Scalar Field Visualization: Topology-based Approach
Review of Scalar Field Visualization

• 2D scalar fields
  – (Direct) color plots
  – Iso-contouring

• 3D scalar fields
  – Direct volume rendering (DVR)
  – Iso-surfacing

- Diagram:
  - Space
  - Scalar function
  - Data Value
  - Polygonal Geometry
  - Rendered Image
Pros vs. Cons

• **Color plots and DVR**
  
  – **Pros.**
    
    • Provide the information of the whole data
    • Different from color plots, DVR typically classifies the volume into different components
  
  – **Cons.**
    
    • Lack of details

• **Iso-contouring and Iso-surfaceing**
  
  – **Cons.**
    
    • Only some subsets of the data
    • Sensitive to noise
  
  – **Pros.**
    
    • Information reduction if done properly
    • Provide flexible rendering options
    • Show the detailed structure and patterns of the field.

• **Which one should we choose is application-dependent**
Why Spatial Segmentation

• Classifying spatial domain based on the scalar values is always desired.
  – Reduce the information overloaded
  – Identify unique features and properties

• Classification in DVR
  – Transfer function design

Are there any other region segmentation method?
There are Other Features

• Other than “same material has similar property and different materials have different properties”, there are some features that can be defined by the scalar values within a small neighborhood of certain spatial position

• Consider a height field, what places do people care about?
1D Case

- Let us get back to the simple 1D case
1D Case

• Let us find out the local minimum/maximum
  Zero derivatives
1D Case

- They partition the domain into monotonic regions
How About 2D Case?

Pre-image of an iso-value: Iso-contours
We Want to Extract Similar Information

Q: Which iso-contours are interesting?
Q: Summarize the evolution of iso-contours?
Topology

- These local minimum and maximum are called “critical points” of the scalar functions.

- Their connection forms the topology of the scalar field, which provides a partition scheme of the spatial domain.

- Each segment has the equivalent homogeneous behavior, e.g. monotonic for 1D case.

- This is similar for 2D and 3D scalar fields
Scalar Field Analysis

• Here is a more formal definition

• Given a scalar field \( f \)
  – Gradient vector
    \[ \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{bmatrix} \]
  – When not zero
    – Points in the direction of quickest ascend
    – Always perpendicular to the iso-contours (or level sets) of \( f \)

• If \( \nabla f (p) = 0 \),
  – \( p \) is a critical point
  – \( f(p) \) is a critical value
Scalar Field Analysis

• A critical point \( p \) is isolated if there exists a neighborhood of \( p \) such that \( p \) is the only critical point in the neighborhood

• Classification of fundamental critical points in 2D

Local minima

Saddle

Local maxima
Scalar Field Analysis

• A function is a **Morse function** if it is smooth and all of its critical points are isolated and non-degenerate
  – Typically a good assumption for scientific data
  – A non-Morse function can be made Morse by adding small but random noise
Example – dunking a doughnut

\[ f(p) = z \] (height function)

Shape analysis is a special case of scalar field analysis.
Example – dunking a doughnut
Example – dunking a doughnut
Example – dunking a doughnut
Example – dunking a doughnut
Example – dunking a doughnut
How Does it Work?
How Does it Work?

Level sets obtaining by sweeping along Z direction
How Does it Work?

Critical Points
Reeb Graph
Reeb Graph

- Vertices of the graph are critical points
- Arcs of the graph are connected components (cylinders in domain) of the level sets of $f$, contracted to points

- Two-step algorithm
  - Locate critical points
  - Connect critical points
Reeb graphs and genus

• The number of loops in the Reeb graph is equal to the surface genus

• To count the loops, simplify the graph by contracting degree-1 vertices and removing degree-2 vertices
Some More Reeb Graph Examples
Additional Reading for Reeb Graph


Morse-Smale Complex
Morse-Smale Complex-1D
Morse-Smale Complex-1D

Ascending manifold
Origin = minimum
Morse-Smale Complex-1D

Descending manifold
Dest = maximum
Morse-Smale Complex-1D

Morse-Smale cell
Origin = minimum
and
Dest = maximum
Morse-Smale Complex-2D
Morse-Smale Complex-2D
Morse-Smale Complex-2D

Descending Manifold
Morse-Smale Complex-2D

Cell of the Morse-Smale complex
Morse-Smale Complex-2D

Decomposition into monotonic regions
Combinatorial Structure 2D

• Nodes of the MS complex are exactly the critical points of the Morse function
• Saddles have exactly four arcs incident on them

All regions are quads
• Boundary of a region alternates between saddle-extremum
• 2k minima and maxima
Combinatorial Structure 2D

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3D MS Complex cell
Detection of Critical Points

Local computation, very efficient

Regular point

Lower link is a hemisphere / topological disk
Detection of Critical Points

Lower link is empty or a sphere
Detection of Critical Points

Lower link consists of two caps or an annular ring
Detection of Critical Points

3D saddles can have two distinct configurations

1-saddle  2-saddle
Applications

Peak-valley decomposition
Applications

Molecular surface segmentation
Applications

Rayleigh-Taylor turbulence analysis
Applications

Quadrangulation of surfaces
Additional Reading of MS Complexes


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