chi-parameter values for each cluster.

The fittest polygons with respect to fitness function φ were obtained at chi=43 for subregion 1, chi=24 for subregion 2, chi=21 for subregion 3, and chi=20 for subregion 4. Figure 14 depicts the generated polygons for each subregion; as can be seen the proposed methodology worked very well on all of the subregions and created a representative polygon for each.

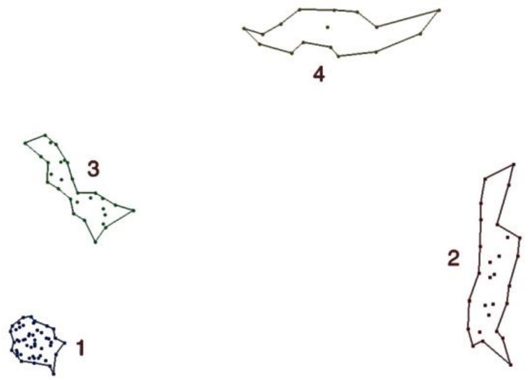


Figure 14: Polygons generated by Characteristic shapes using fitness function φ.

Table 2. Properties of each polygon in Figure 14

	Area	Perimeter	Emptiness	Complexity
P1	4355	277	0.145	0.203
P2	13007	727	0.108	0.1
P3	6585	511	0.091	0.124
P4	12212	679	0.107	0.084

Next, to illustrate the benefits of the two fitness functions and their underlying measures, we quantitatively compare the polygons in Figure 14 with polygons that have been obtained by running Characteristic shape with chi=10 (Figure 15) and chi=80 (Figure 16). Tables 2, 3, and 4 report the complexity, emptiness, perimeter, and area of each polygon in Figures 14, 15, and 16, respectively. For the emptiness calculation, we set θ to 1.25 times the average area of the triangles in DT(D).

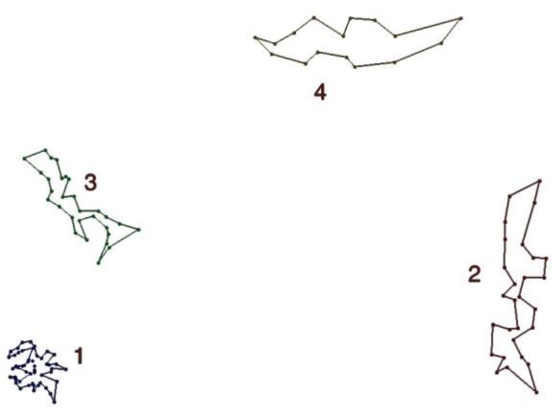


Figure 15. Generated polygons for chi=10

Table 3. Properties of each polygon in Figure 15

	Area	Perimeter	Emptiness	Complexity
P1	2505	498	0.08	0.464
P2	9528	866	0.093	0.258
P3	4547	634	0.052	0.289
P4	11478	706	0.107	0.122

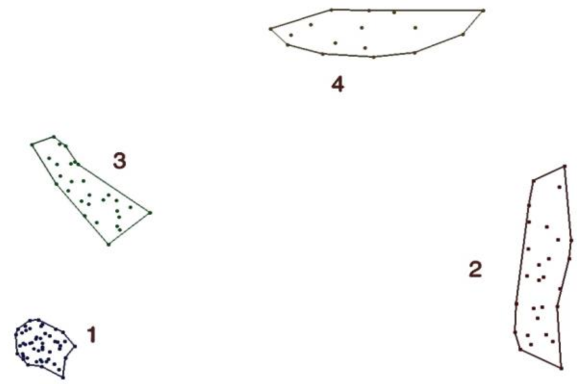


Figure 16. Generated polygons for chi=80

Table 4. Properties of each polygon in Figure 16

	Area	Perimeter	Emptiness	Complexity
P1	4696	268	0.162	0.015
P2	16890	687	0.157	0.012
P3	9661	479	0.168	0.016
P4	15839	645	0.147	0.001

For chi=10, the generated polygons are very tight for subregions 1, 2 and 3, whereas for chi=80 the generated polygons are very large for subregions 2, 3 and 4. It can also be seen as the chi-parameter value increases polygon complexity decreases, whereas polygon emptiness increases; for example, for the polygon created for the second area its emptiness increases from 0.093 to 0.108 to 0.157, whereas the polygon complexity decreases from 0.464 to 0.258 to 0.012. It also can be observed that the polygon for chi=10 is much more complex than the polygon selected by fitness function φ, but this significant increase in polygon complexity did not lead to a significant reduction in polygon emptiness which just dropped from 0.108 to 0.093. Finally, for each of the four subregions a different value for parameter chi was optimal, which motivates the need to use fitness functions to guide parameter selection.

We also created polygons using the fitness function ϕ defined in equation 1 with the same dataset by setting C[2] to 0.35 and obtained the polygons shown in Figure 17:

[2] Due to space limitations we will not report results for other C values, and also do not analyze the results in more detail; however, they have been reported at [18].

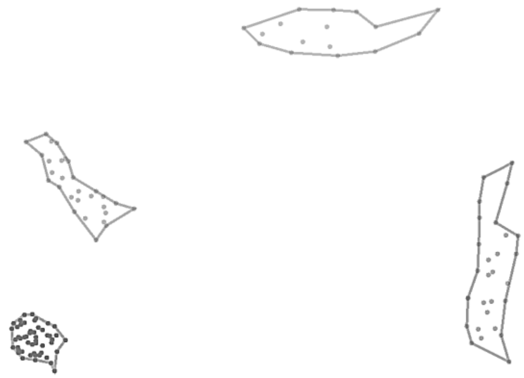


Figure 17. Polygons generating using fitness function ϕ

By increasing/decreasing the C parameter less/more complex polygons can be created depending on application needs. In general, using ϕ as the fitness function provides for more control over the polygons created.

5.3 Step 3: Post-processing

Although, the generated polygons in Figure 14 are good representatives of each region, they can be made smoother by applying the post-processing algorithms defined in our methodology. None of the polygons overlap, so we will not apply the polygon-merge operation. Besides, there are no very short edges in these polygons. However, there are some almost-parallel edges in polygons and when we merge those edges, we get the polygons depicted in Figure 18:

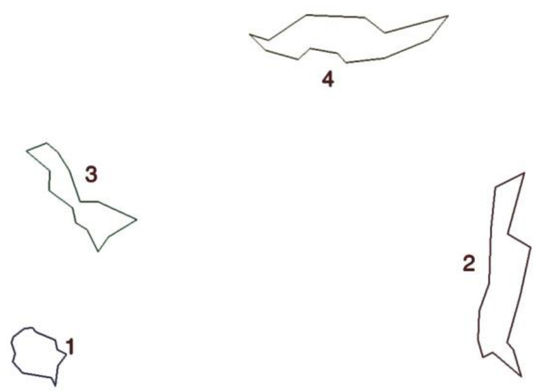
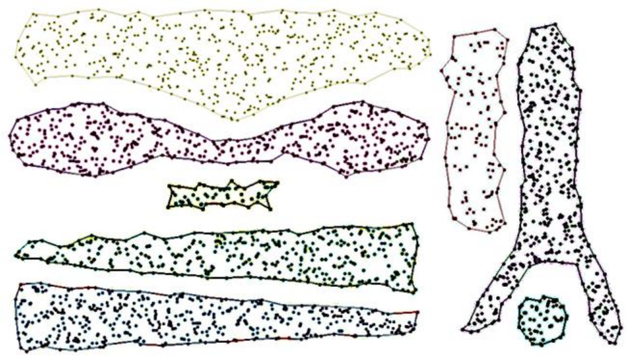


Figure 18. Polygons after post-processing

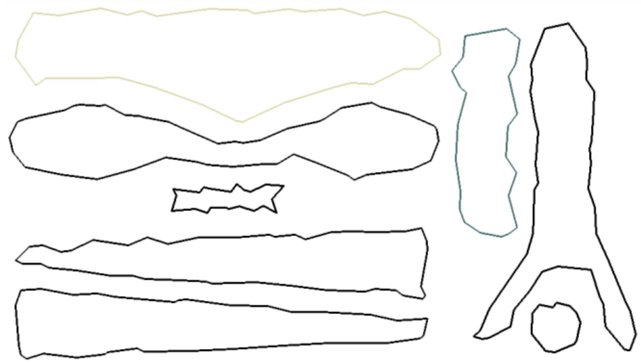
It is hard to see the difference visually, however the total number of edges in the polygon model decreased from 68 to 55. In more complex polygons with hundreds of edges, the change is more significant.

5.4 Experiments on the Complex 8 dataset

We tested our methodology on Complex 8 dataset. The clusters and generated polygons which were obtained by using fitness function φ are shown in Figure 19a. We applied our post-processing algorithms on these polygons. Polygons after the post-processing step are shown in Figure 19b. When we merged the almost-parallel edges, the total numbers of edges for all polygons decreased from 376 to 258 which correspond to a 31% decrease. Yet, the polygons are almost same, none of the polygons' area changed more than 1%. On the other hand, only 1 or 2 short edges were removed in each polygon.



(a) Complex 8 dataset clusters and generated polygons



(b) Polygons after post-processing

Figure 19. Complex 8 dataset results

In this section, we tested each step of our methodology and we observed that the proposed pre-processing algorithms were effective at detecting subregions and for removing outliers. We verified that pre-processing step is essential for generating acceptable polygon models. In polygon-generating step, we used a novel fitness function and generated very good polygon models. We also demonstrated the use of measuring emptiness, complexity, perimeter, and area of a polygon, to assess its quality to serve as a polygon model. Our post-processing functions proved to be effective at smoothing and simplifying the polygons without affecting their shapes significantly.

6. CONCLUSION

In this paper, we proposed a three-step comprehensive methodology for creating smooth simple polygons for spatial clusters. Our methodology employs pre-processing algorithms to remove outliers and detect subregions in a cluster. The Pre-processing step ensures that the generated polygons in the second step of the methodology are good representatives of the cluster. We employ two different clustering algorithms for pre-processing, AutoClust and SNN.

As popular polygon model generation algorithms have input parameters that are difficult to select, we introduced two novel fitness functions to automate parameter selection. We are not aware of any other work that uses this approach. The first fitness function balances the complexity of the polygon generated and the degree the polygon contains empty areas with respect to a point set D. The second fitness function seeks to find the "optimal" balance between the area and perimeter of the polygon generated. The methodology uses the Characteristic shape algorithm in conjunction with those two fitness functions. We also claim that the proposed fitness functions can be used in conjunction with other polygon generating algorithms, such as the concave hull algorithms in R and in PostGIS.

We also proposed novel post-processing algorithms that merge overlapping polygons and make the generated polygons smoother. The complexity of the generated polygon models decrease dramatically as a result of our post-processing methods, yet the shapes of the polygons are not affected significantly.

Each step of the methodology was tested with different datasets and our methodology proved to be effective at creating desired polygon models. SNN and AutoClust successfully detected subregions and removed outliers automatically. When used with our polygon fitness functions, the Characteristic shapes algorithm generated very accurate polygon models on the pre-processed clusters. Lastly, post-processing step simplified the generated polygon models quite significantly.

As a future work, we plan to extend our methodology to allow for holes in polygons. We also plan to generalize our fitness functions to be used in conjunction with other existing polygon generating algorithms.

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