A Visualization Framework for Multi-scale Coherent Structures in Taylor-Couette Turbulence – Supplemental Document

Duong B. Nguyen, Rodolfo Ostilla Monico, and Guoning Chen

Abstract—In this supplemental document, we provide (1) an example demonstrating the effect of different neighborhood sizes (i.e., different values of \( m \)) for the KDE computation, (2) an example showing how different iso-values affect the surface extraction for the representation of large-scale structures, and (3) a full resolution of the Q-criterion field for the TCF with \( R_\Omega = 0.0 \).

Index Terms—Flow visualization, Taylor-Couette turbulence, coherent structures

1 KDE PARAMETER DISCUSSION

In this section we provide a more detailed discussion and an example regarding the effect of kernel size \( m \) to the smoothness of the distance field computed using the feature level set method. We also include an example showing the iso-surfaces extracted from the obtained KDE field for the representation of the large-scale structures in a TCF.

Effect of neighborhood size \( m \). As described in the paper, a local neighborhood centered at each vertex \( x \) is constructed for the density estimation in our KDE computation. The size of this neighborhood is controlled by parameter \( m \). For example, if \( m = 5 \), a \( 5 \times 5 \times 5 \) neighborhood around a central vertex \( x \) will be considered for the density estimation. Similar to other kernel-based smoothing strategy, the larger the neighborhood, the smoother the output field will be, we observe similar behavior with the parameter \( m \) of our KDE computation. Figure 1 shows a few iso-surfaces extracted from the KDE density field computed with different values of \( m \) (i.e., neighborhood with different sizes). The iso-values used to extract these surfaces are the middle values of the data ranges of the obtained KDE density fields. With the increase of \( m \), the extracted surfaces become smoother. However, when \( m = 20 \) or larger, the individual surfaces extracted become connected, violating our goal of well separation of the structures. In addition, the overly smoothed field leads to larger surfaces that include more small-scale structures, which is undesired. In practice, we found that \( m = 10 \) works the best for all three TCFs we experimented with.

Effects of different iso-values. Another parameter (or threshold) that the user needs to specify in our framework is the iso-value that is used to extract the iso-surfaces from the obtained KDE density field for the representation of the large-scale structures. Figure 2 shows the iso-surfaces extracted with different iso-values (i.e., 0.01, 0.06, and 0.08, respectively) from the same KDE density field. As can be seen, smaller value tends to result in surfaces enclosing smaller volume area, while larger value leads to surfaces enclosing more volume region. Surfaces enclosing smaller volume region may miss important large-scale structures, while surfaces enclosing too many regions may include more small-scale structures. This is shown by the accuracy measurements (TP and FP) of the three surfaces. In particular, the TP and FP for the result with iso-value 0.01 are 96.2% and 18.7%, respectively, while the TP and FP for the result with iso-value 0.08 are 82.3%, and 3.2%, respectively. For comparison, the TP and FP for the result with iso-value 0.06 are 94.4% and 7.3%. Specifically, the iso-value 0.06 is the middle value of the data range of the obtained density field.

2 THE ISSUE OF USING Q-CRITERION FOR TCF WITH \( R_\Omega=0 \)

Due to the space limit, we have cropped the visualization of the small-scale vortices extracted by using the threshold \( Q > 2 \) for a TCF simulation with \( R_\Omega=0 \) in the paper. The full resolution of the picture is shown in Figure 3. The vortices cover almost the entire domain, making the separation of the large- and small-scale structures using attribute \( Q \) impossible.
Fig. 1. The effect of the size $m$ of the neighborhood $P$ in the KDE computation to the obtained surface representation of large-scale structures for the TCF with $R_\Omega = 0.1$. From the left to the right, $m = 5, 10, 20$, respectively. The corresponding iso-values are (a) 0.04, (b) 0.06, (c) 0.035.

Fig. 2. The iso-surface extracted with different threshold values from the KDE field for the representation of large-scale coherent structures for the TCF with $R_\omega = 0.1$, (a) 0.01 (b) 0.06, (c) 0.08, respectively. The red fibers are the small-scale features extracted with the threshold $Q > 2.0$. With a small value (a), the obtained iso-surfaces are big and include more small-scale structures (the green circle). On the other hand, if the threshold value is too large (c), the surfaces may miss a large amount of large-scale features (the yellow circle). The result in (b) provides the best coverage among all three values. In particular, the TP and FP values for the result with iso-value 0.01 are 96.2% and 18.7%, respectively, while the TP and FP values for the result with iso-value 0.08 are 82.3% and 3.2%, respectively. For comparison, the TP and FP values for the result with iso-value 0.06 (middle) are 94.4% and 7.3%.
Fig. 3. Visualization of vortices extracted by using the threshold $Q > 2$ for a TCF simulation with $R_{\infty}=0$. It is impossible to visually distinguish the difference between the large- and small-scale structures as the vortices are distributed everywhere in the spatial domain, and the Taylor rolls are not clearly formed.