

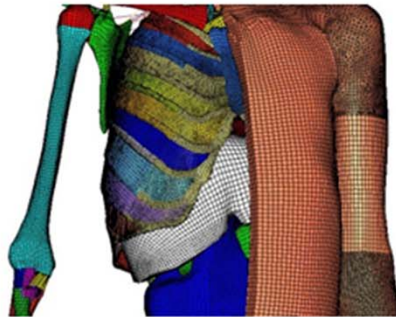
# Hexahedral Meshing with Varying Element Sizes

Kaoji Xu, Xifeng Gao, Zhigang Deng, Guoning Chen  
University of Houston

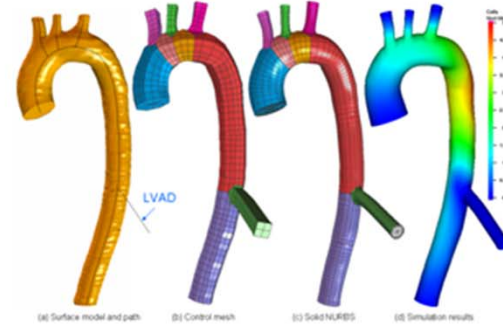


# Why Hexahedral (Hex-) Meshes?

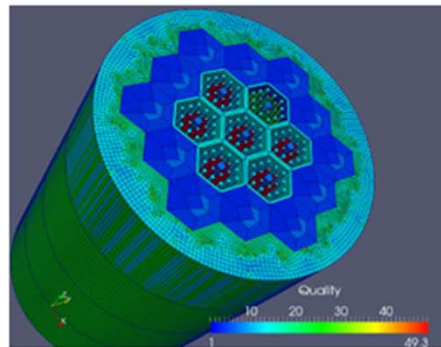
## Medical



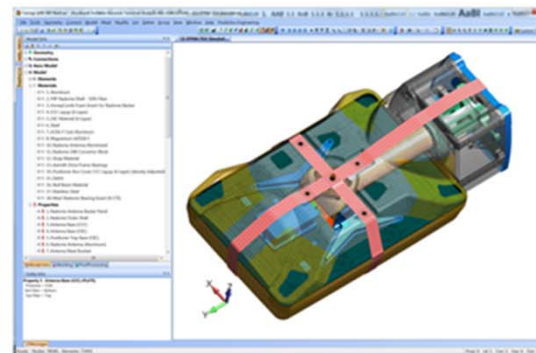
Biomechanics [Wayne State University]



Simulation-based medical planning [Zhang, et al. 2007]



Nuclear reactor manufacturing [Kitware]

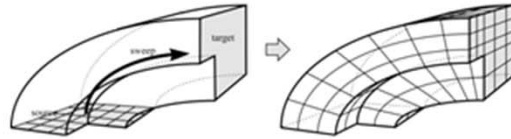


FEA Simulation [Predictive engineering]

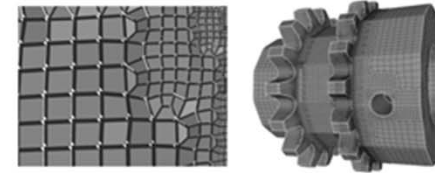
## CAD

# Existing Hex-Meshing Techniques

## Sweeping/Octree



[Owen, et al. 1998]

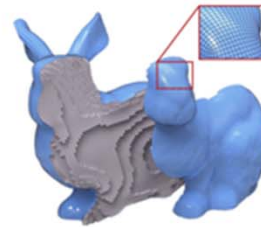


[Maréchal, et al. 2009]

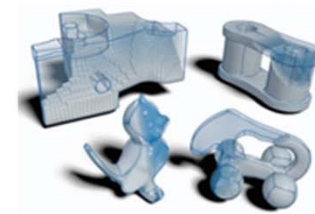
## Frame Field



[Nieser, et al. 2011]

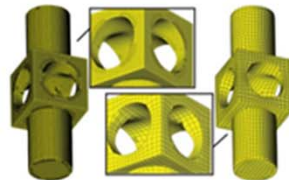


[Li, et al. 2012]

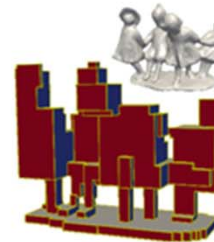


[Jiang, et al. 2013]

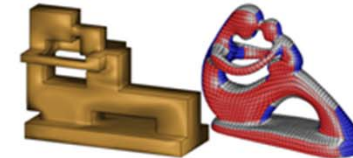
## Polycube



[Gregson, et al. 2011]



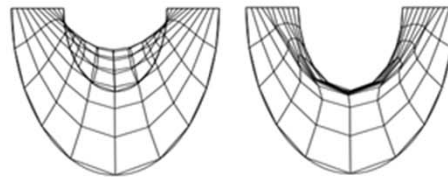
[Huang, et al. 2014]



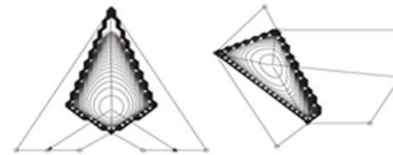
[Livesu, et al. 2013]

# Existing Hex-Meshing Optimization Techniques

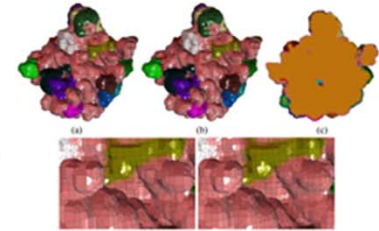
## Geometric Optimization



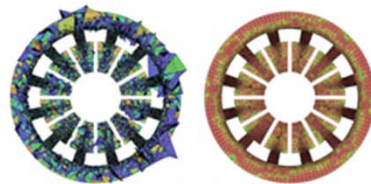
[Knupp, et al. 2000]



[Knupp 2003]



[Zhang, et al. 2009]

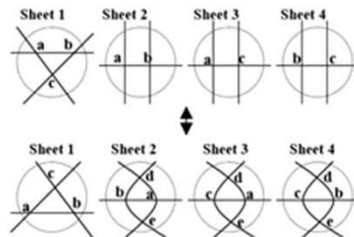


[Ruiz-Gironés, et al. 2014]

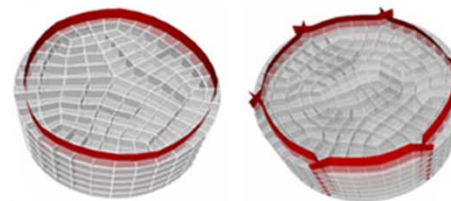


[Livesu, et al. 2015]

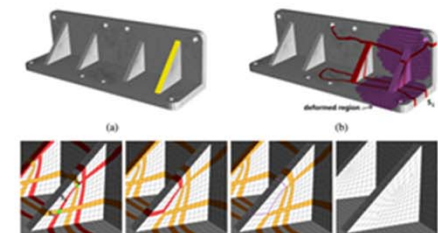
## Simplification



[Tautges, et al. 2003]



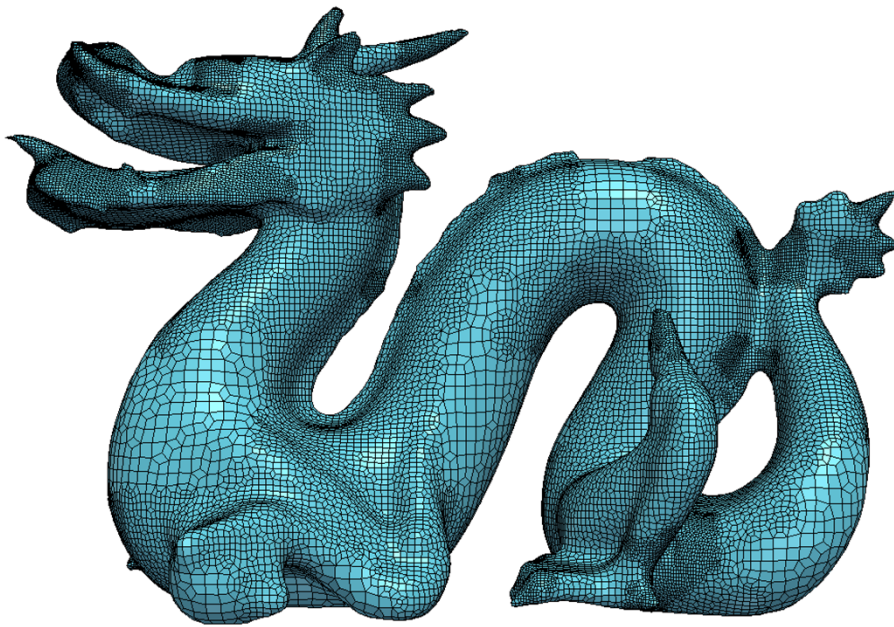
[Ledoux, et al. 2010]



[Zhu, et al. 2014]

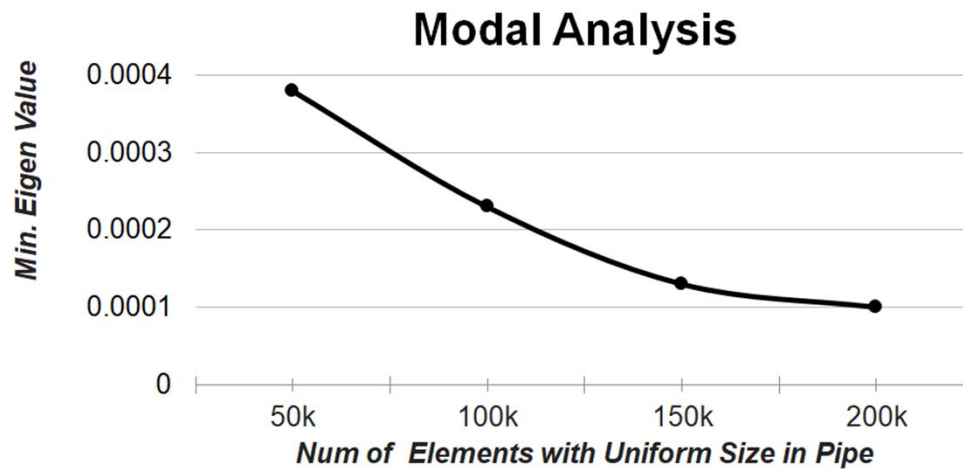
# Why Adaptive Meshes?

Adaptive meshes: Element sizes adapt to the feature sizes.

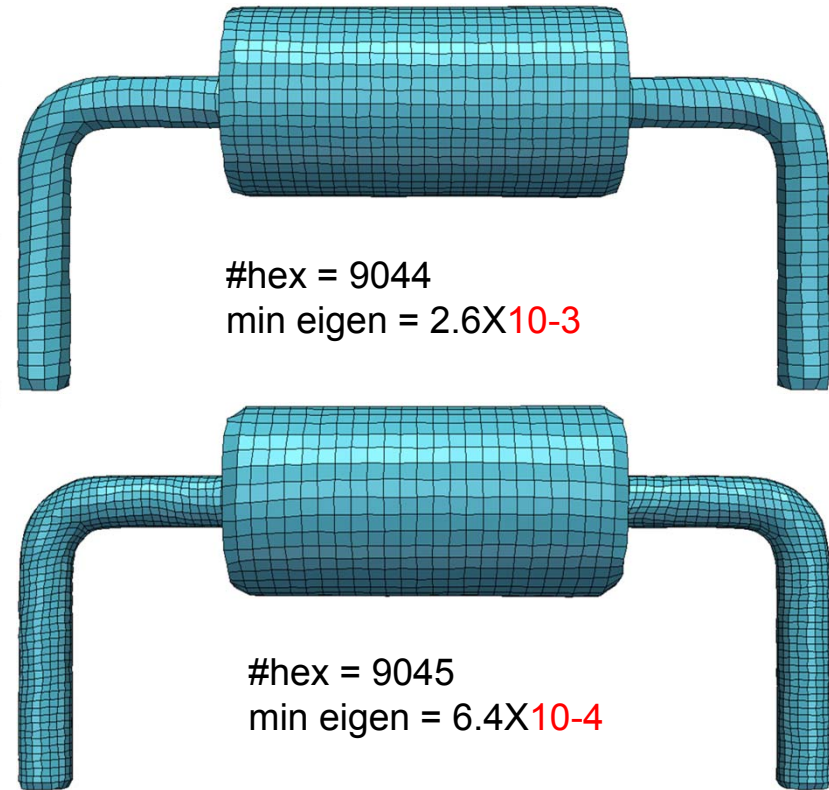


Octree

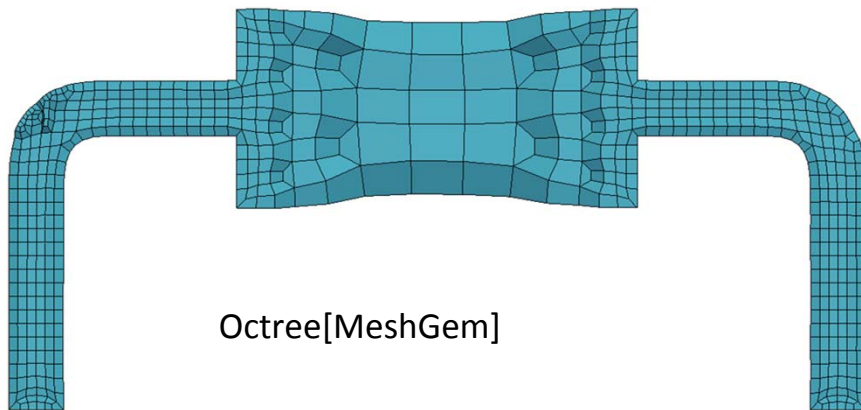
# Meshing quality in the linear elasticity problem



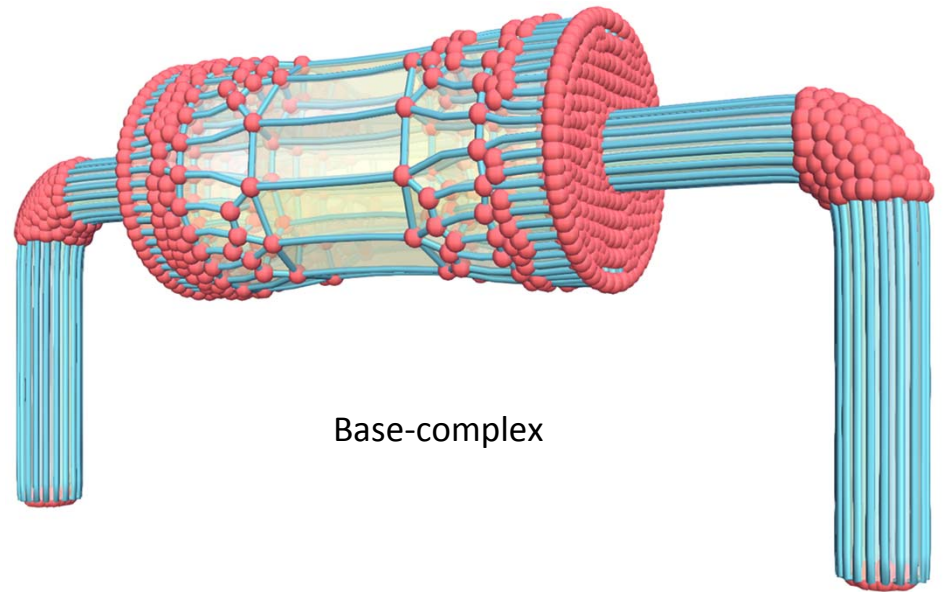
*The small the min. eigenvalue, the more accurate in modal analysis*



# What's the problem? Singularities



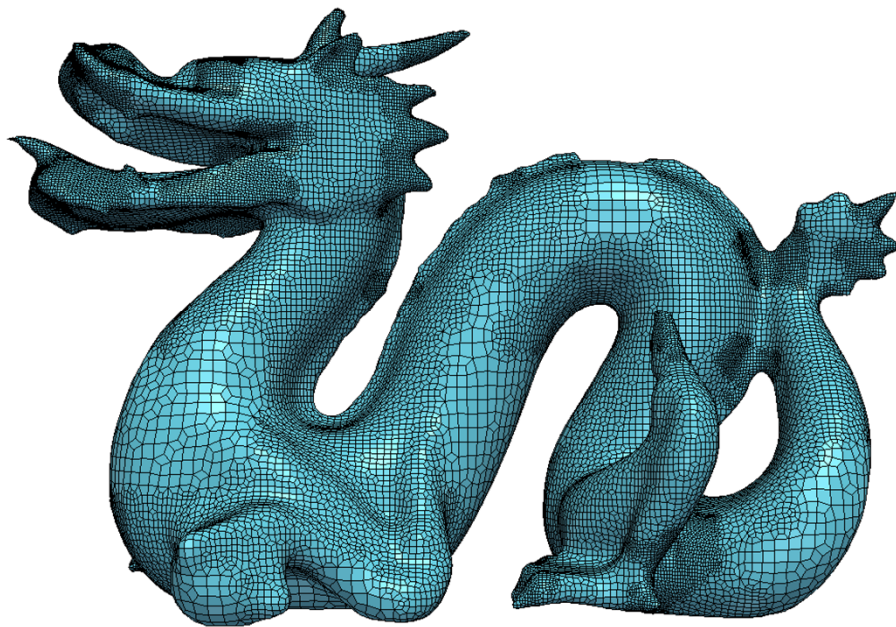
Octree[MeshGem]



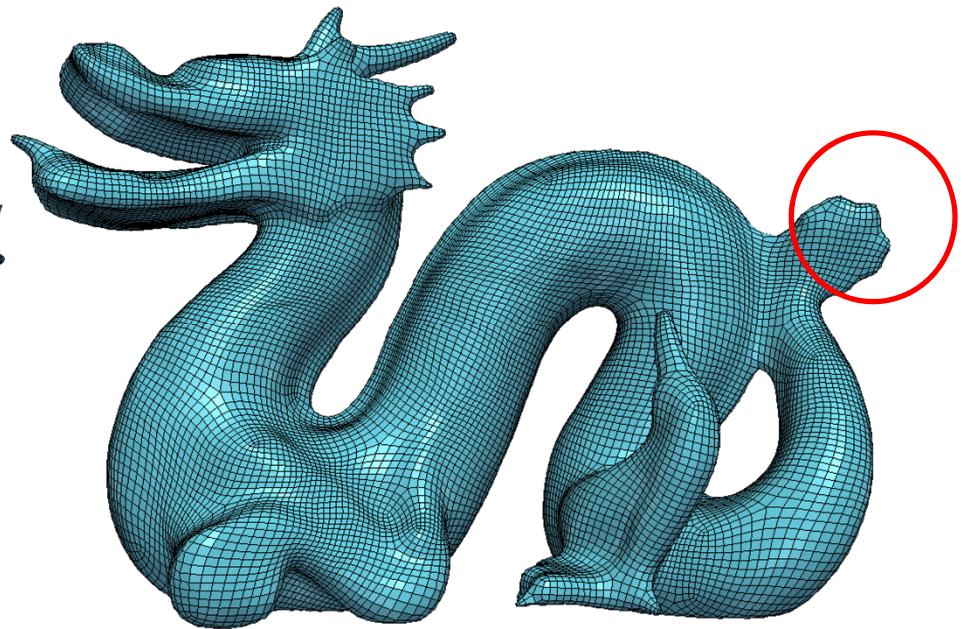
Base-complex

# What's the problem? Feature Loss

Polycube and frame field based method tend to smooth out surfaces.



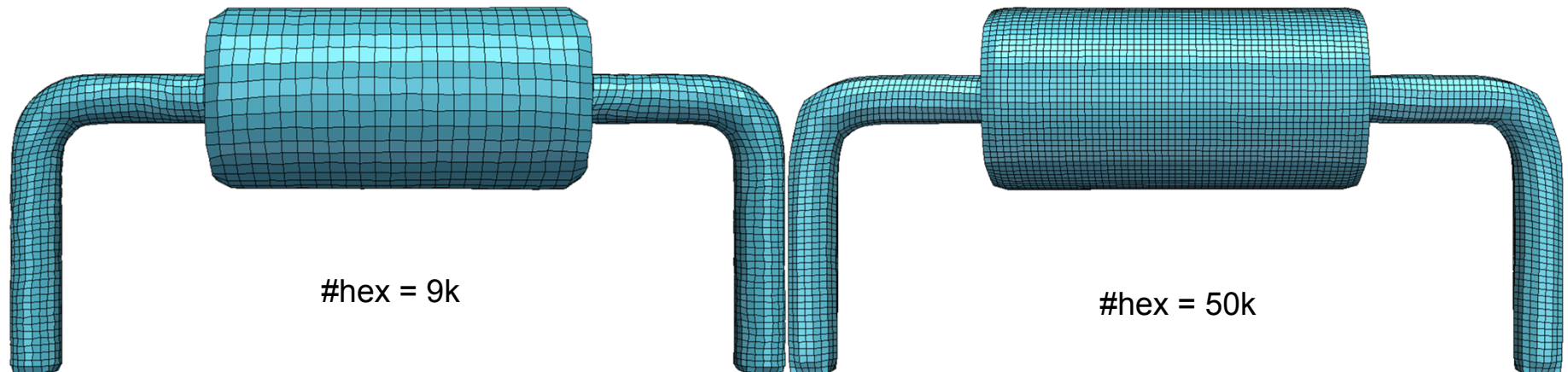
Octree



L1-PolyCube

# What's the problem? Too dense!

L1-PolyCube



#hex = 9k

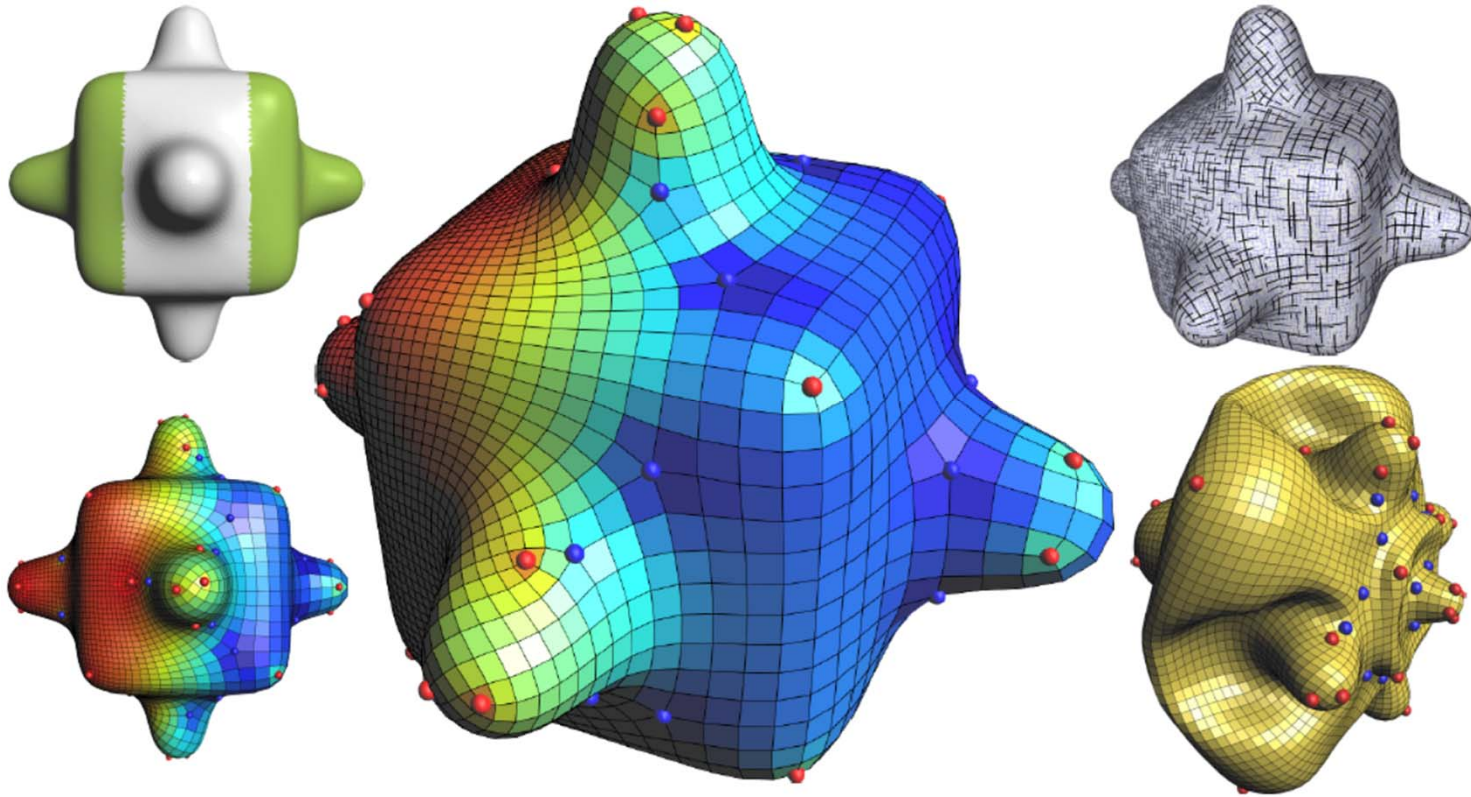
preferred

#hex = 50k

Not preferred

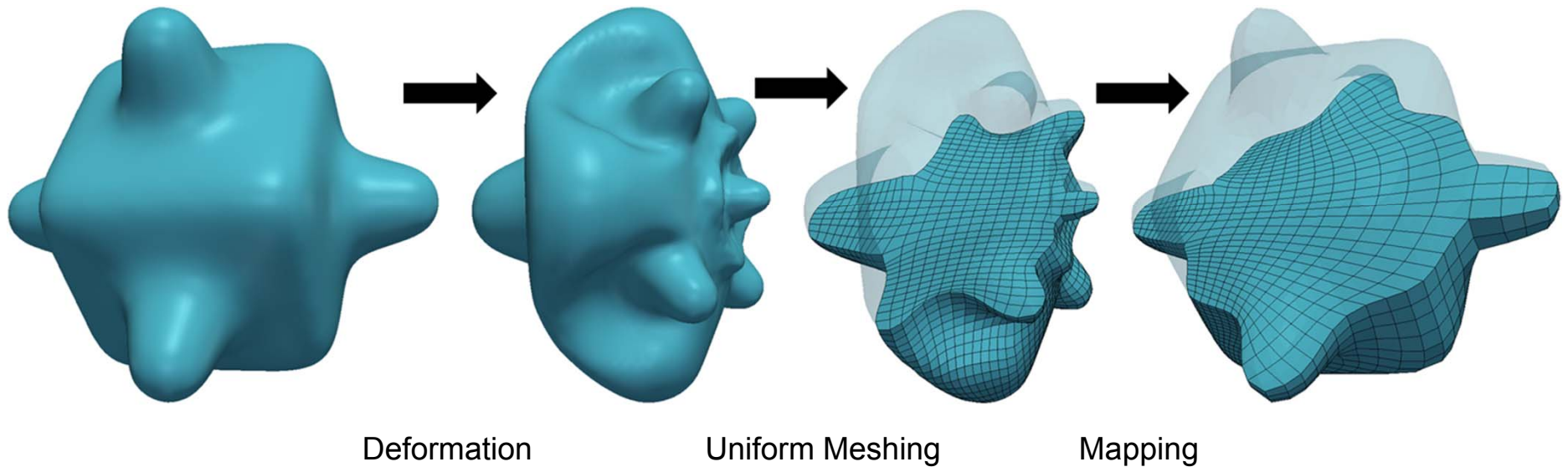
Is there a way to generate adaptive hex-meshes with simple structure?

# Inspiration - Anisotropic Quad Meshing

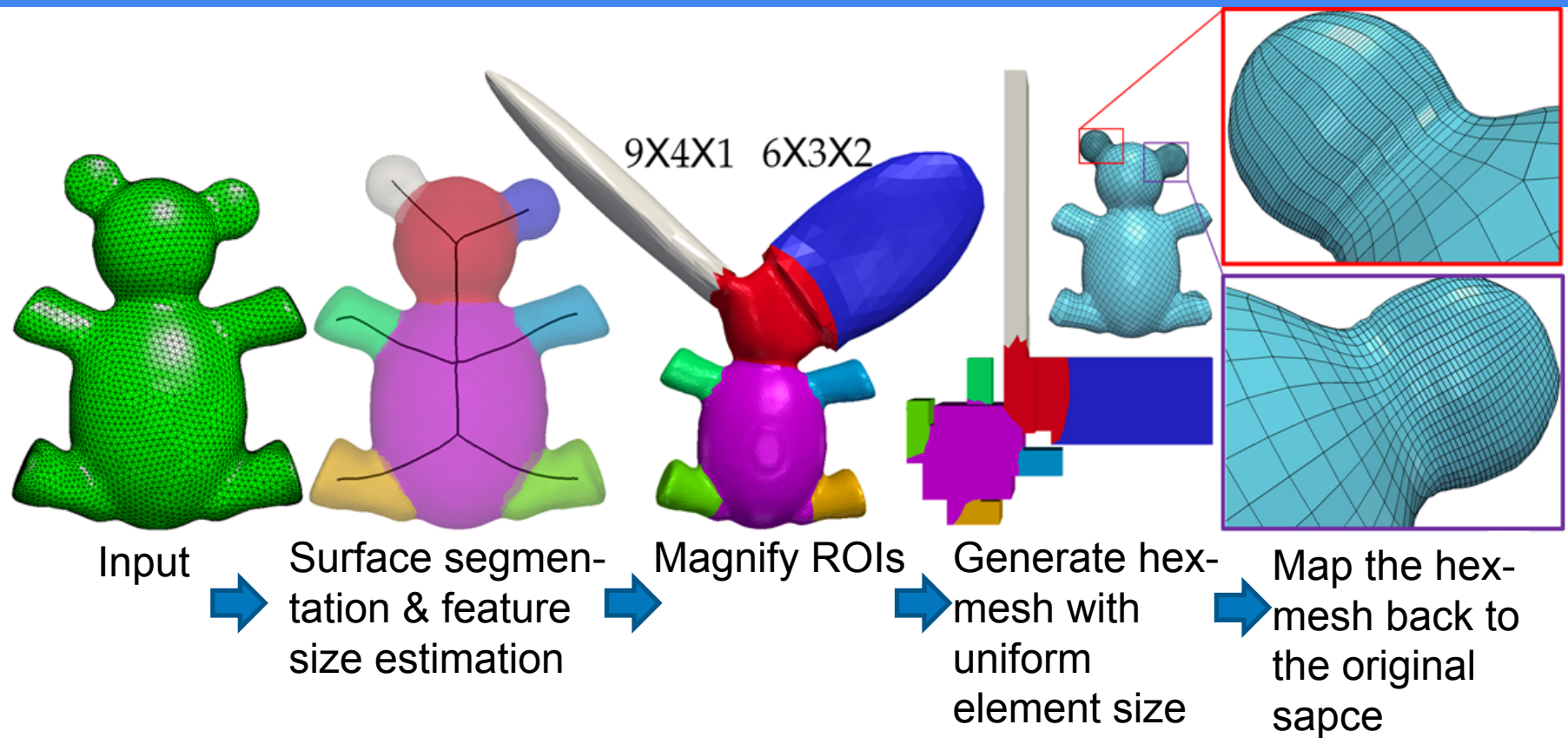


Anisotropic Quad Meshing [Panozzo et al. 2014 - Frame Fields Anisotropic and Non-Orthogonal Cross Fields]

# Our Adaption - Anisotropic Hex-Meshing

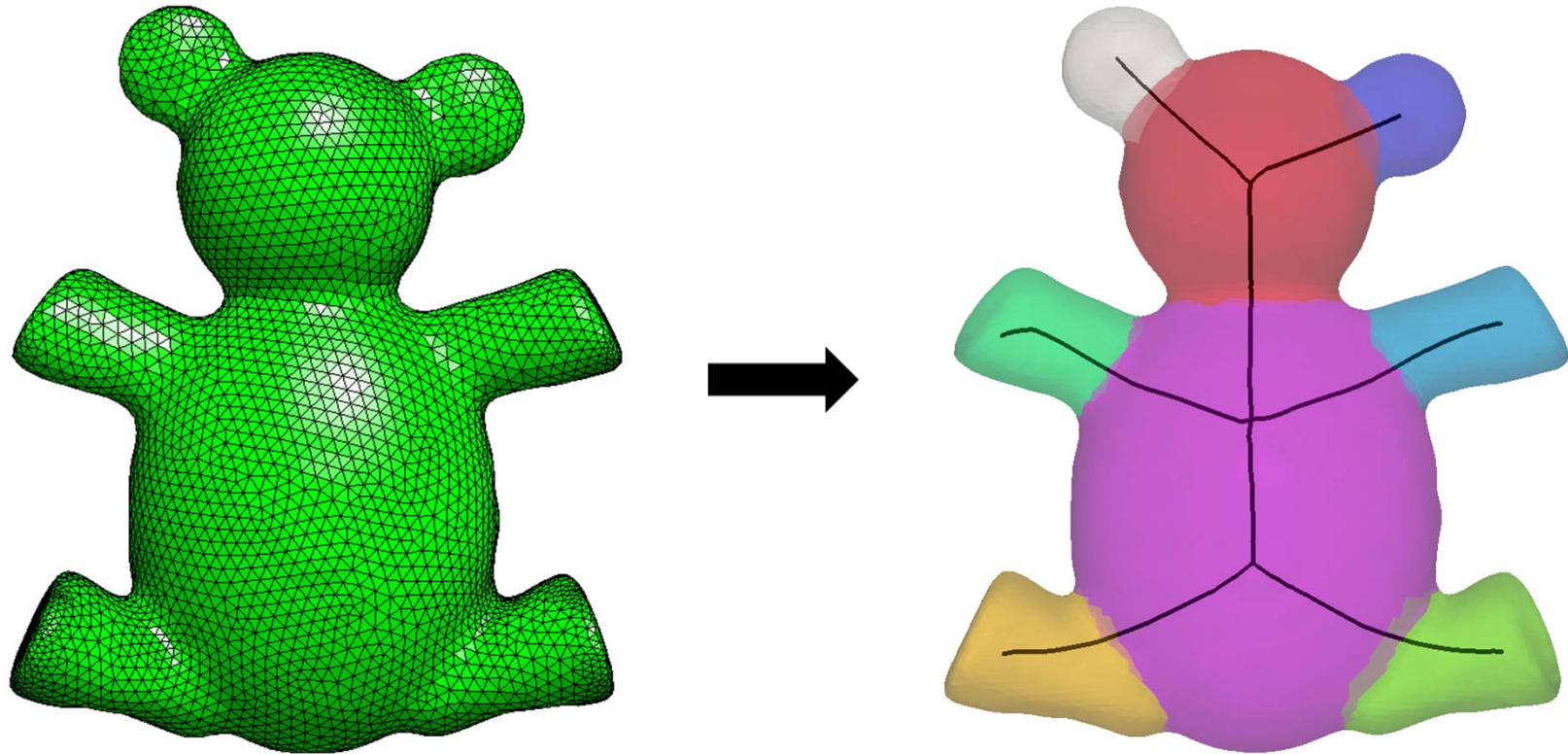


# Our Pipeline



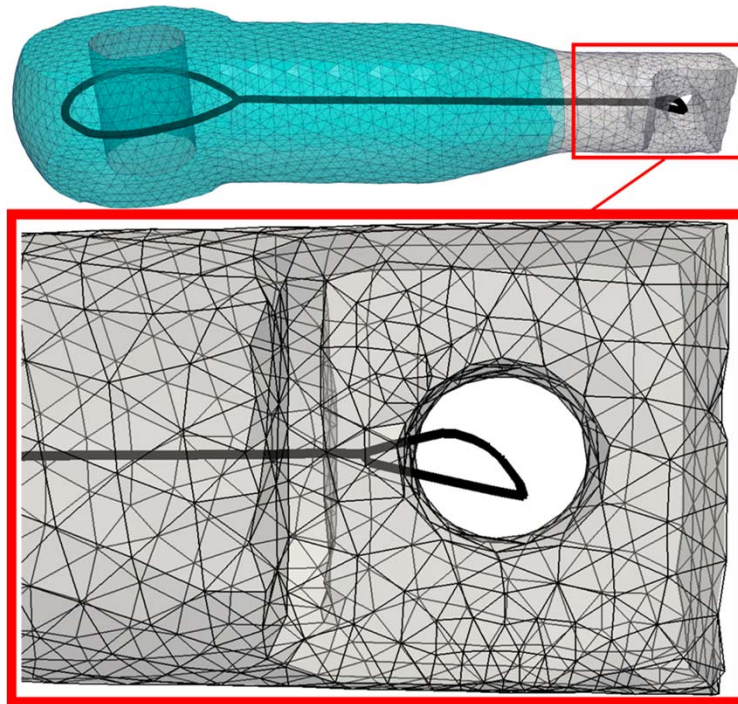
# One Realization of the Pipeline

# Segmentation through Skeletonization

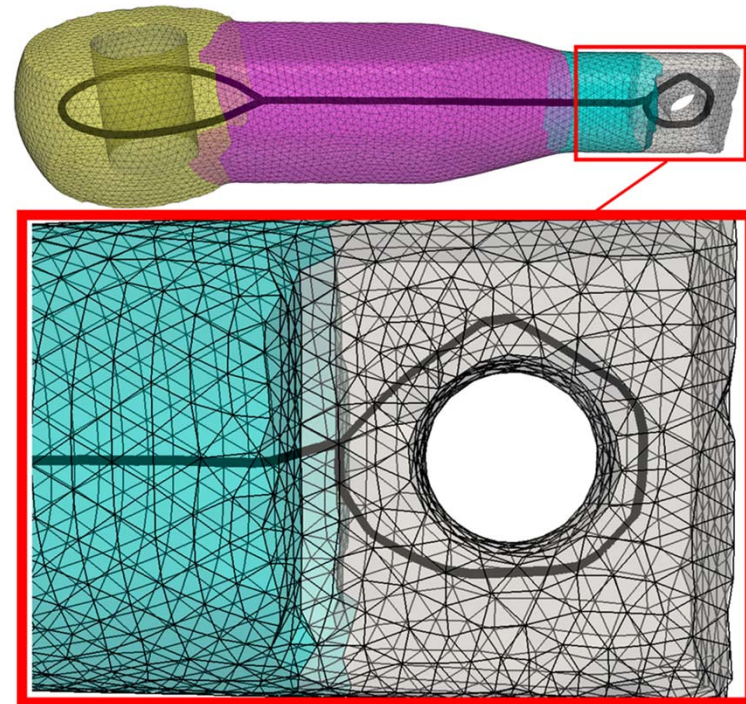


[Tagliasacchi et.al 2012 - Mean curvature skeletons]

# Input Triangle Mesh Segmentation



5k triangles



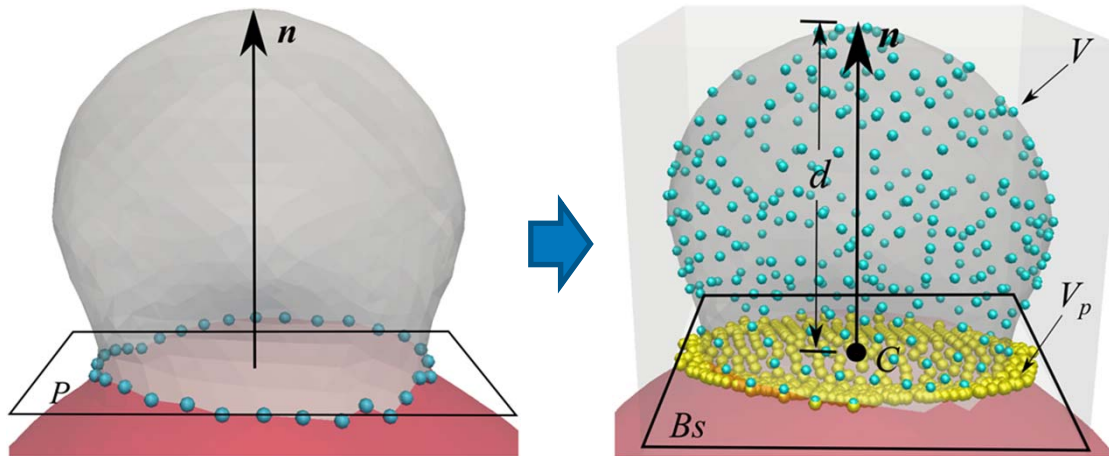
13k triangles

# Identify Regions-of-Interest (ROIs)

Segments with feature size smaller than the average feature size of the entire mesh are candidates of ROIs.

The user can also manually select ROIs from these candidates, but this is not required.

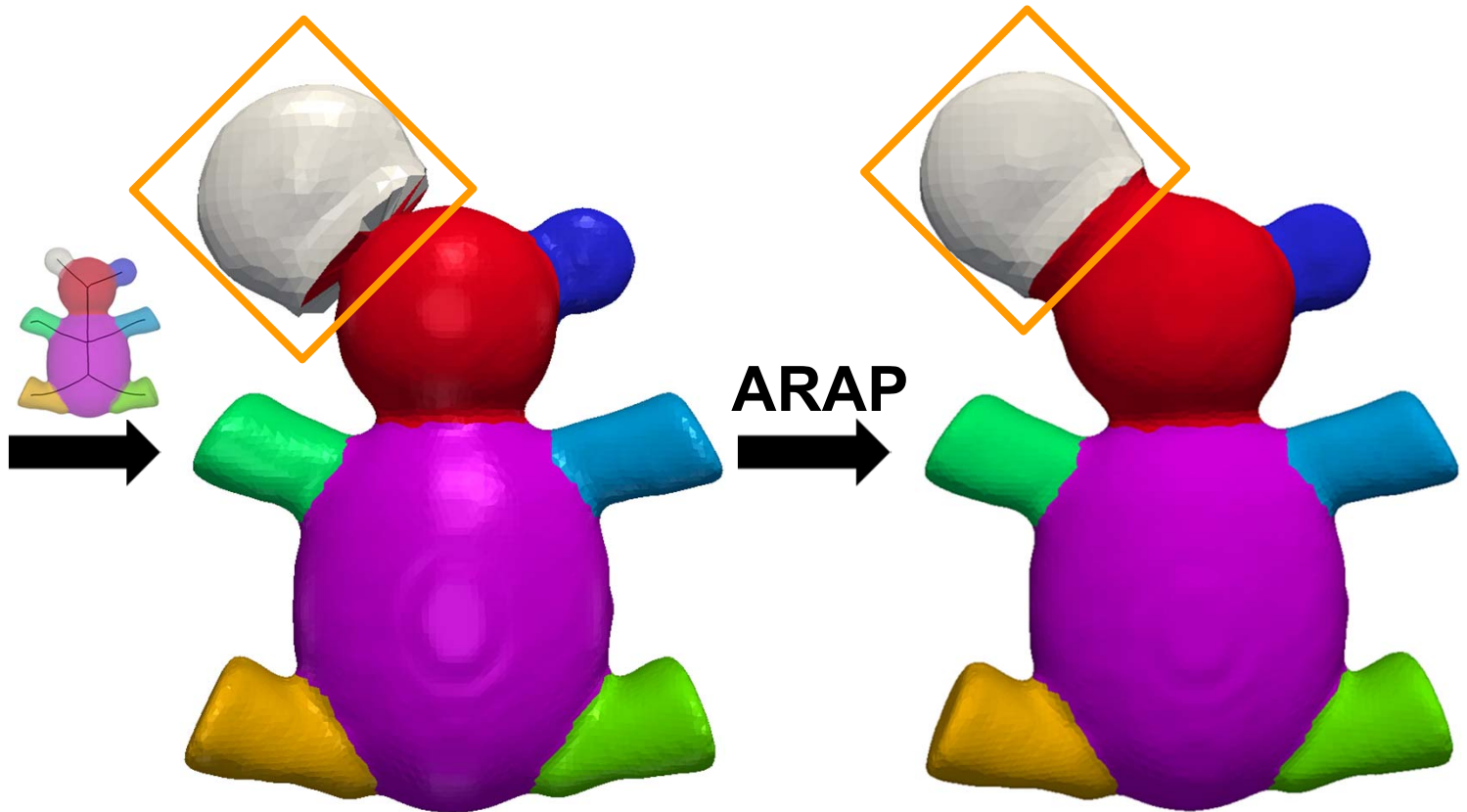
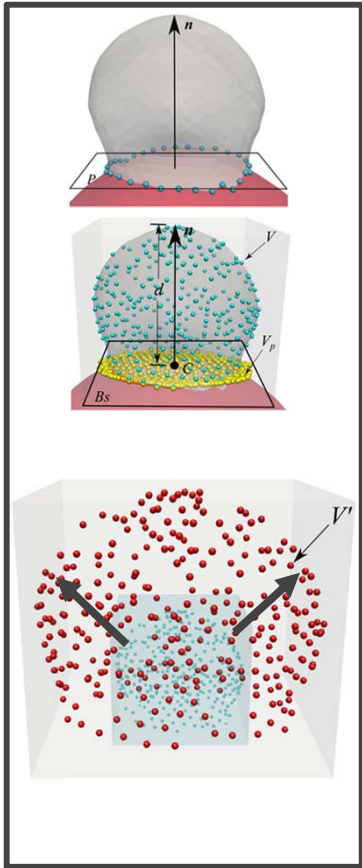
# Magnification – Determine Bounding Box



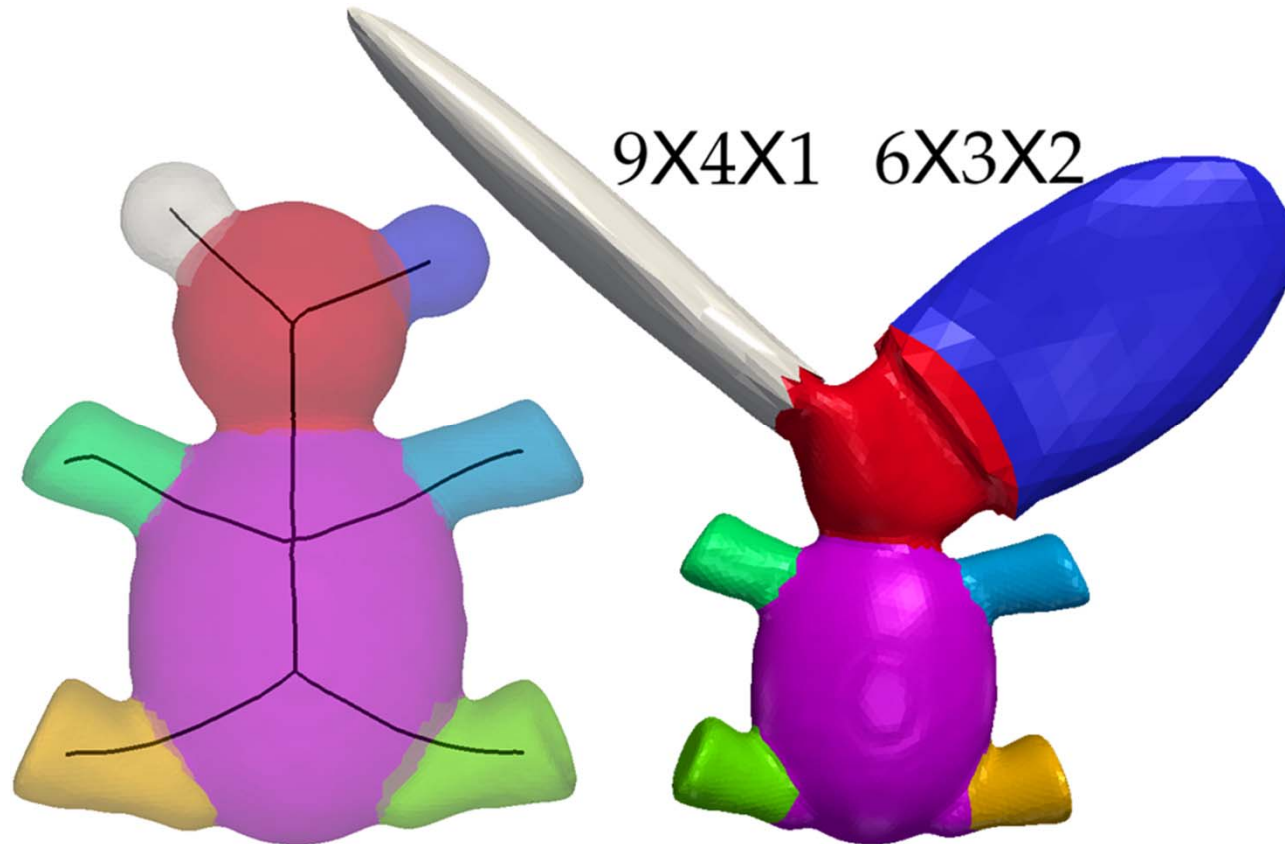
Scaling factor  $s_{avg}/s_{ROI}$

Or user-specified

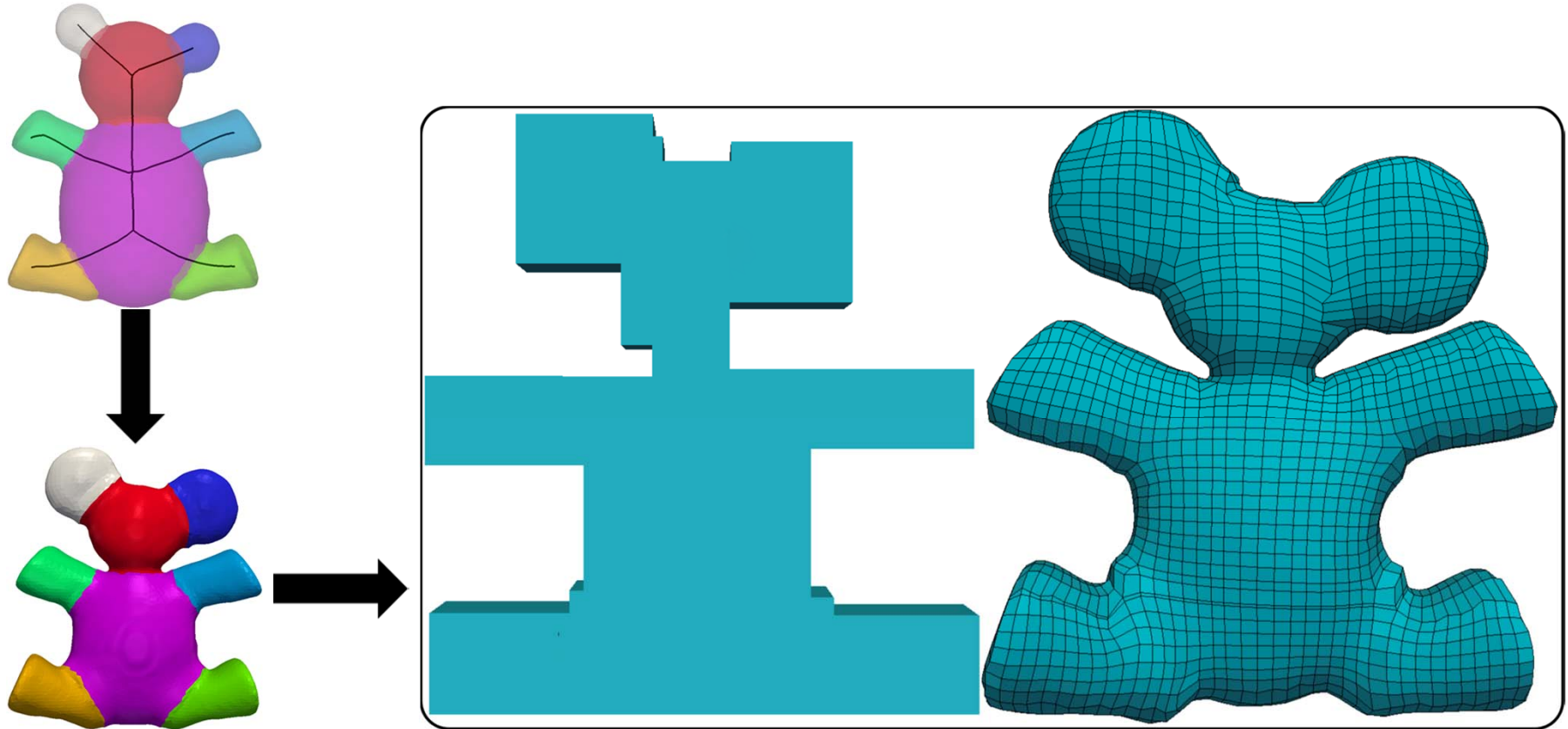
# Magnification - Surface ARAP



# Anisotropic Magnification



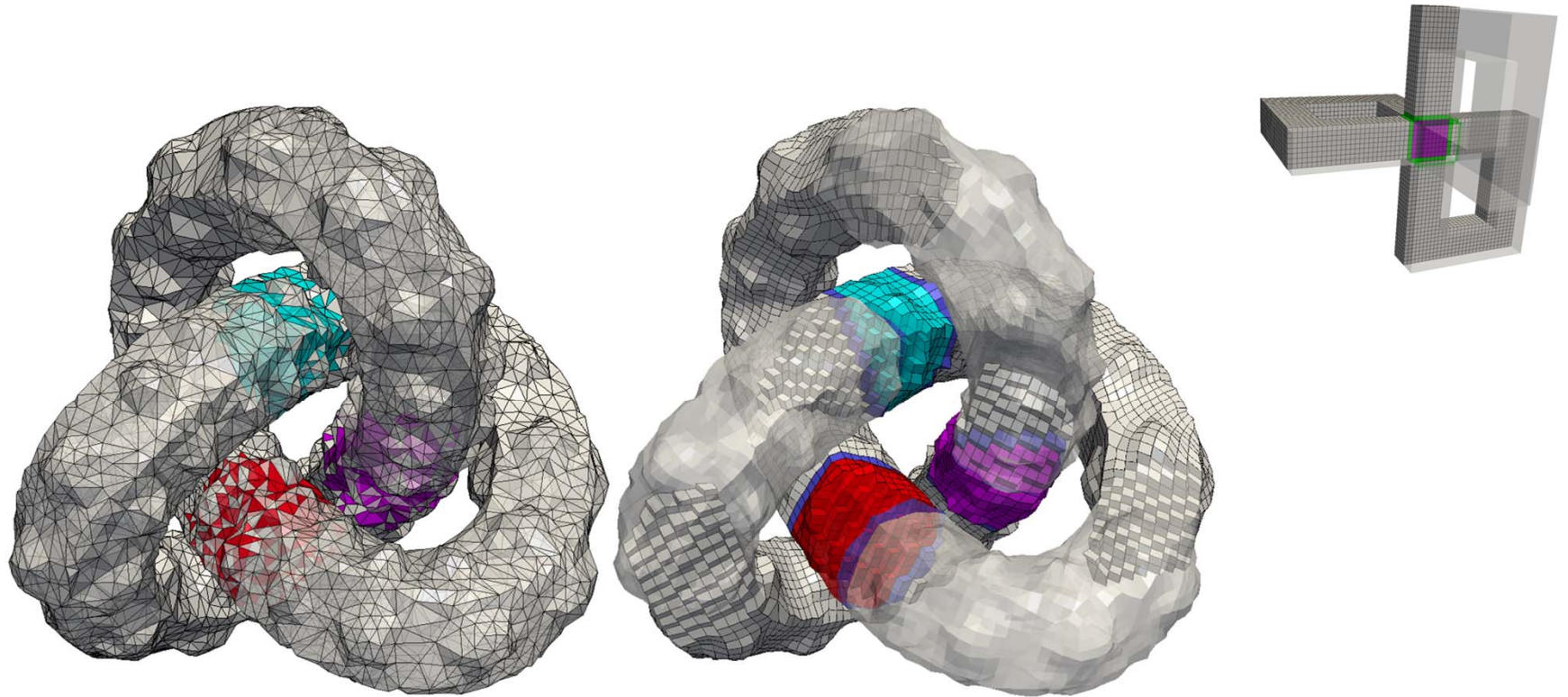
# Hex-Meshing Using L1-PolyCube



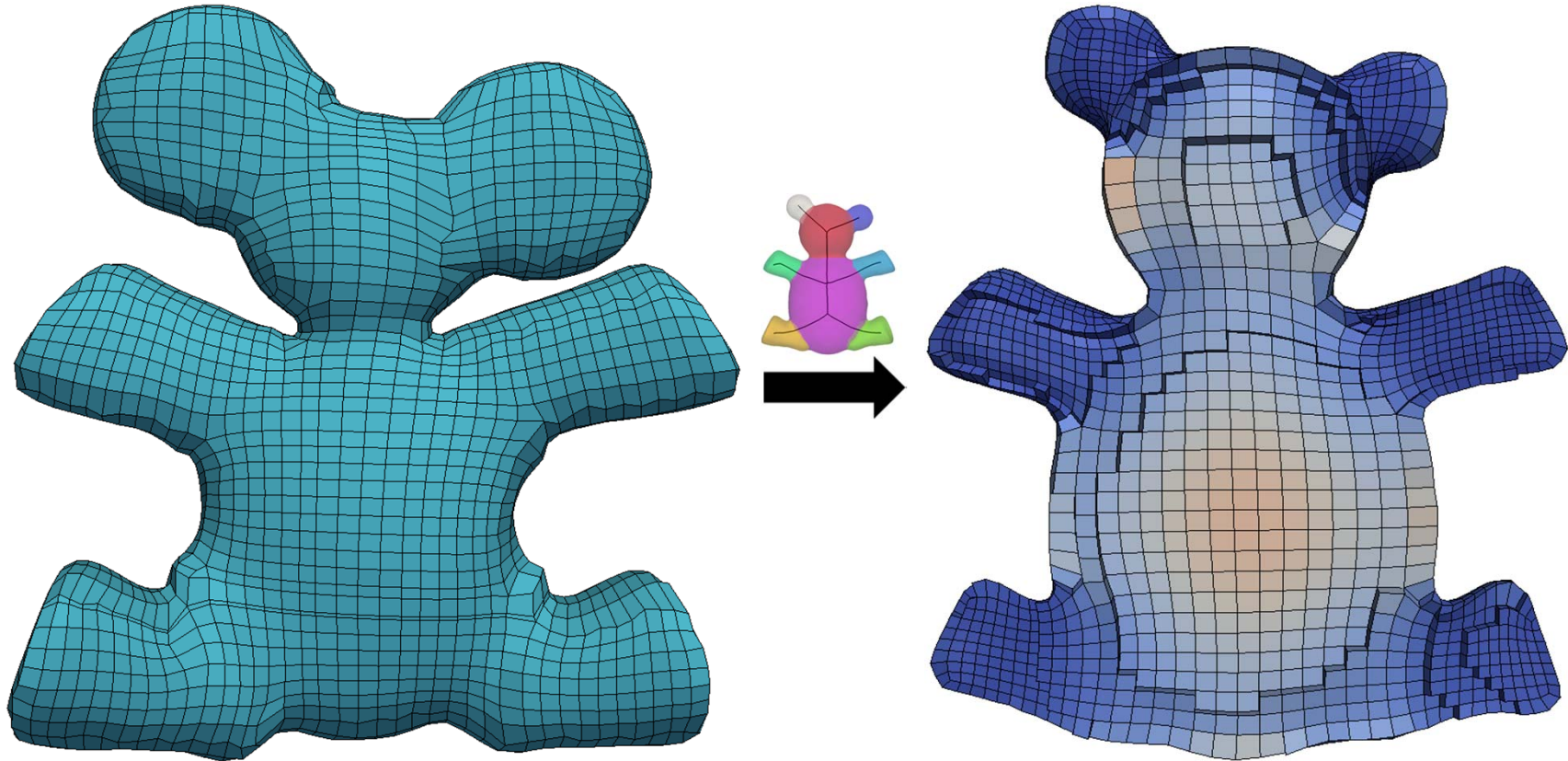
# Issues of Using Polycube Methods



# Issues of using polycube methods

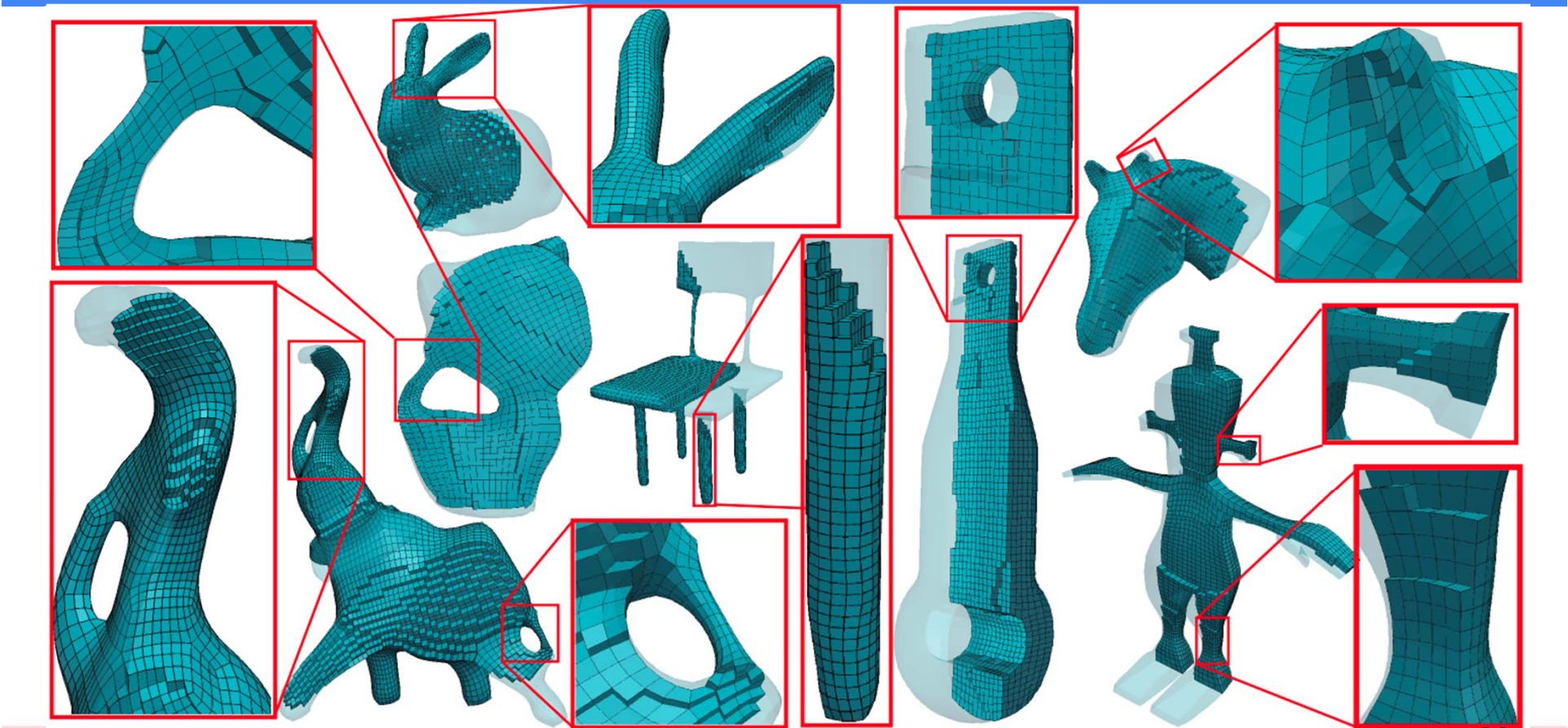


# Mapping Back to the Original Space

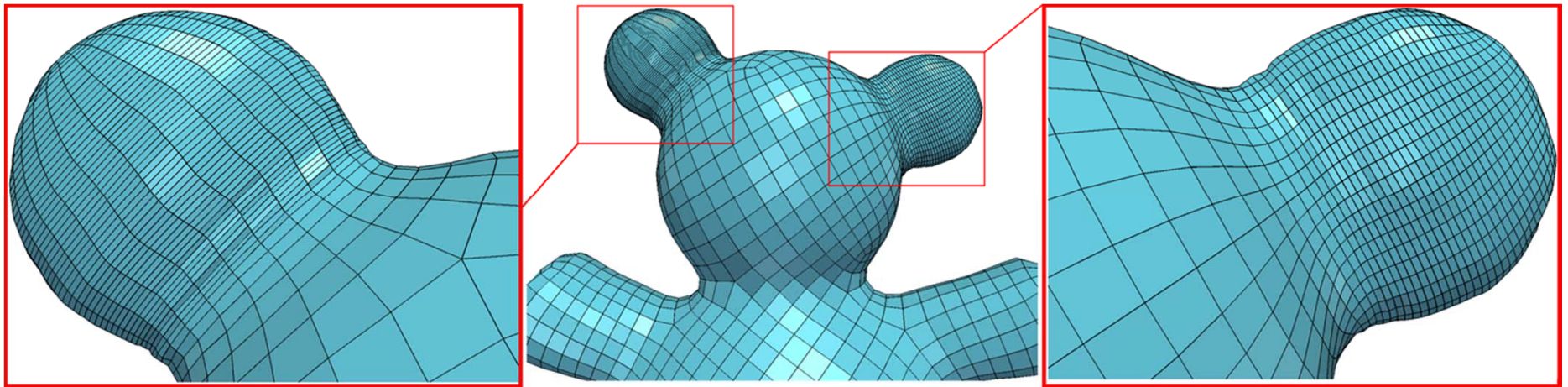
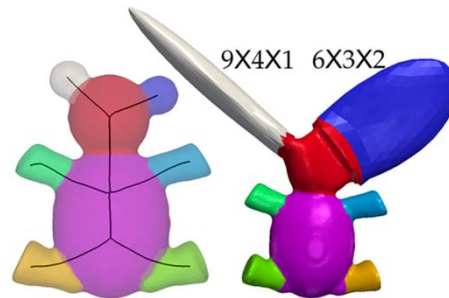


# Results

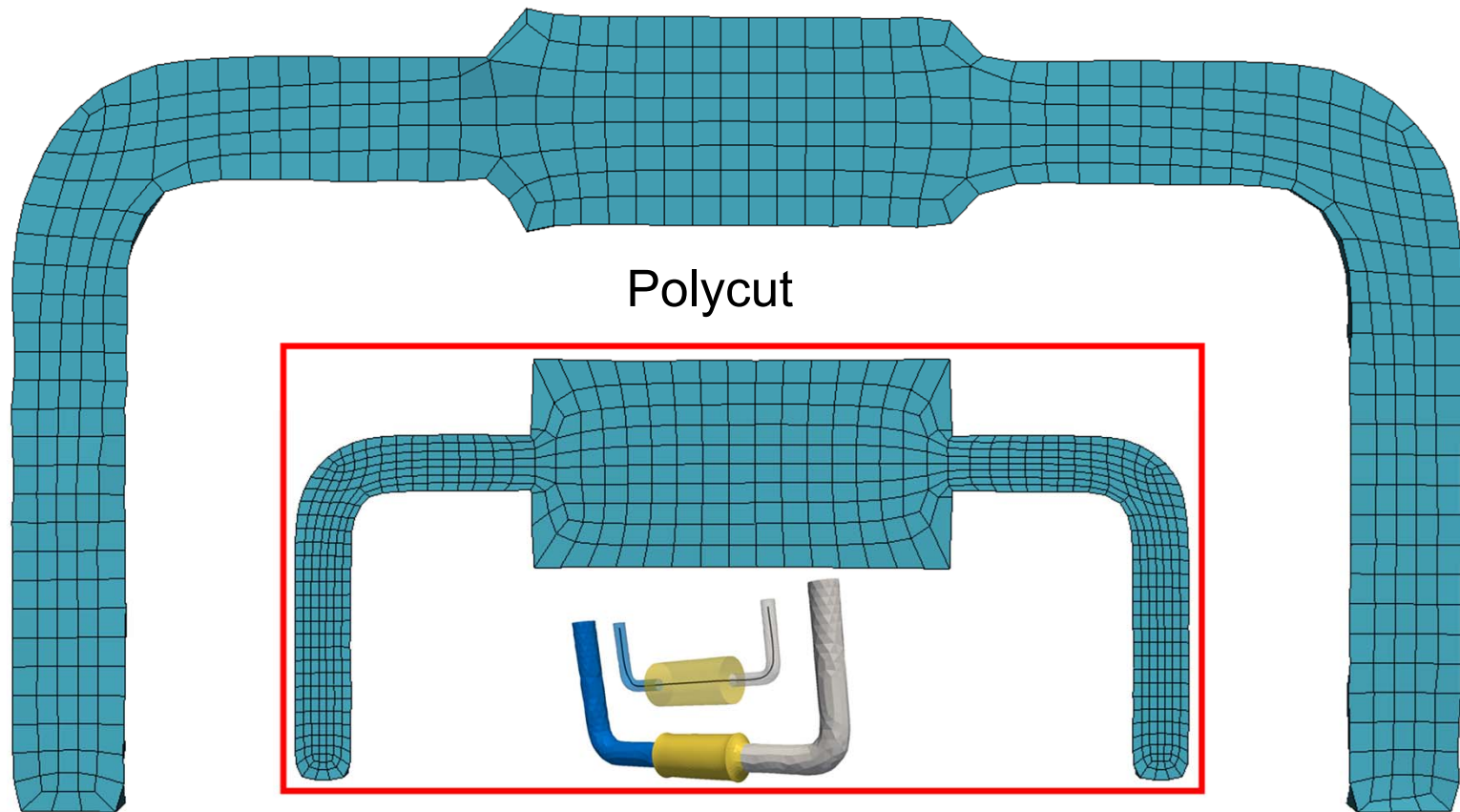
## Results



# Result - Anisotropic Meshes

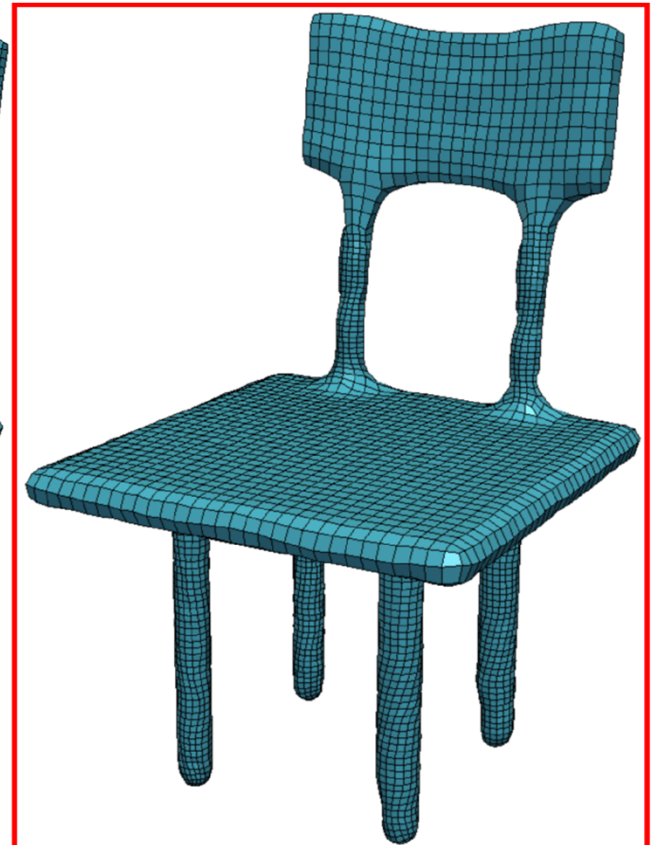
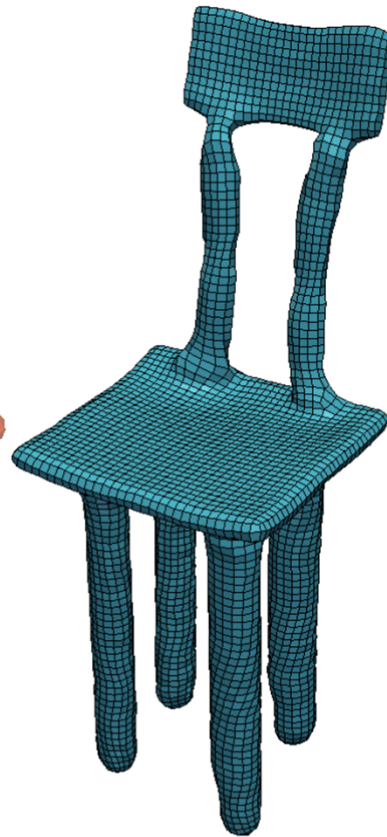
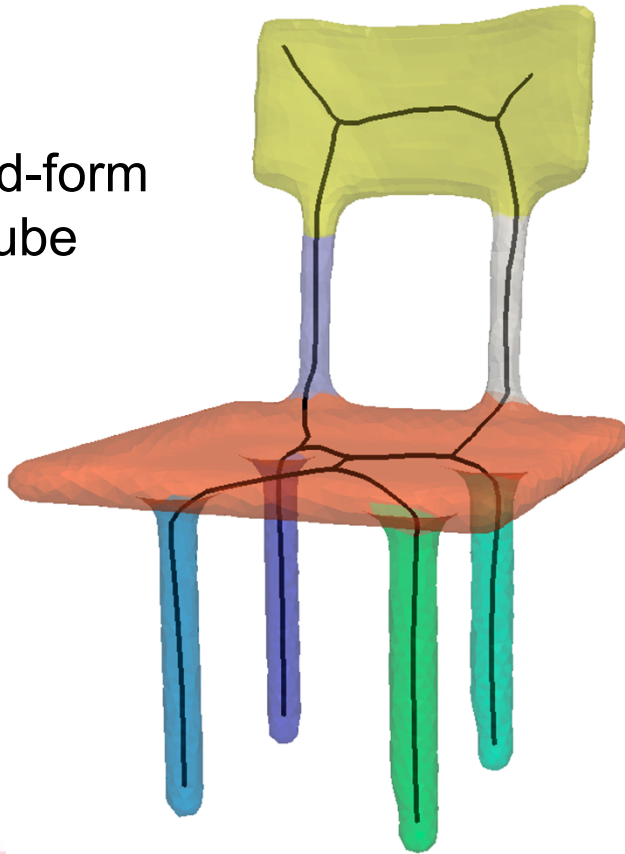


# Result via Different Hex-Meshing Techniques

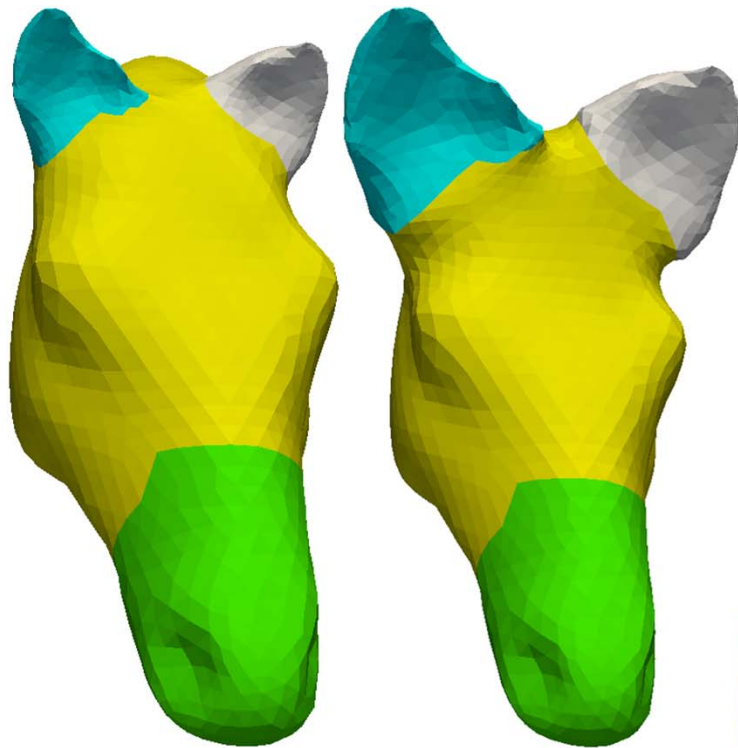


# Result via Different Hex-Meshing Techniques

Closed-form  
Polycube

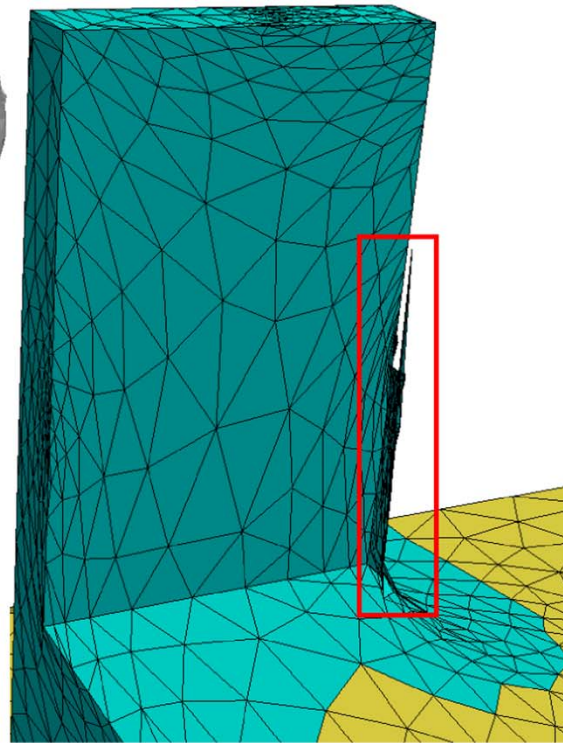


# Improve Polycubes

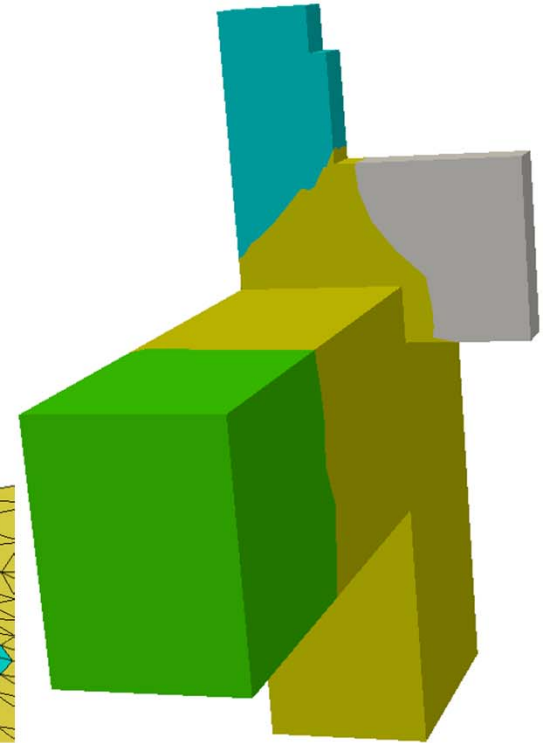


before

after

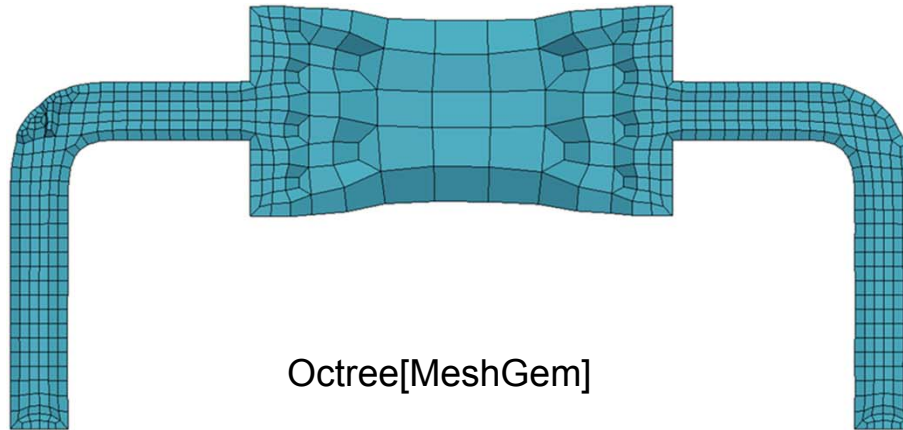


before

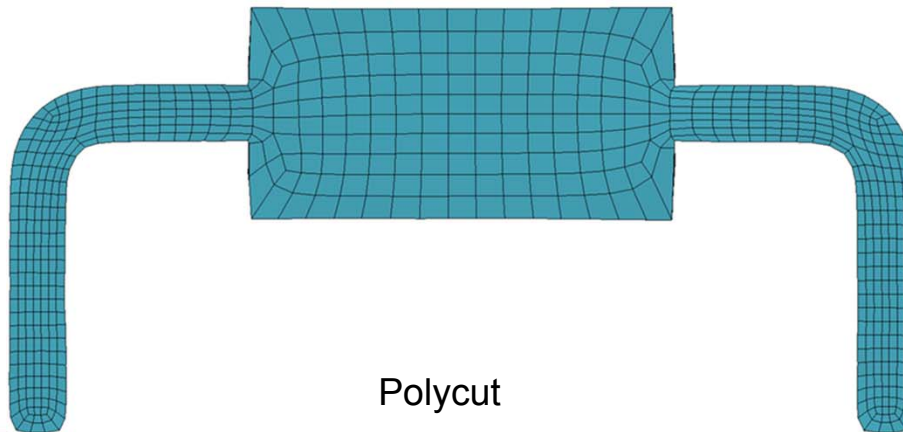


after

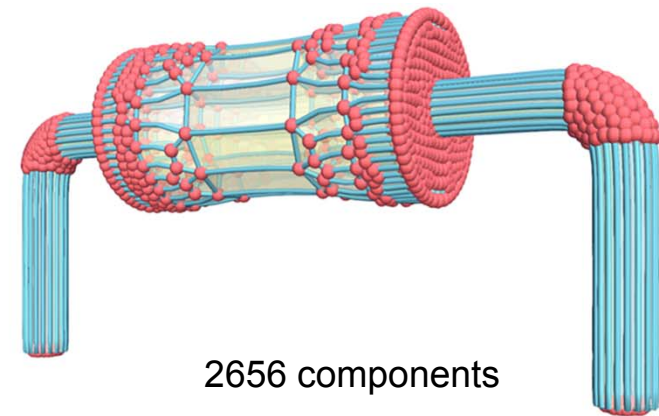
# Singularities comparison



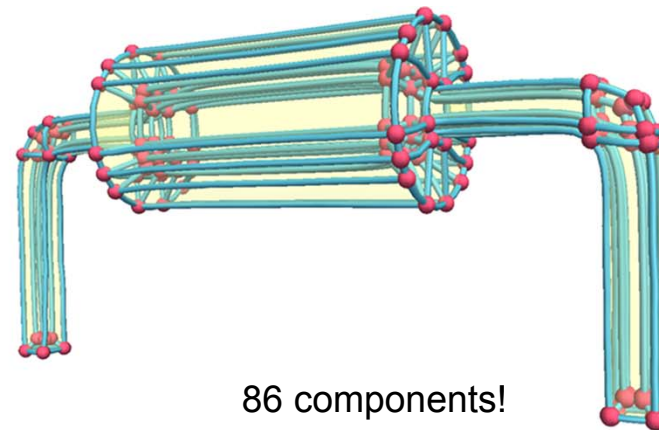
Octree[MeshGem]



Polycut

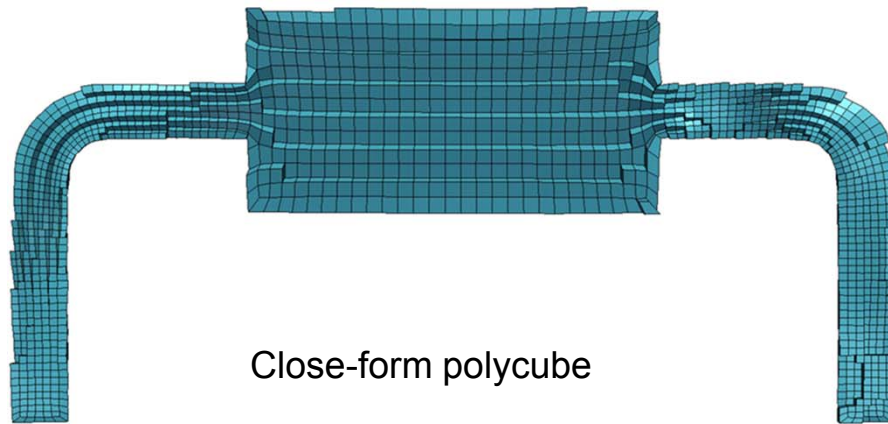


2656 components

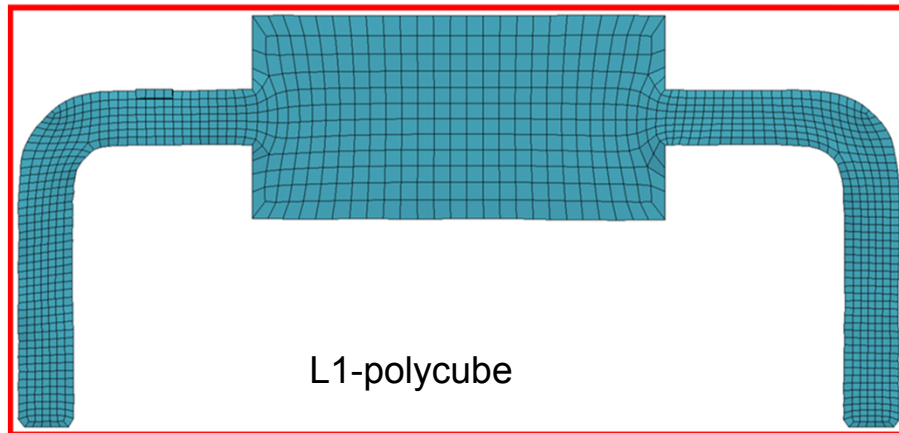


86 components!

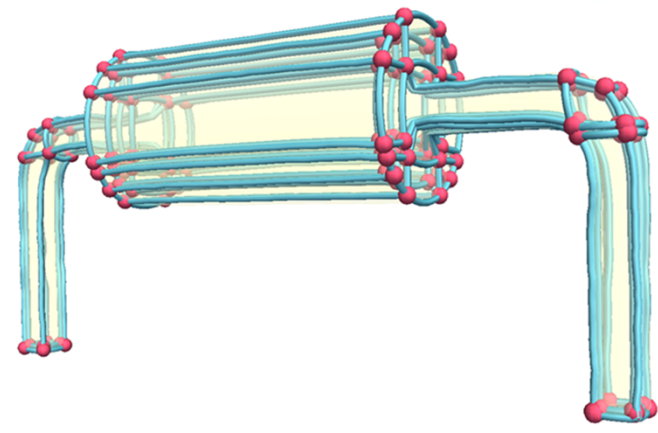
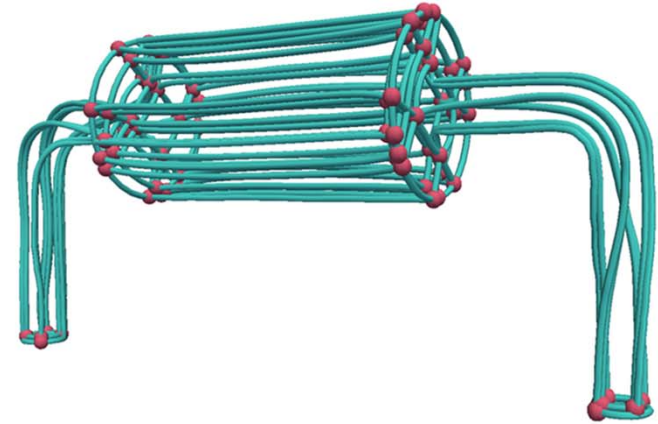
# Singularities comparison



Close-form polycube



L1-polycube



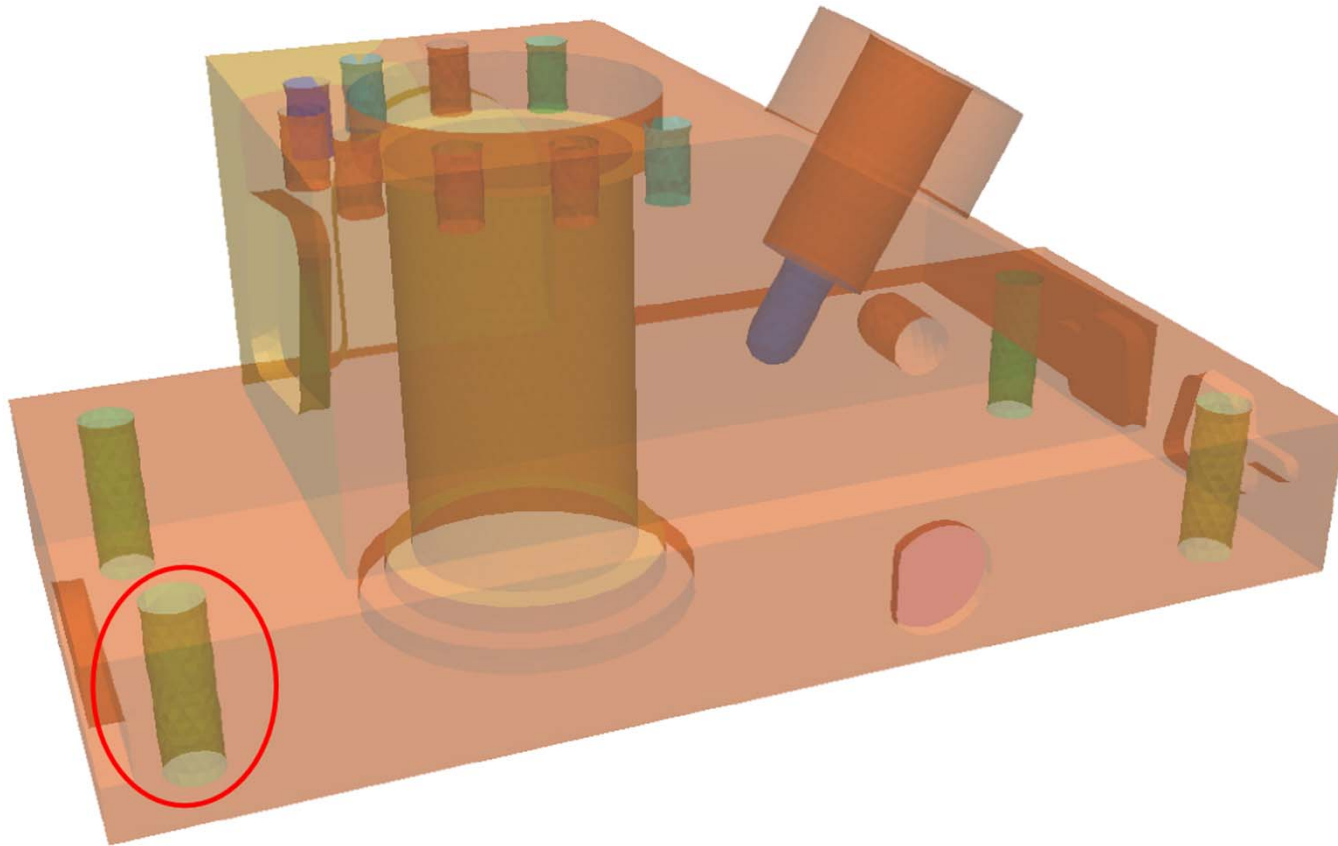
# Performance

| Model              | #Tet   | #Tri  | #Seg | #Scal | #Sin  | #Com  | #Hex  | MSJ / ASJ   | H Dis | S Time | P Time  | H Time |
|--------------------|--------|-------|------|-------|-------|-------|-------|-------------|-------|--------|---------|--------|
| bumpycube          | –      | 39936 | –    | –     | 60    | 121   | 16937 | 0.266/0.905 | 2.65  | –      | 18m27s  | 16s    |
| bear               | 98548  | 14626 | 8    | 2.2   | 188   | 467   | 7697  | 0.443/0.918 | 3.13  | 1.3s   | 209m31s | 55s    |
| bear*              | 10912  | 4588  | –    | –     | 140   | 104   | 10700 | 0.470/0.935 | 2.43  | –      | 1m53s   | 10s    |
| bunny              | 24169  | 7098  | 3    | 2.0   | 60    | 77    | 14571 | 0.275/0.900 | 2.92  | 0.5s   | 6m27s   | 19s    |
| chair              | 14163  | 7490  | 8    | 2.0   | 160   | 266   | 9659  | 0.274/0.918 | 1.81  | 0.6s   | 2m43s   | 50s    |
| chair <sup>o</sup> | 14163  | 7490  | –    | –     | 12005 | 35223 | 41664 | 0.076/0.864 | 1.08  | –      | –       | 1.2s   |
| elephant           | 119321 | 24806 | 9    | 1.8   | 526   | 5827  | 23002 | 0.242/0.887 | 1.51  | 1.9s   | 193m10s | 110s   |
| horse              | 41831  | 7506  | 4    | 2.0   | 78    | 124   | 7523  | 0.259/0.876 | 6.14  | 0.5s   | 14m19s  | 21s    |
| kitty              | 4521   | 1946  | 2    | 2.0   | 80    | 129   | 7124  | 0.291/0.887 | 2.56  | 0.2s   | 36s     | 6s     |
| pipe               | 16285  | 4442  | 3    | 2.0   | 80    | 86    | 9045  | 0.419/0.933 | 1.59  | 0.2s   | 2m37s   | 12s    |
| pipe <sup>o</sup>  | 16285  | 4442  | –    | –     | 1847  | 2656  | 5571  | 0.092/0.790 | 3.92  | –      | –       | 1.1s   |
| pipe*              | 16285  | 4442  | –    | –     | 80    | 69    | 7168  | 0.258/0.925 | 1.45  | –      | 2m32s   | 14s    |
| robot              | 27606  | 9032  | 10   | 1.9   | 196   | 598   | 8013  | 0.254/0.941 | 1.80  | 0.6s   | 6m30s   | 25s    |
| rod                | 70178  | 13818 | 4    | 1.5   | 48    | 66    | 11448 | 0.063/0.932 | 1.14  | 1.2s   | 53m53s  | 48s    |

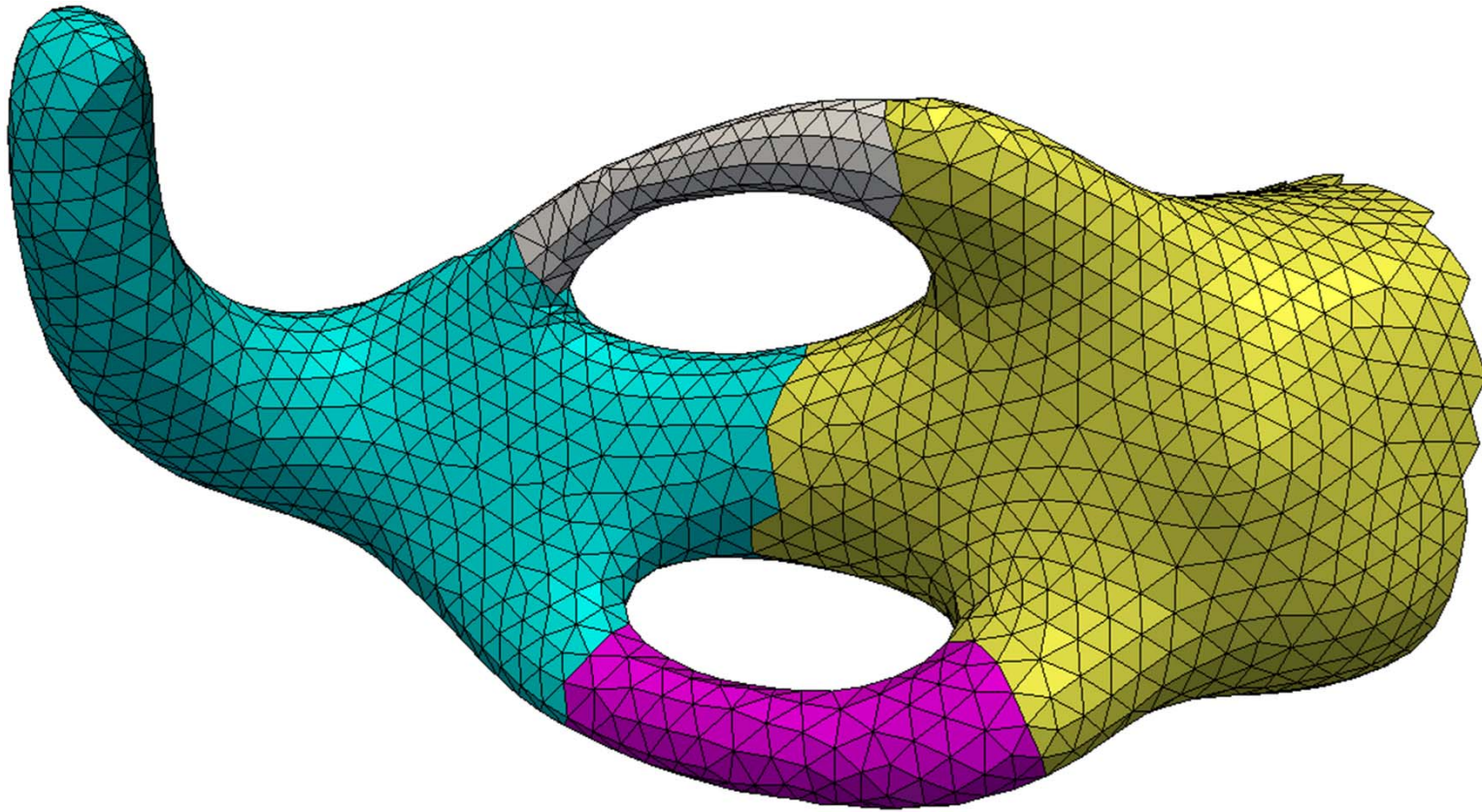
S Time, P time and H time show the timing for segmentation, L1-polycube construction, and hex-mesh extraction.

# Limitations

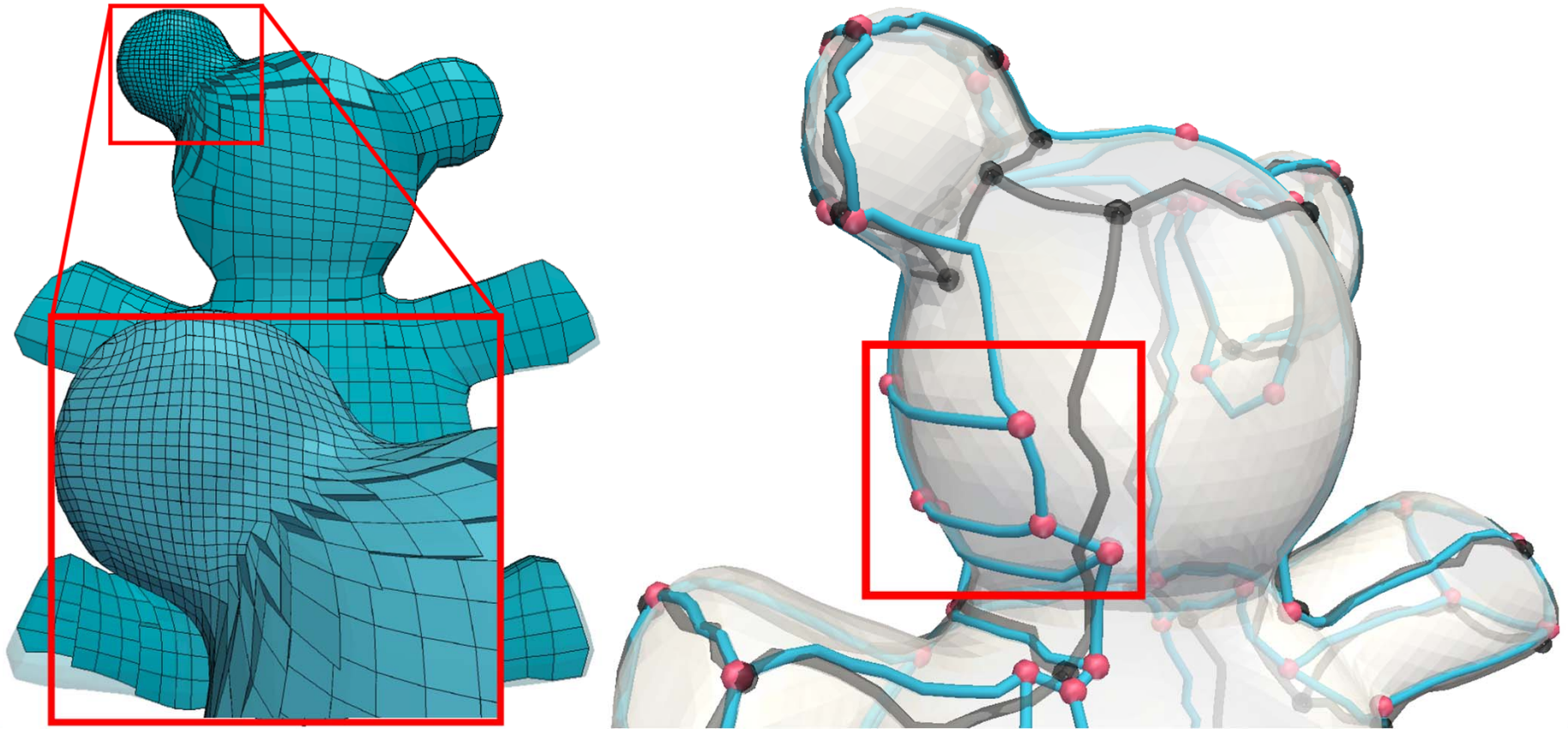
# Limitation 1



## Limitation 2



## Limitation 3



# Acknowledgment



We would like to thank Xianzhong Fang, Jin Huang, Marco Livesu, and Yang Liu for helping generate hex-meshes with their respective methods for comparison. This work was partially supported by NSF IIS-1553329.

*Thank you for  
your attention!*