Review of Texture-based Methods

• Employs texture synthesis and image processing techniques to provide global, continuous, dense, and visually pleasing representations without constructing intermediate geometry.

• LIC family is the most popular texture-based technique

• IBFV is an easy but flexible technique

• Both can be extended to (2.5D) surface flow visualization

• IBFV is more computationally efficient than LIC

• Extending to 3D volumetric data visualization is possible but challenging due to the occlusion
a point in the flow field — the counterpart of a pixel in the output LIC image

locate a set of pixels hit by the streamline

index the input noise for the texture values

obtain the value of the target pixel in the LIC image via texture convolution

weighting is governed by a low-pass filter

\[
\frac{\text{texture}[i] \times \text{weight}[i]}{\sum \text{weight}[i]}
\]
IBFV

http://www.win.tue.nl/~vanwijk/ibfv/
Texture-based Methods on Surfaces

Surface LIC

IBFVS

(Detlev Stalling, ZIB, Germany)

ISA
Volumetric Texture
Arrows vs. Streamlines vs. Textures

Streamlines: selective
Arrows: simple

Textures: 2D-filling
Vector Field Visualization: Feature-based

Goal: how to compute Jacobian of flow; know what features in vector fields are of interest; what are the common physical features; how to use information of Jacobian to extract some of the relevant features.
What features are in flows?

Features are highly application dependent

- Non-topology (physics-based)
- Topology
Physics-based Feature Extraction

Their computation is mostly local
Vector Field Gradient

• Consider a vector field

\[
\frac{d\mathbf{x}}{dt} = V(\mathbf{x}) = \mathbf{f}(x, y, z) = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}
\]

• Its gradient is

\[
\nabla V = \begin{bmatrix}
\frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} & \frac{\partial f_x}{\partial z} \\
\frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} & \frac{\partial f_y}{\partial z} \\
\frac{\partial f_z}{\partial x} & \frac{\partial f_z}{\partial y} & \frac{\partial f_z}{\partial z}
\end{bmatrix}
\]

It is also called the **Jacobian** matrix of the vector field. Many feature detection for flow data relies on Jacobian.
What is the Jacobian of the following vector field?

\[ V = \begin{cases} 
  vx = -y \\
  vy = \frac{1}{2} x
\end{cases} \]
What is the Jacobian of the following vector field?

\[ V = \begin{cases} 
v x = -y \\
v y = \frac{1}{2} x 
\end{cases} \]

\[ \nabla V = \begin{bmatrix} 0 & -1 \\
\frac{1}{2} & 0 \end{bmatrix} \]
What is the Jacobian of the following vector field?

\[
V = \begin{cases} 
  v_x = 10x + 10y + 12340 \\
  v_y = 12x - 19y - 12000 
\end{cases}
\]
What is the Jacobian of the following vector field?

\[ V = \begin{cases} 
 vx = 10x + 10y + 12340 \\
 vy = 12x - 19y - 12000 
\end{cases} \]

\[ \nabla V = \begin{bmatrix} 
 10 \\
 12 \\
 10 \\
 -19 
\end{bmatrix} \]
Divergence and Curl

- **Divergence** measures the magnitude of outward flux through a small volume around a point

\[
div V = \nabla \cdot V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}
\]

\[
\nabla = \begin{bmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}
\end{bmatrix}
\]

- **Curl** describes the infinitesimal rotation around a point

\[
curl V = \nabla \times V = \begin{bmatrix}
\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} & \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} & \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}
\end{bmatrix}
\]

\[
\nabla \cdot (\nabla \times \phi) = 0
\]

\[
\nabla \times (\nabla \phi) = \vec{0}
\]
Divergence and Curl

- Divergence measures the magnitude of outward flux through a small volume around a point
  \[
  \text{div } V = \nabla \cdot V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}
  \]

\[
\nabla = \left[ \begin{array}{ccc}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
\end{array} \right]
\]

- Curl describes the infinitesimal rotation around a point
  \[
  \text{curl } V = \nabla \times V = \begin{bmatrix}
\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} & \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} & \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}
\end{bmatrix}
\]

\[
\nabla \cdot (\nabla \times V) = 0
\]

\[
\nabla \times (\nabla \phi) = \vec{0}
\]
Divergence and Curl

• Divergence- measures the magnitude of outward flux through a small volume around a point

\[ \text{div } V = \nabla \cdot V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} \]

\[ \nabla = \left[ \begin{array}{ccc} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{array} \right] \]

• Curl- describes the infinitesimal rotation around a point

\[ \text{curl } V = \nabla \times V = \left[ \begin{array}{ccc} \frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} & \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} & \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \end{array} \right] \]

\[ \nabla \cdot (\nabla \times V) = 0 \]

\[ \nabla \times (\nabla \phi) = \vec{0} \]

Both are local computation!
Consider a 2D Steady Vector Fields

• Assume a 2D steady **piecewise linear** vector field
  \[
  \frac{dx}{dt} = V(x) = \vec{f}(x, y) = \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} ax + by + c \\ dx + ey + f \end{pmatrix}
  \]

• Its Jacobian is
  \[
  \nabla V = \begin{bmatrix}
  \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\
  \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y}
  \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix}
  \]

• Divergence is \(a + e\)

• Curl is \(-(b - d)\)

Given a vector field defined on a discrete mesh, it is important to compute the coefficients \(a, b, c, d, e, f\) for later analysis.
What are the divergence and curl of the following vector field?

\[ V = \begin{cases} 
  v_x = -y \\
  v_y = \frac{1}{2} x 
\end{cases} \]

\[ \nabla V = \begin{bmatrix} 0 & -1 \\
  \frac{1}{2} & 0 \end{bmatrix} \]
What are the divergence and curl of the following vector field?

\[ V = \begin{cases} 
  v_x = -y \\
  v_y = \frac{1}{2}x 
\end{cases} \]

\[ \nabla V = \begin{bmatrix} 
  0 \\
  \frac{1}{2} \\
  0 
\end{bmatrix} \]

\[ \text{div } V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} = 0 + 0 = 0 \]

\[ \text{curl } V = -\left( \frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x} \right) = -(-1 - \frac{1}{2}) = \frac{3}{2} \]
What are the divergence and curl of the following vector field?

\[ V = \begin{cases} 
    v_x = x + 2y + z + 0.5 \\
    v_y = x - 3z - 1 \\
    v_z = -y + z + 2 
\end{cases} \]
What are the divergence and curl of the following vector field?

\[
V = \begin{cases}
    vx = x + 2y + z + 0.5 \\
    vy = x - 3z - 1 \\
    vz = -y + z + 2
\end{cases} \quad \nabla V = \begin{bmatrix}
    1 & 2 & 1 \\
    1 & 0 & -3 \\
    0 & -1 & 1
\end{bmatrix}
\]
What are the divergence and curl of the following vector field?

\[ V = \begin{cases} 
  v_x = x + 2y + z + 0.5 \\
  v_y = x - 3z - 1 \\
  v_z = -y + z + 2 
\end{cases} \]

\[ \nabla V = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -3 \\ 0 & -1 & 1 \end{bmatrix} \]

\[ \text{div } V = \nabla \cdot V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = 1 + 0 + 1 = 2 \]

\[ \text{curl } V = \nabla \times V = \begin{bmatrix} \frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} \\ \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} \\ \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \end{bmatrix} \]
What are the divergence and curl of the following vector field?

\[ V = \begin{cases} v_x = x + 2y + z + 0.5 \\ v_y = x - 3z - 1 \\ v_z = -y + z + 2 \end{cases} \]

\[ \nabla V = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -3 \\ 0 & -1 & 1 \end{bmatrix} \]

\[ \text{div } V = \nabla \cdot V = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = 1 + 0 + 1 = 2 \]

\[ \text{curl } V = \nabla \times V = \begin{bmatrix} \frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} \\ \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} \\ \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \end{bmatrix} \]

\[ = [-1 - (-3), 1 - 0, 1 - 2] = [2, 1, -1] \]

vorticity vector!!
Examples of Divergence and Curl of 2D Vector Fields

Divergence and curl of a vector field
Rainbow color coding is used.

Examples of vector-valued data reduction!
Volume rendering of vorticity in various flows (images from Google images)
Jacobian and its derived physical quantities can be applied to the extraction of certain physics-based features.
Vortices

There is NO unified definition of vortices!!!!
Vortices

Blood Flow Analysis [Köhler et al. 2013]

Post et al. STAR report 2003

Source:
http://www.onera.fr/cahierdelabo/english/aerod_ind03.htm
Vortices in turbulent flows
Typical vortex detection techniques

- **Region-based** – using one of the following attributes and some ad-hoc thresholds

  \[
  J = S + R, \quad S = \frac{1}{2} [J + J^T], \quad R = \frac{1}{2} [J - J^T]
  \]

  Vorticity

  \[
  Q = \frac{1}{2} (\|R\|^2 - \|S\|^2)
  \]

  \(\lambda_2\) is the second largest eigenvalue of the tensor \(S^2 + R^2\)
Typical vortex detection techniques

- **Line-based** – detecting vortex cores
  
  PV-parallel vector operator
  
  $\mathbf{v} \parallel \nabla \mathbf{v} \cdot \mathbf{v}$
  
  the acceleration is parallel to the velocity
  
  the Jacobian matrix has a complex pair of eigenvalues
Typical vortex detection techniques

- **Line-based** – detecting vortex cores
  
  PV-parallel vector operator
  
  \[ \mathbf{v} \parallel \nabla \mathbf{v} \cdot \mathbf{v} \]

  the acceleration is parallel to the velocity
  
  the Jacobian matrix has a complex pair of eigenvalues

- **Geometric-based** – finding returning streamline using winding angles
Recent vortex detection techniques

**Objectivity-based** – extract optimal reference frame

[Guenther et al., SIGGRAPH 2017]

**Vortex boundary extraction**

[Elliptic LCS, Haller et al. 2016]

**Machine learning approach**

[Berenjkoub et al., IEEE VIS 2020]
Separation Flows

Separation flow on delta wing surface [Tricoche et al. AIAA 2004]
Separation Flows
Separation and attachment line detection is again based on the parallel vector operator (in 2D)

PV-parallel vector operator

\[ \mathbf{v} || \nabla \mathbf{v} \cdot \mathbf{v} \]

both eigen-values of the Jacobian matrix are real number, leading to real eigen-vectors

Figure 3: a) Separation and attachment lines on a solid surface of the “bluntfin” data set (black lines). The white arrows indicate the projected velocity \( \mathbf{v} \), the black arrows the field \( \mathbf{w} = (\nabla \mathbf{v}) \cdot \mathbf{v} \). b) The same lines compared to an LIC of \( \mathbf{v} \)

[Peikert and Roth, IEEE VIS 99]
Coherent structures in turbulent flows are in different scales and tangled in space and time

Images from [Nguyen et al, IEEE VIS 2020]
Helmholtz decomposition

\[ V = \nabla \varphi + \nabla \times A \]

\( \nabla \varphi \) is the gradient of a scalar field \( \varphi \)

Divergence free (or no-solenoidal field)
Helmholtz decomposition

\[ V = \nabla \phi + \nabla \times A \]

Curl (or rotation) free

Divergence free

Hodge decomposition

\[ V = \nabla \phi + \nabla \times A + \gamma \]

Curl (or rotation) free

Divergence free

Harmonic
Example: Helmholtz decomposition

curl-free  general  divergence-free
Additional Materials
