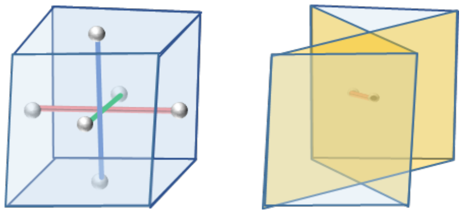
Evaluating Hex-mesh Quality Metrics via Correlation Analysis

While analytical expressions of most of the quality metrics in Table 1 of the paper can be found in [SEK*07], we list below the formulas of metrics that are highly related to the condition number. We also describe the quality metric of Dimension that is not obvious to derive from [SEK*07].

Notations. $\vec{X_i}$ is a principal axis of a hexahedral element and L_i is the magnitude of $\vec{X_i}$, where $i \in \{0, 1, 2\}$.



A hexahedron has three principal axes, each of which is pointing to the center of a face from the center of its opposite face (left of the figure). The cross-derivative of $\vec{X_{ij}}$ is pointing from the center of one diagonal face to the center of the other one, where the diagonals of the two diagonal faces are on the two hex-faces of X_k (right of the figure). L_{ij} denotes the length of $\vec{X_{ij}}$.

Diagonal is the ratio of the minimum diagonal length to the maximum diagonal length of a hexahedron. A hexahedron has four diagonals.

Edge Ratio is the ratio of the longest edge to the shortest edge of a hexahedron. A hexahedron has twelve edges.

Maximum Edge Ratio measures the largest aspect ratio of two principle axes, which is computed as $\max\{\frac{L_0}{L_1}, \frac{L_1}{L_0}, \frac{L_0}{L_2}, \frac{L_2}{L_0}, \frac{L_1}{L_2}, \frac{L_2}{L_1}\}$.

Skew measures the degree to which a pair of normalized principle axes are parallel, which is computed as$\max\{|\frac{\vec{X_0}}{||\vec{X_0}||} \cdot \frac{\vec{X_1}}{||\vec{X_1}||}|, |\frac{\vec{X_0}}{||\vec{X_0}||} \cdot \frac{\vec{X_2}}{||\vec{X_2}||}|, |\frac{\vec{X_1}}{||\vec{X_1}||} \cdot \frac{\vec{X_2}}{||\vec{X_2}||}|\}$.

Taper measures the maximum ratio of a cross-derivative to its shortest associated principal axis, which is computed as $\max\{\frac{L_{01}}{min\{L_0,L_1\}}, \frac{L_{02}}{min\{L_0,L_2\}}, \frac{L_{12}}{min\{L_1,L_2\}}\}$.

Dimension is computed based on the finite element gradient operator and is defined as $\mathcal{D} = \dfrac{V}{2\sqrt{B \cdot B}}$ [TF89], where V is the volume of the hexahedron and B is the discrete gradient operator evaluated over the hexahedral element. B is computed as $B = [B_{ix}, B_{iy}, B_{iz}]^T$ and $i \in [0, 1, 2, 3, 4, 5, 6, 7]$. Given that $B_0x = y_2((z_6 - z_3) - (z_4 - z_5)) + y_3(z_2 - z_4) + y_4((z_3 - z_8) - (z_5 - z_2)) + y_5((z_8 - z_6) - (z_2 - z_4)) + y_6(z_5 - z_2) + y_8(z_4 - z_5)$, other terms of B_{ix} can be evaluated using the same formula by permuting the nodes according to Table 1 and subsequently, B_{iy} and B_{iz} are computed by permuting the coordinate axes according to Table 2.

Table 1: Nodal permutations

0	1	2	3	4	5	6	7
1	2	3	0	5	6	7	4
2	3	0	1	6	7	4	5
3	0	1	2	7	4	5	6
4	7	6	5	0	3	2	1
5	4	7	6	1	0	3	2
6	5	4	7	2	1	0	3
7	6	5	4	3	2	1	0

Table 2: Coordinate axes permutations

x	y	z
y	z	x
z	x	y