What is an iso-contour?
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A set of points in the data that have the same scalar values

What are the advantages of iso-contouring?
Scalar Field Visualization – 3D

Cutting Planes & Iso-surfacing

Goal: know the simple cutting plane based visualization and the construction of iso-surfacing
3D Cut Planes
Reduce 3D volume to 2D slices
Here, we focus on axis-aligned cut planes, assuming the 3D volume is also axis-aligned.
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Now let us look at z dimension.
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Now let us look at z dimension.

z = 0 defines an XY planes with all voxels whose z index is zero.
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Let us concern with a uniform volume (or 3D image), which consists of individual voxels. Its dimension is $NX \times NY \times NZ$.

Now let us look at the $z$ dimension with different $z$ values (indices), you get different $XY$ planes.
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Now let us look at the \(z\) dimension.

This shows an \(XY\) cut plane (perpendicular to \(z\)) can be constructed by specifying a \(z\) value between \([0, NZ-1]\).
In VTK

Use the following to get the dimension of the 3D image data or structured grid

```python
dim = reader.GetOutput().GetDimensions()
```

# Create a mapper and assign it to the corresponding reader
xy_plane_Colors = vtk.vtkImageMapToColors()
xy_plane_Colors.SetInputConnection(reader.GetOutputPort())
xy_plane_Colors.SetLookupTable([you color look up table])
xy_plane_Colors.Update()
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# Create an **image actor** for the XY plane
xy_plane = vtk.vtkImageActor()
xy_plane.GetMapper().SetInputConnection(xy_plane_Colors.GetOutputPort())
xy_plane.SetDisplayExtent(0, dim[0]-1, 0, dim[1]-1, current_zID, current_zID)
# Current_zID is a user-input integer within the range of [0, zdim-1]
In VTK

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```

YZ and XZ cut planes can be similarly added!!!

This is one task of your assignment 3.
You also need to play with the transfer function for the color plots shown in the individual cut planes
Trilinear Interpolation

This is useful, for example, if we have passed an oblique cutting plane through a 3D mesh of points and are trying to interpolate scalar values from the 3D mesh to the 2D plane.

\[ S(t, u, v) = (1-t)(1-u)(1-v)S_0 + t(1-u)(1-v)S_1 + (1-t)u(1-v)S_2 + tu(1-v)S_3 + (1-t)(1-u)vS_4 + t(1-u)vS_5 + (1-t)uvS_6 + tuvS_7 \]
 Iso-surfacing
Iso-Surfaces: Applications

A contour line is often called an *iso-line*, that is a line/curve of equal value. When hiking, for example, if you could walk along a single contour line of the terrain, you would remain at the same elevation.

An iso-surface is the same idea, only in 3D. It is a surface of equal value.

Sometimes the shapes of the iso-surfaces *have a physical meaning*, such as bone, skin, different layers of earth etc. (e.g., the left example above). Sometimes the shape just *helps turn an abstract notion into something physical* to help us gain insight (e.g., the other two examples).
(Pseudo-) Iso-surface Construction: Wireframe Iso-surface
Here’s the situation: we have a 3D grid of data points. At each node, we have an X, Y, Z, and a scalar value S. We know the Transfer Function. We also have a particular scalar value, $S^*$, at which we want to draw the iso-surface(s).
Wireframe Iso-Surfaces

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Once you have done Marching Squares for contour lines, doing wireframe iso-surfaces is amazingly easy.

The strategy is that you pick your $S^*$, then draw $S^*$ contours on all the parallel $XY$ planes. Then draw $S^*$ contours on all the parallel $XZ$ planes. Then draw $S^*$ contours on all the parallel $YZ$ planes. And, then you’re done.

What you get looks like it is a connected surface mesh, but in fact it is just independent curves. It is easy to program (once you’ve done Marching Squares at least), and looks good. Also, it’s fast to compute.
Overall Logic for a Wireframe Iso-Surfacing

for( k = 0; k < numZ; k++ )
{
       //Compute Marching Squares on the plane XY_k
}

for( i = 0; i < numX ; i++ )
{
       //Compute Marching Squares on the plane YZ_i
}

for( j = 0; j < numY; j++ )
{
       //Compute Marching Squares on the plane XZ_j
}

....
Render all the obtained contours
Iso-surface Construction: Marching Cubes

Similar to Marching Squares, we go through individual cubes to construct a patch of the iso-surface
• For simplicity, we shall work with zero level \((s^* = 0)\) iso-surface, and denote

There are **EIGHT** vertices, each can be positive or negative - so there are \(2^8 = 256\) different cases!
These two are easy!

There is no portion of the iso-surface inside the cube!
Iso-surface Construction - One Positive Vertex - 1

Intersections with edges found by inverse linear interpolation (as in iso-contouring)
Joining edge intersections across faces forms a triangle as part of the iso-surface
Isosurface Construction - Positive Vertices at Opposite Corners
Iso-surface Construction:
Marching Cubes

• One can work through all 256 cases in this way - although it quickly becomes apparent that many cases are similar.
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• For example:
  – 2 cases where all are positive, or all negative, give no iso-surface
  – 16 cases where one vertex has opposite sign from all the rest
Iso-surface Construction: Marching Cubes

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• For example:
  – 2 cases where all are positive, or all negative, give no isosurface
  – 16 cases where one vertex has opposite sign from all the rest

• In fact, there are only 15 topologically distinct configurations
Case Table
8 Above
0 Below

1 case
7 Above
1 Below
1 case
6 Above
2 Below
3 cases
5 Above
3 Below

3 cases
4 Above

4 Below

7 cases
4 Above

4 Below

7 cases

4 connected
4 Above

4 Below

7 cases

1 opposite pairs
- 4 Above
- 4 Below

7 cases

1 vertex opposite to triplet
4 Above

4 Below

7 cases

1 isolated vertices
Marching Cubes – Look-up Table

• Connecting vertices by triangles
  – Triangles shouldn’t intersect
  – To be a closed manifold:
    • Each vertex used by a triangle “fan”
    • Each mesh edge used by 2 triangles (if inside grid cell) or 1 triangle (if on a grid face)
    • Each mesh edge on the grid face is shared between adjacent cells

• Look-up table
  – \(2^8=256\) entries
  – For each sign configuration, it stores indices of the grid edges whose vertices make up the triangles
Additional Readings

• Marching Cubes:
  • “Marching cubes: A high resolution 3D surface construction algorithm”, by Lorensen and Cline (1987)
    – over 17,000 citations on Google Scholar

• Dual Contouring:
  • “Dual contouring of hermite data”, by Ju et al. (2002)
    – over 800 citations on Google Scholar
  • “Manifold dual contouring”, by Schaefer et al. (2007)
In VTK

use the `vtkMarchingCubes()` filter and its function `SetValue(0, iso-value)`
Polygonal Iso-Surfaces
- An Alternative

Marching Cubes is difficult, and so we will look at another approach.
Polygonal Iso-Surfaces: Data Structure
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`bool FoundEdgeIntersection[12]`
One entry for each of the 12 edges.
`false` means $S^*$ did not intersect this edge
`true` means $S^*$ did intersect this edge
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Node EdgeIntersection[12]
If an intersection did occur on edge #i, Node[i] will contain the interpolated x, y, z, nx, ny, and nz.

```cpp
struct Node{
    float x, y, z, nx, ny, nz;
};
```
Polygonal Iso-Surfaces: Data Structure

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bool FoundEdgeConnection[12][12]
A true in entry [i][j] or [j][i] means that Marching Squares has decided there needs to be a line drawn from Cube Edge #i to Cube Edge #j
Both entry [i][j] and [j][i] are filled so that it won’t matter which order you search in later.

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```
Polygonal Iso-Surfaces: Algorithm

*Strategy in ProcessCube():*

1. Use `ProcessCubeEdge()` 12 times to find which cube edges have S* intersections.

2. Return if no intersections were found anywhere.

3. Call `ProcessCubeQuad()` 6 times to decide which cube edges will need to be connected. This is Marching Squares like we did it before, but it doesn’t need to re-compute intersections on the cube edges in common. `ProcessCubeEdge()` already did that. This leaves us with the `FoundEdgeConnection[][]` array filled.

4. Call `DrawCubeTriangles()` to create triangles from the connected edges.
Polygonal Iso-Surfaces: Algorithm

Strategy in `DrawCubeTriangles()`:

1. Look through the `FoundEdgeConnection[][][]` array for a Cube Edge #A and a Cube Edge #B that have a connection between them.
2. If can’t find one, then you are done with this cube.
Polygonal Iso-Surfaces: Algorithm

*Strategy in *DrawCubeTriangles*():*

1. Look through the `FoundEdgeConnection[][][]` array for a Cube Edge #A and a Cube Edge #B that have a connection between them.
2. If can’t find one, then you are done with this cube.
3. Now look through the `FoundEdgeConnection[][][]` array for a Cube Edge #C that is connected to Cube Edge #B. *If you can’t find one, something is wrong.*
4. **Draw a triangle** using the `EdgeIntersection[]` nodes from Cube Edges #A, #B, and #C. Be sure to use `glNormal3f()` in addition to `glVertex3f()`.
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5. Turn to false the FoundEdgeConnection[][] entries from Cube Edge #A to Cube Edge #B.
6. Turn to false the FoundEdgeConnection[][] entries from Cube Edge #B to Cube Edge #C.
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5. Turn to false the FoundEdgeConnection[][] entries from Cube Edge #A to Cube Edge #B.
6. Turn to false the FoundEdgeConnection[][] entries from Cube Edge #B to Cube Edge #C.
7. Toggle the FoundEdgeConnection[][] entries from Cube Edge #C to Cube Edge #A. If this connection was there before, we don’t need it anymore. If it was not there before, then we just invented it and we will need it again.
Polygonal Iso-Surfaces: Why Does this work?

Take this case as an example. The intersection points A, B, C, and D were found and the lines AB, BC, CD, and DA were found because Marching Squares will have been performed on each of the cube’s 6 faces.
Polygonal Iso-Surfaces: Why Does this work?

Take this case as an example. The intersection points A, B, C, and D were found and the lines AB, BC, CD, and DA were found because Marching Squares will have been performed on each of the cube’s 6 faces.

At this point, we could just draw the quadrilateral ABCD, but this will likely go wrong because it is surely non-planar. So, starting at A, we break out a triangle from the edges AB and BC (which exist) and the edge CA (which doesn’t exist, but we need it anyway to complete the triangle).
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This leaves `FoundEdgeConnection[][]` for CA, CD, and AD all set to `true`, which will cause the algorithm to find them and connect them into a triangle next.
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Similarly, you can process the intersections at edges E, F, and G, and edge segments EF, FG, and GE to form a triangle.
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Polygonal Iso-Surfaces: Surface Normals

We would very much like to use lighting when displaying polygonal iso-surfaces, but we need surface normals at all the triangle vertices. Because there really isn’t a surface there, this would seem difficult, but it’s not.

The normal is a vector that shows how the iso-surface is changing. How “something is changing” is called the gradient. So, the surface normal to a volume is:

\[ \vec{n} = \left( \frac{\partial S}{\partial x}, \frac{\partial S}{\partial y}, \frac{\partial S}{\partial z} \right) = \nabla S \]

\[ \frac{dS}{dx} = \frac{S_{i+1,j,k} - S_{i-1,j,k}}{2\Delta x} \]
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\]

Prior to the iso-surface calculation, you compute the surface normals for all the nodes in the 3D mesh. You then interpolate them along the cube edges when you create the iso-surface triangle vertices.
Acknowledgment

• Thanks for materials from
  – Dr. Mike Bailey