Time-Dependent (Unsteady) Flow Analysis

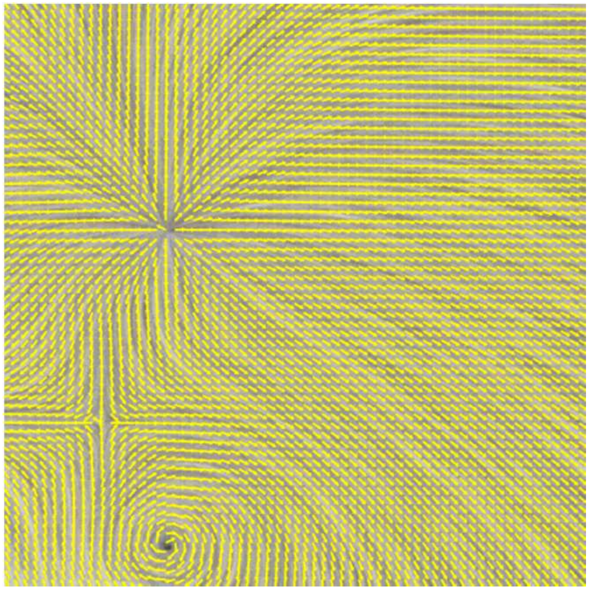
What is Different?

Steady (time-independent) flows:

- flow itself constant over time
- $\mathbf{v}(\mathbf{x})$, e.g., laminar flows
- simpler case for visualization

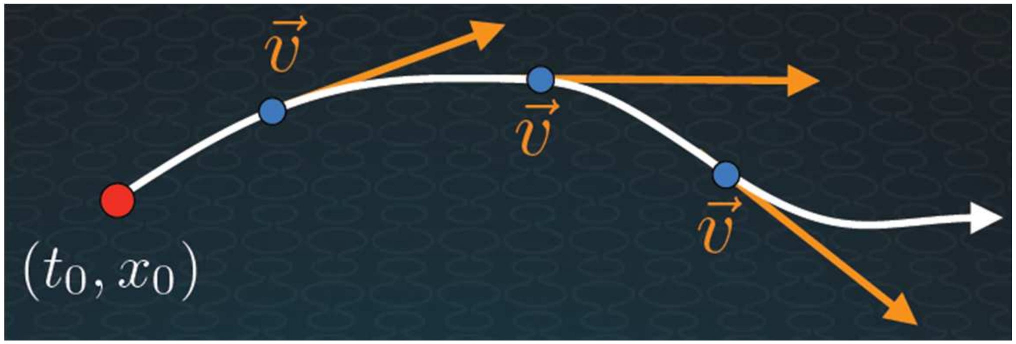
Time-dependent (unsteady) flows:

- flow itself changes over time
- $\mathbf{v}(\mathbf{x},t)$, e.g., turbulent flow
- more complex case



Mathematical Framework

- An unsteady **vector field**
  - is a continuous vector-valued function $\vec{v}(\boldsymbol{x}, t)$ on a manifold X
  - can be expressed as a system of ODE $\frac{d\boldsymbol{x}}{dt} = \vec{v}(\boldsymbol{x}, t)$
  - is a map $\varphi : R \times X \to X$
  - with initial condition $\boldsymbol{x}(t_0) = \boldsymbol{x}_{\mathbf{0}}$, its solution is called an integral curve, trajectory, or orbit.



Stream, Path, and Streaklines

Terminology:

- **Streamline**: a curve that is everywhere tangent to the steady flow (release 1 massless particle)

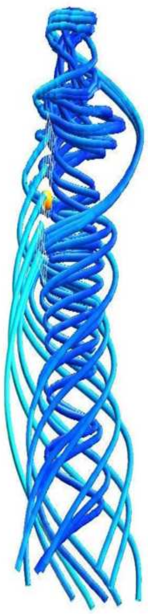
  $\mathbf{s}(t) = \mathbf{s}_0 + \int_{0 \le u \le t} \mathbf{v}(\mathbf{s}(u))\, du$

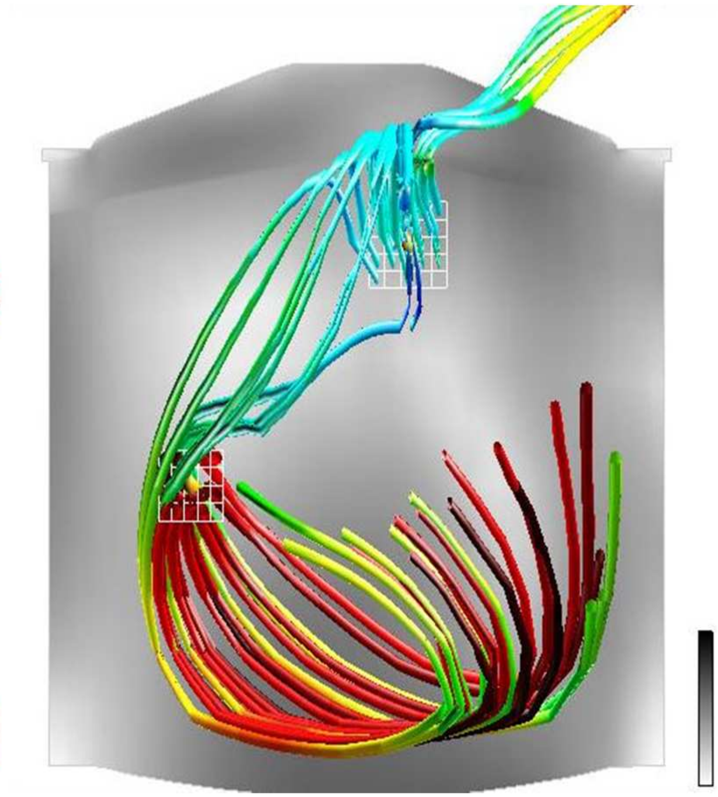
- **Pathline**: a curve that is everywhere tangent to an unsteady flow field (release 1 massless particle)

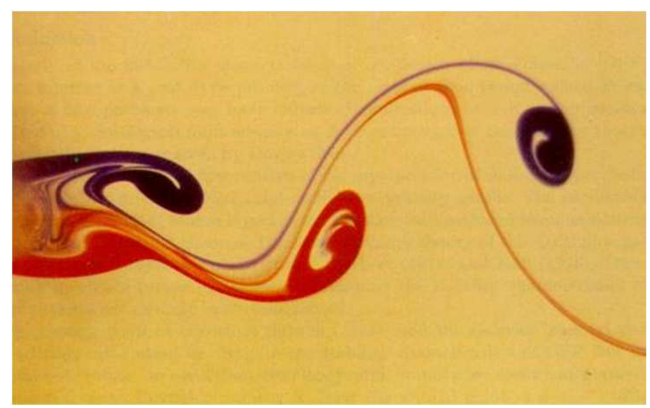
  $\mathbf{s}(t) = \mathbf{s}_0 + \int_{0 \le u \le t} \mathbf{v}(\mathbf{s}(u), \boldsymbol{u})\, du$

- **Streakline**: a curve traced by the continues release of particles in unsteady flow from the **same position in space** (release infinitely many massless particles)

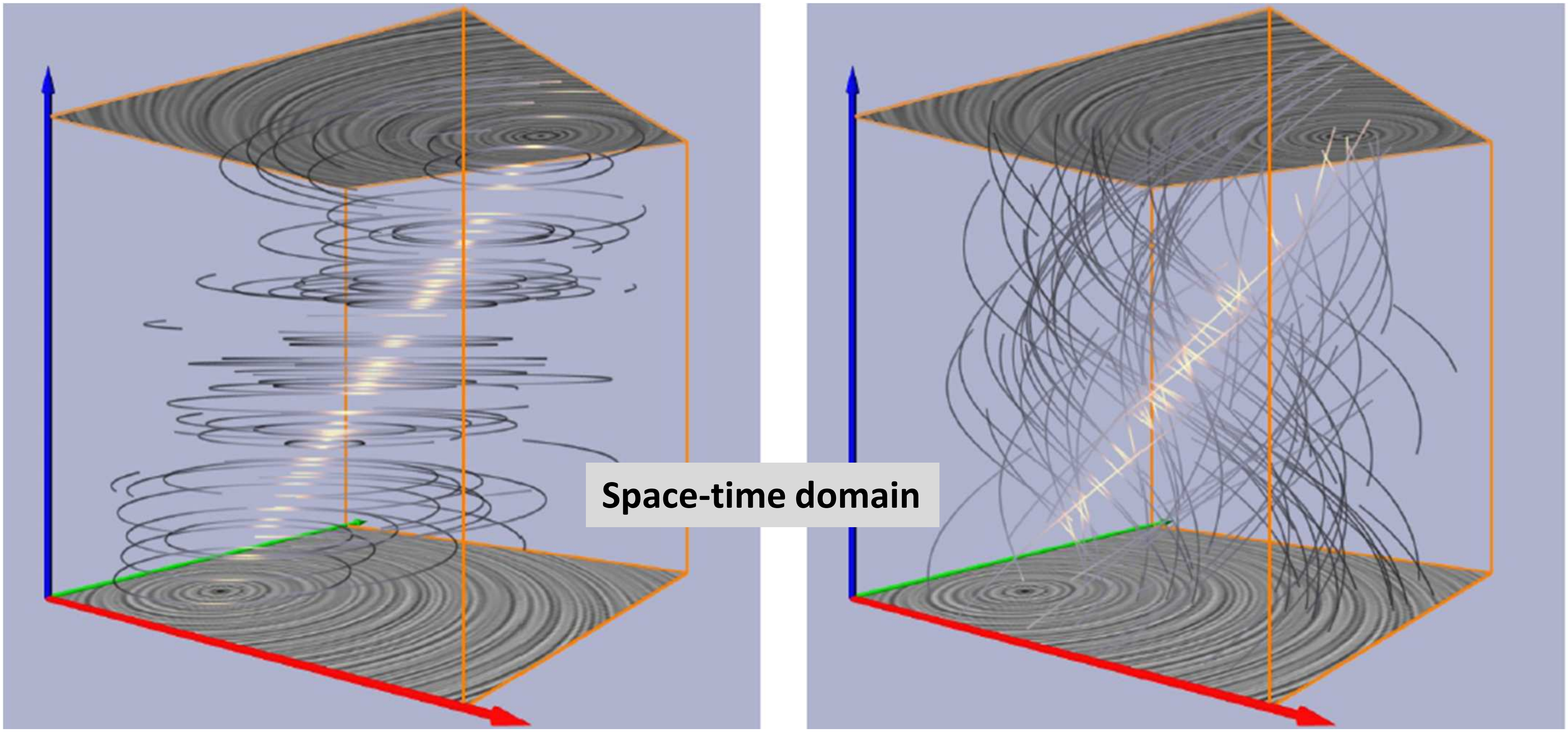
- They are identical under the steady flow







Difference Between Streamlines and Pathlines



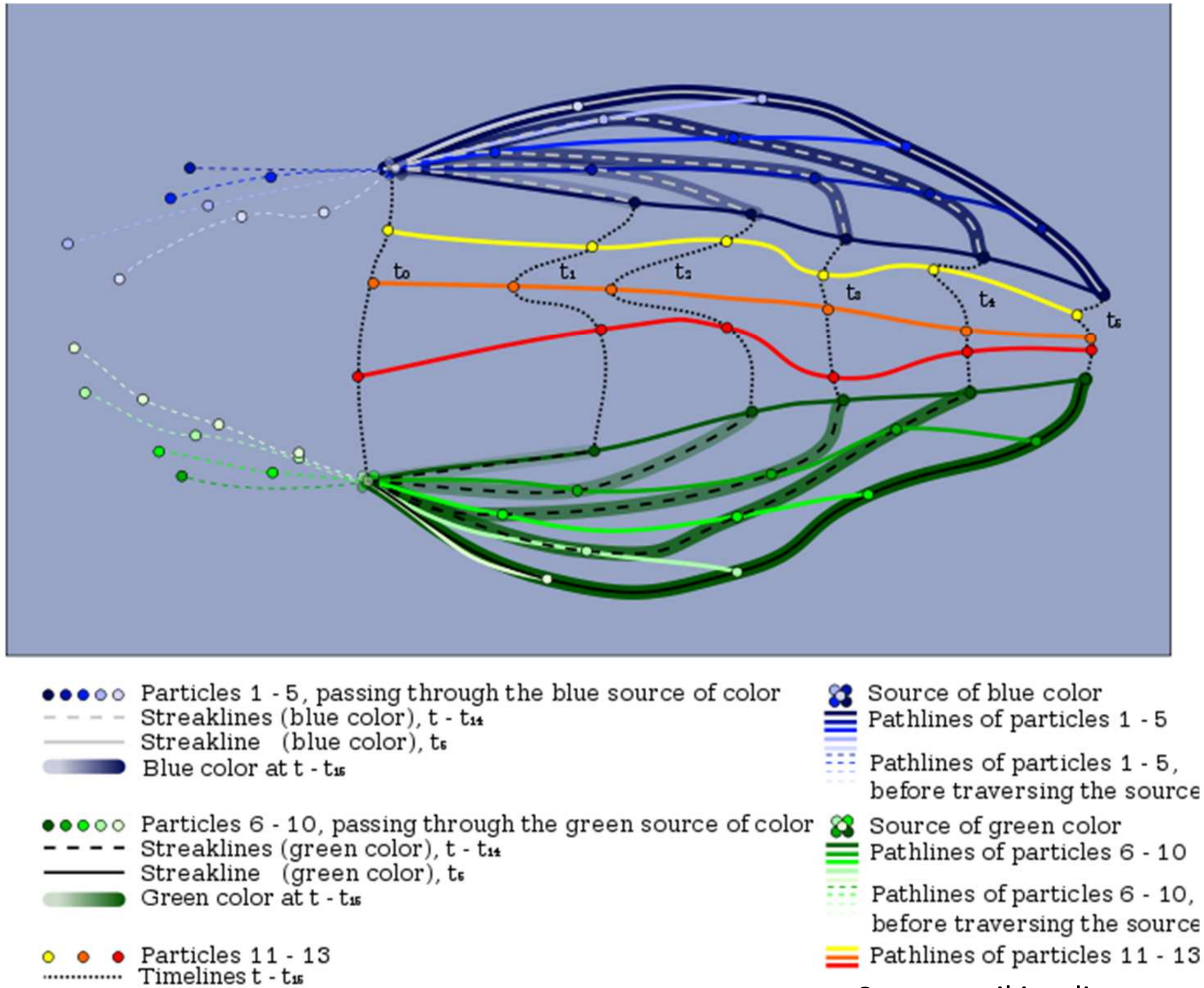
A moving center over time: (left) streamlines; (right) path lines
Source: Weinkauf et al. Vis2010

Streaklines

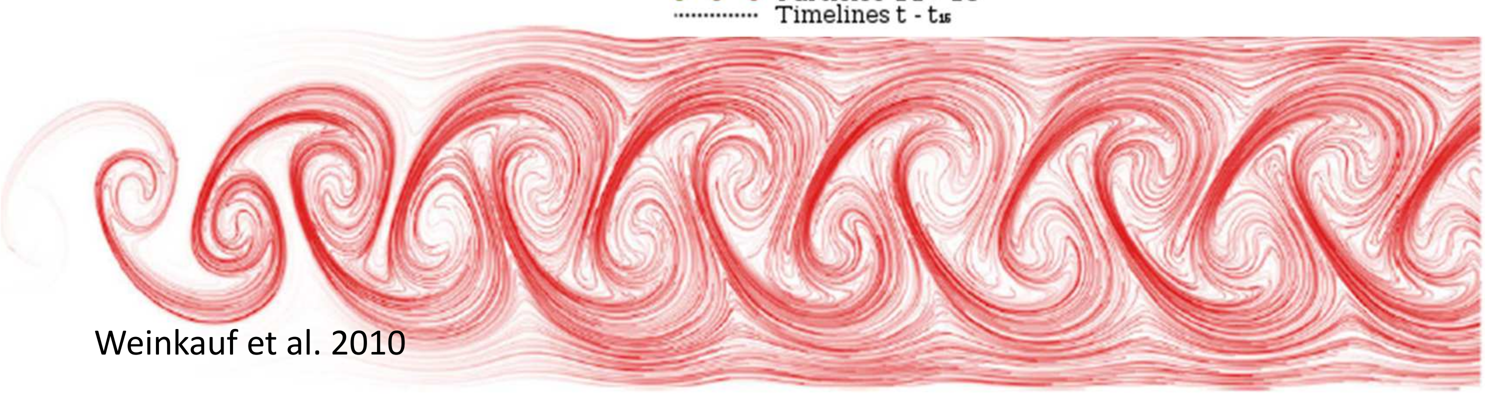
- Not tangent curves to the vector fields
- Union of the current positions of particles released at the same point in space

Timelines

- Union of the current positions of particles released at the same time in space



Source: wikipedia



Weinkauf et al. 2010

Time-Dependent Vector Field Analysis

Different Points of View

- Streamline-based
  - Extract the so-called “instantaneous” topology at each time step and keep track of the “evolution” of this topology over time
  - Feature tracking
- Pathline-based
  - Classify the behaviors of different pathlines
- Streakline-based
  - Extension of the pathline-based approach

Track Topology Evolution

- Parameter dependent topology:
  - “Fixed points” (no more fixed) move, appear, vanish, transform
  - Topological graph connectivity changes

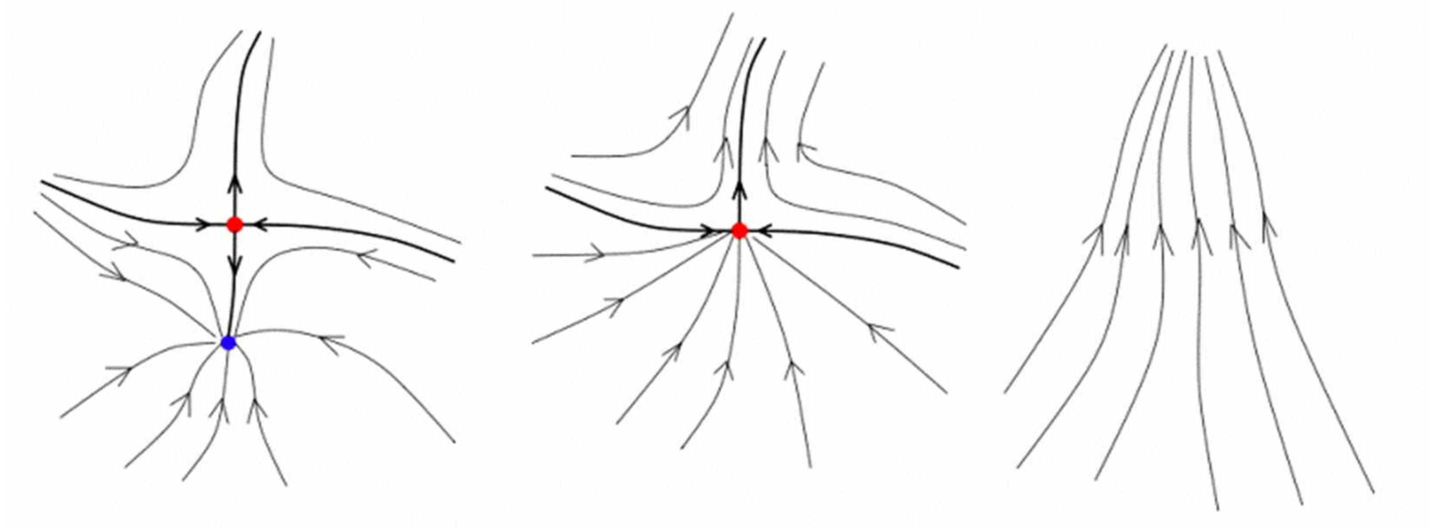
- Structural stability: topology is stable w.r.t. small but arbitrary changes of parameter(s) if and only if
  - 1) Number of fixed points and closed orbits is finite and all are hyperbolic
  - 2) No saddle-saddle connection (unstable)

Bifurcations

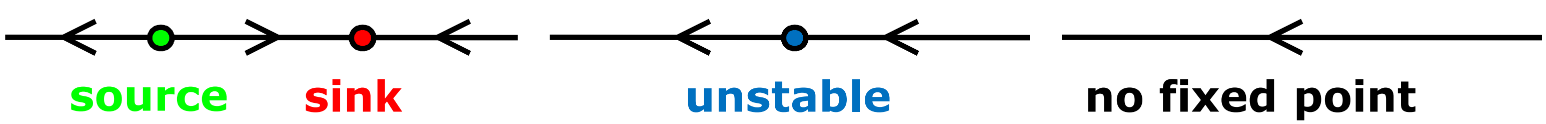
- Transition from one stable structure to another through *unstable state*
- Bifurcation value: parameter value inducing the transition
- Local vs. global bifurcations

Local Bifurcations

- Transformation affects local region
- Fold bifurcation: saddle + sink/source

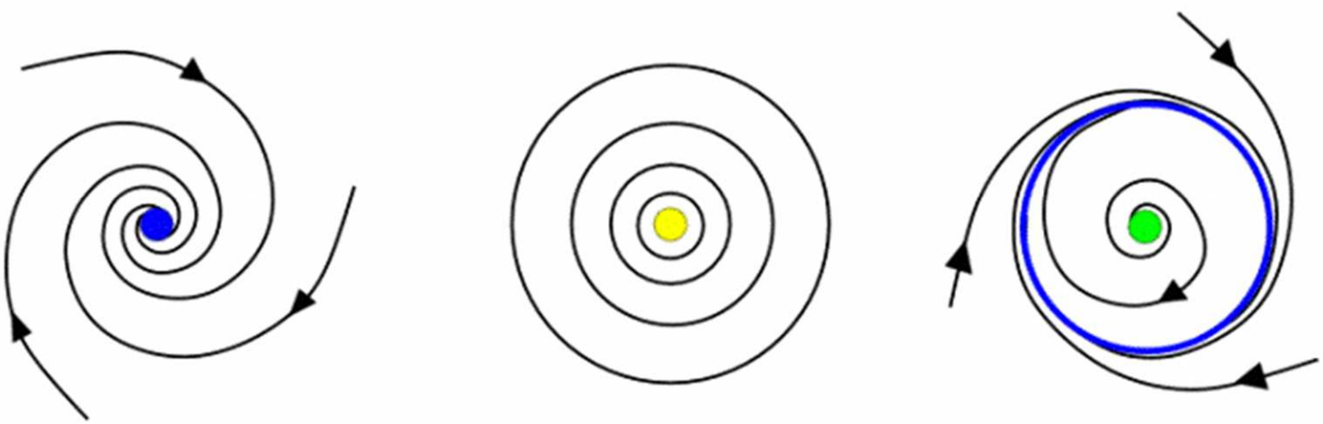


1D equivalent:



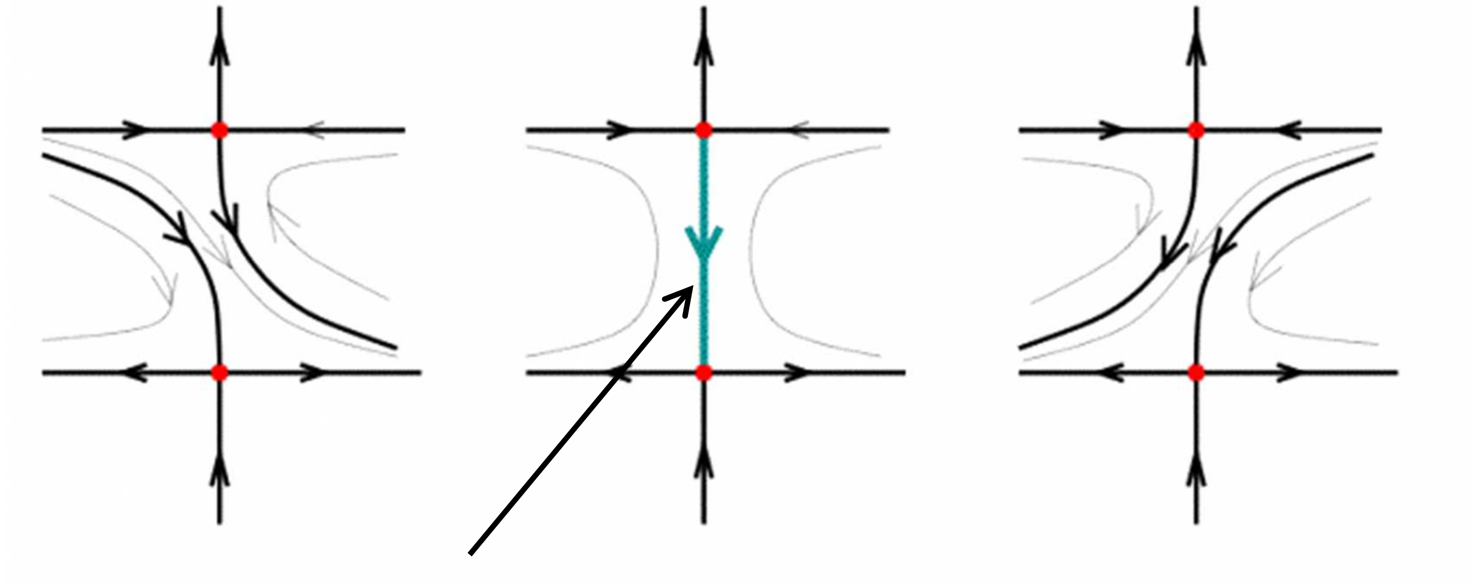
Local Bifurcations

- Transformation affects local region
- Hopf bifurcation: sink/source + closed orbit



Global Bifurcations

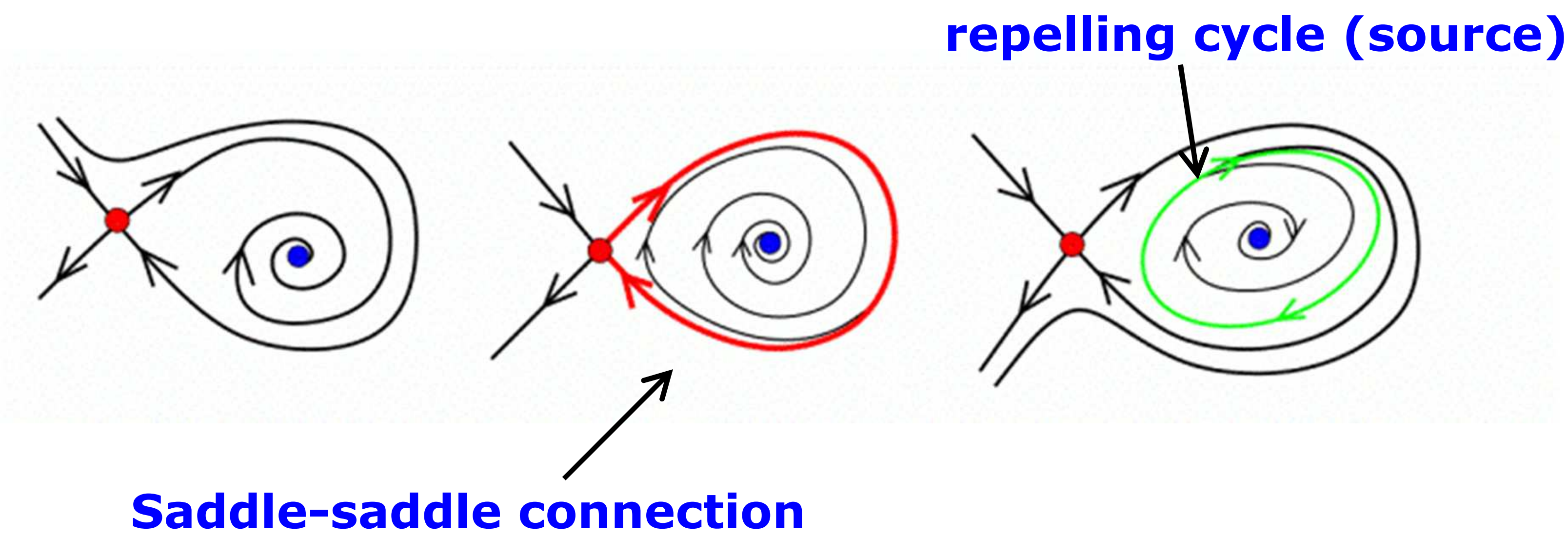
- Affects overall topological connectivity
- Basin bifurcation



Saddle-saddle connection

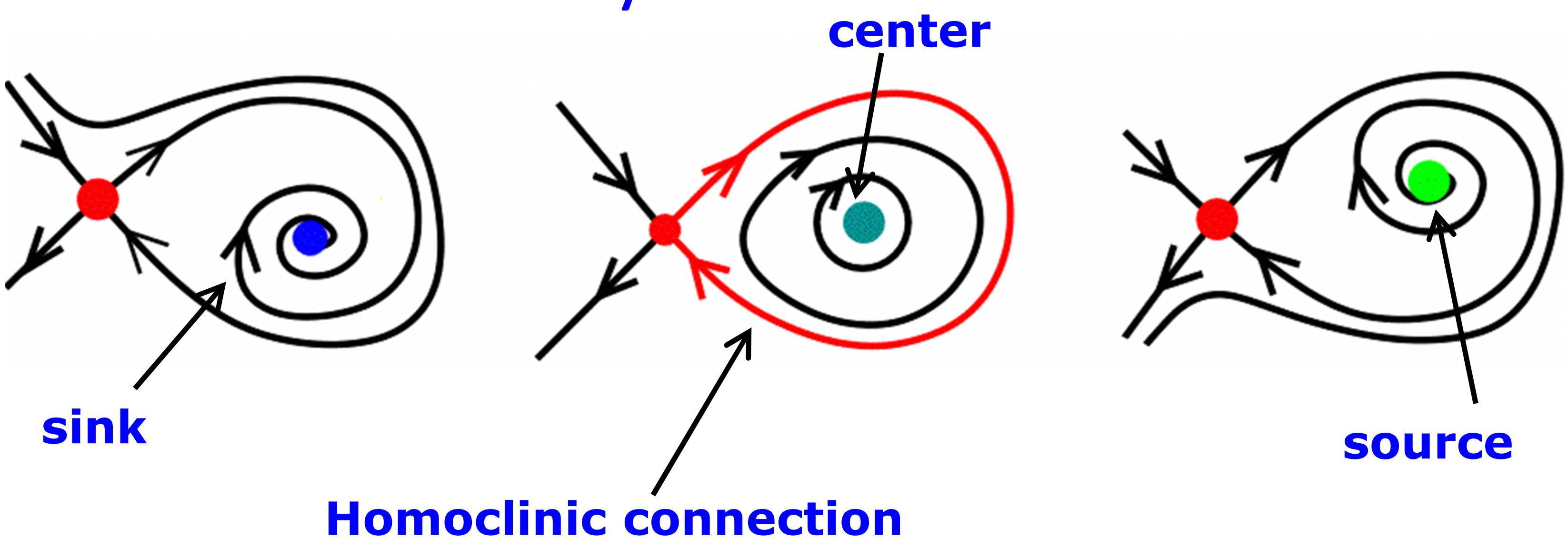
Global Bifurcations

- Modifies overall topological connectivity
- Homoclinic bifurcation



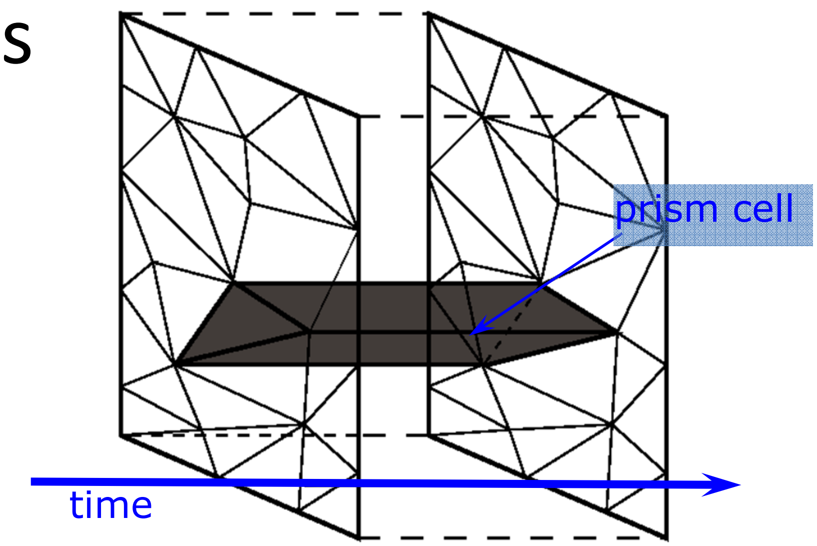
Global Bifurcations

- Modifies overall topological connectivity
- Periodic blue sky



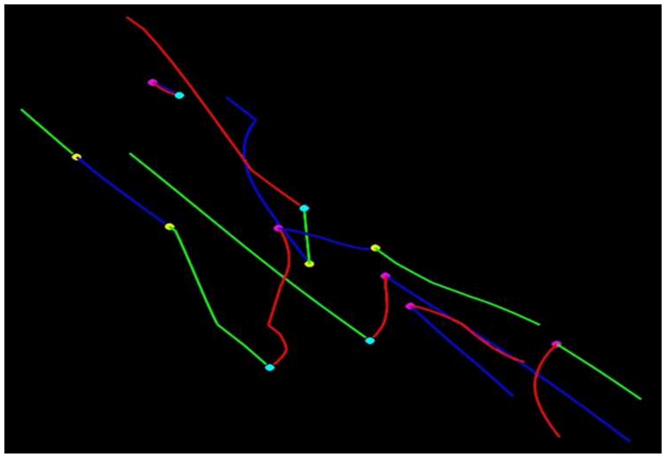
2+1D Topology

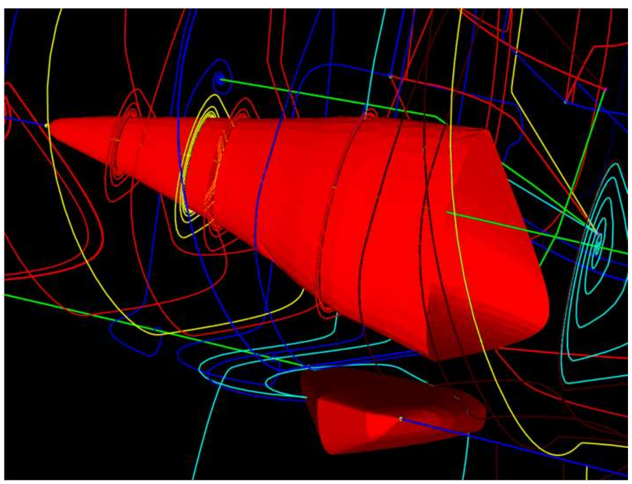
- Time-wise interpolation
- Cell-wise tracking over 2+1D grid
- Detect local bifurcations

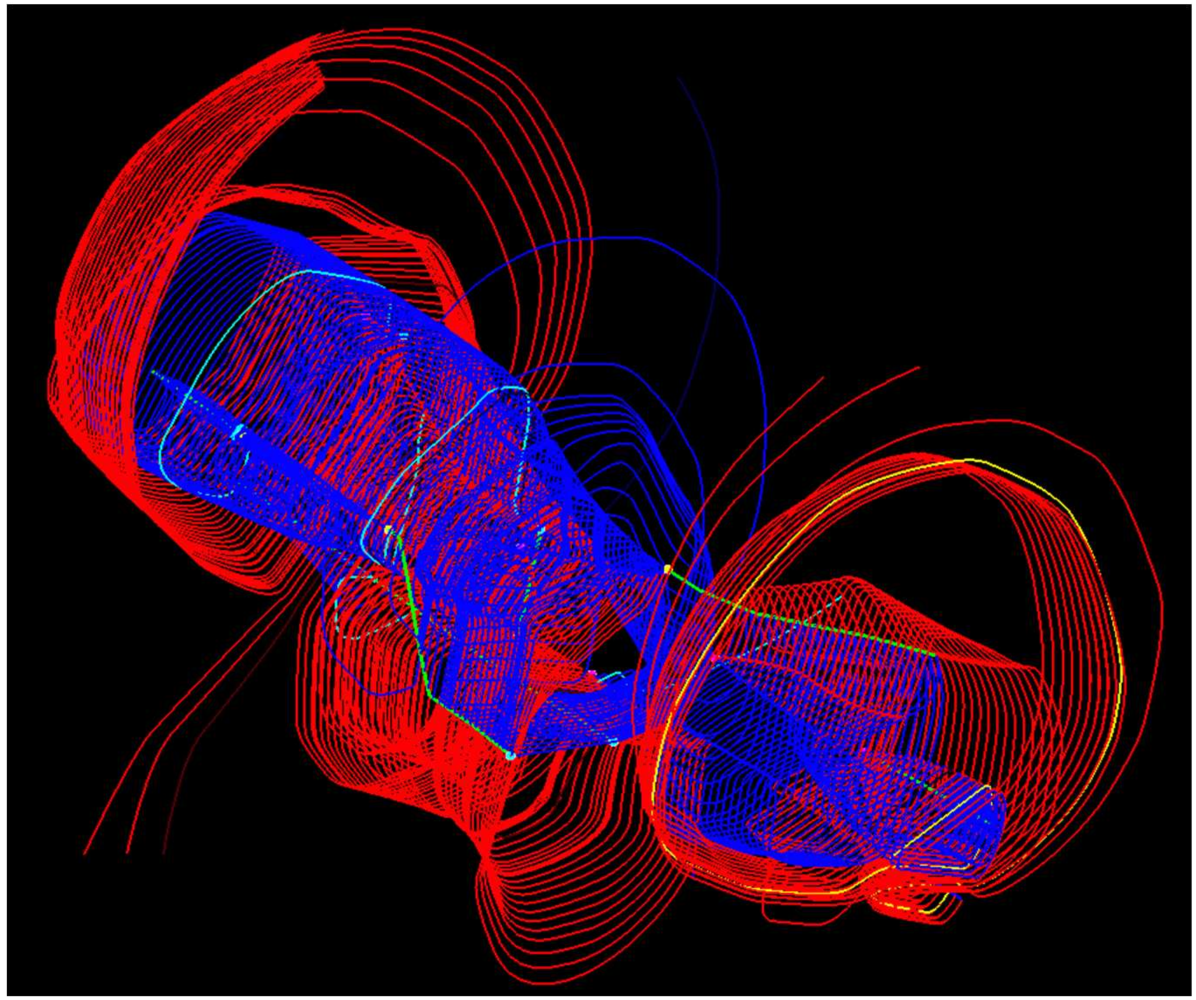


- Track associated separatrices (surfaces)

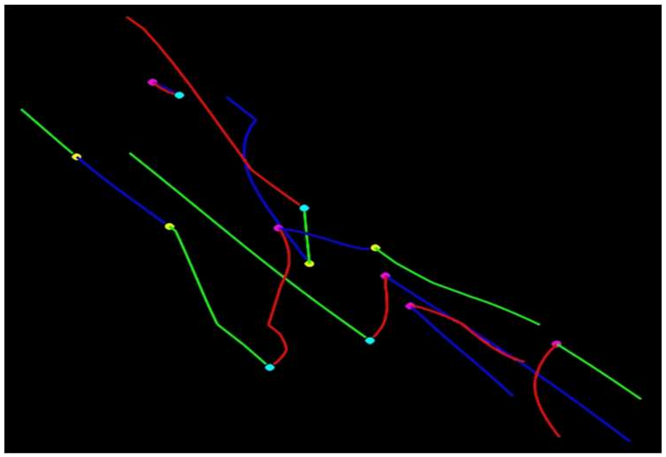
2+1D Topology

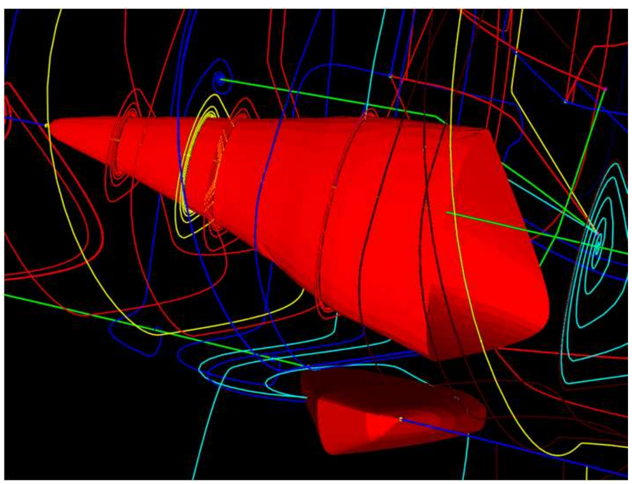


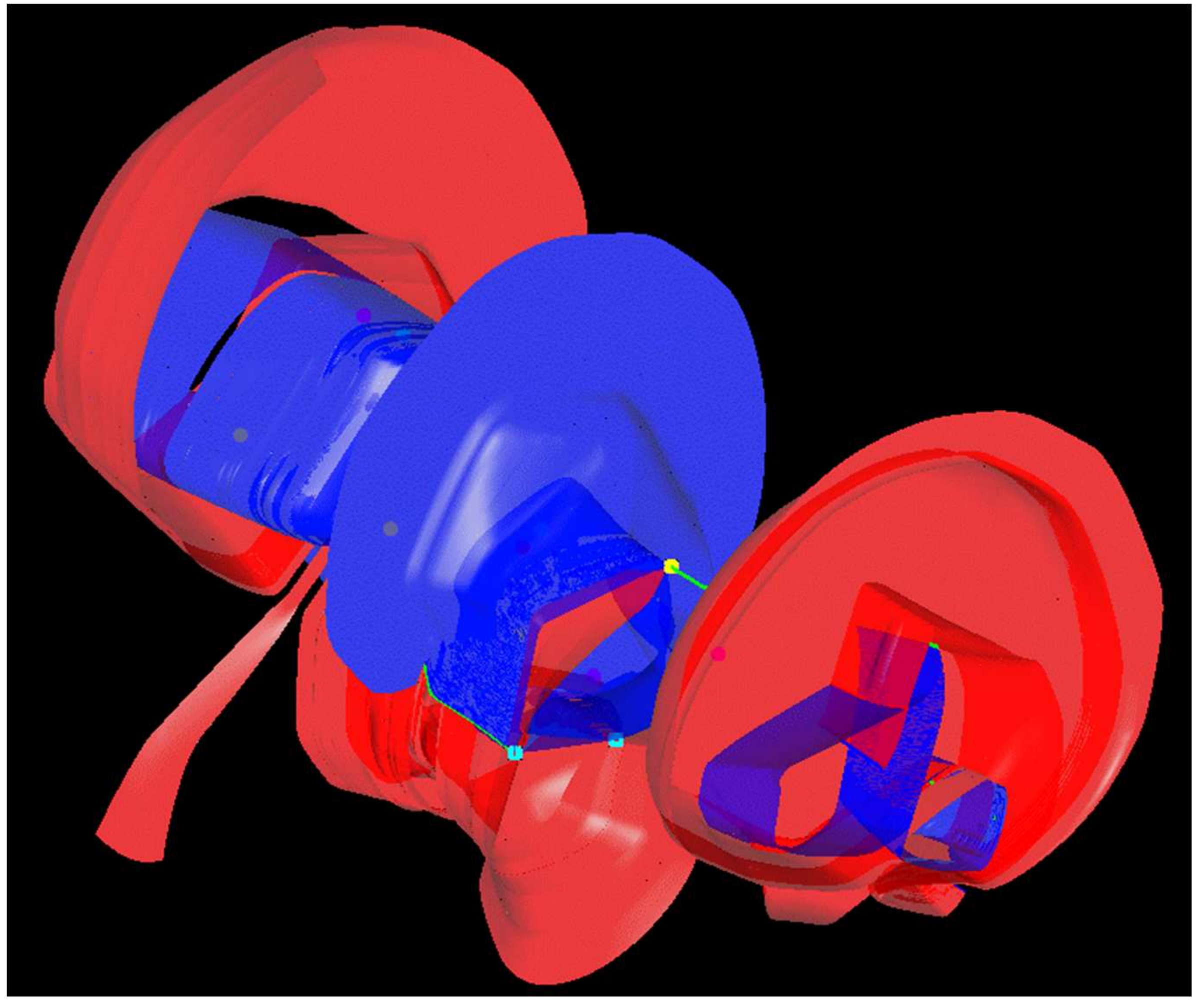




2+1D Topology







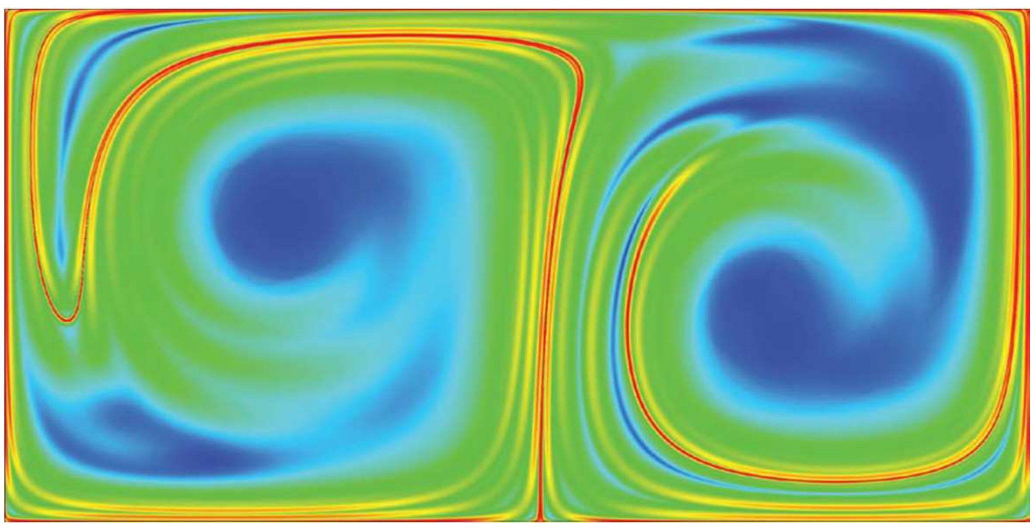
Pathline-based Topology

- Issue of the instantaneous topology tracking
  - It is not easy to compute and has to be computed for each time step
  - To see the result, animation must be used
  - Segmentation does not reveal the true segmentation for coherent flow over time

- Any solutions?
  - Pathline-oriented
  - Finite-Time Lyapunov Exponent

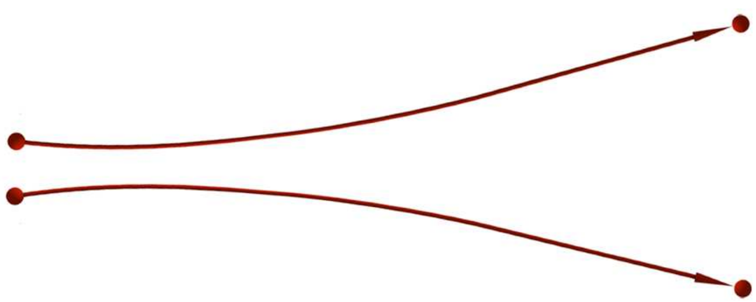
Finite-Time Lyapunov Exponent

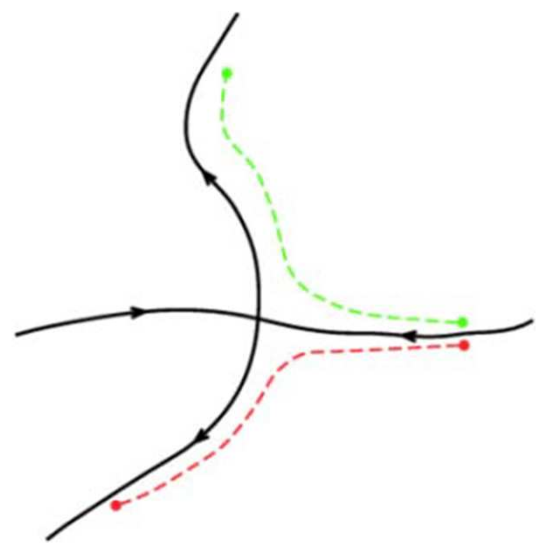
- FTLE Overview:
  - Provide a single **scalar field** that accounts for the integration over time
  - Provide an **average** that indicates the actual transport
  - Ridges separate regions of coherent flow (**ideally no flow across ridges**), called Lagrangian Coherent Structures (LCS)



Finite-Time Lyapunov Exponent

- Obtaining the scalar field
  - Observe particle trajectories
  - Measure the divergence between trajectories, i.e. how much flow stretch





[Shadden]

Finite-Time Lyapunov Exponent

- Description
  - Lyapunov exponents describe rate of separation or stretching of two infinitesimally close points over time in a dynamical system
  - FTLE refers to the largest Lyapunov exponent for only a **limited time** and is measured **locally**
  - *Largest exponent is governing the behavior of the system, smaller ones can be neglected*

Finite-Time Lyapunov Exponent

- Example: unsteady double-gyre

http://mmae.iit.edu/shadden/LCS-tutorial/examples.html

[Shadden]

Finite-Time Lyapunov Exponent

- Other examples

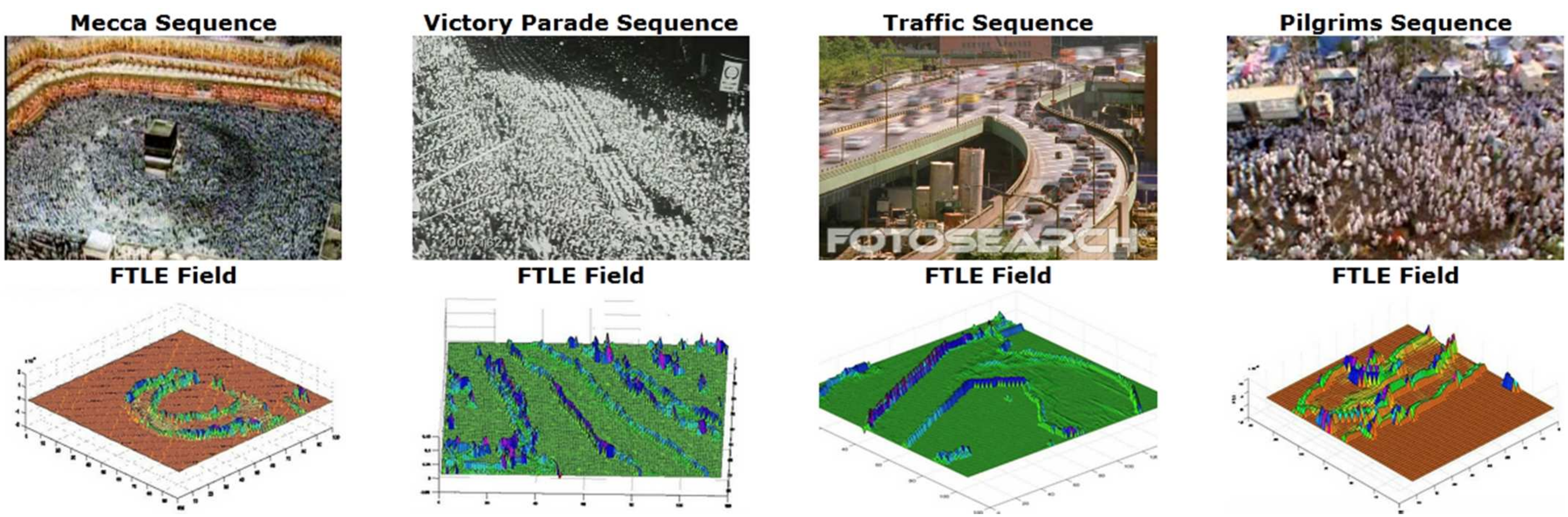
Ocean transport http://mmae.iit.edu/shadden/mbay

Biotransport http://mmae.iit.edu/shadden/jellyfish

[Shadden]

Finite-Time Lyapunov Exponent

- Further Applications
  - Detection of crowd flow in videos



http://www.cs.ucf.edu/~sali/Projects/CrowdSegmentation/index.html

[Ali et al.]

Finite-Time Lyapunov Exponent

- FTLE ridges are approximately material structures
- Non-zero cross-flux across FTLE ridges
- Accuracy increases with integration time
- Problem: data sets often bounded with time

Additional Readings

- Frits H. Post, Benjamin Vrolijk, Helwig Hauser, Robert S. Laramee, and Helmut Doleisch, **The State of the Art in Flow Visualisation: Feature Extraction and Tracking,** in *Computer Graphics Forum (CGF)*, Vol. 22, No. 4, 2003, pages 775-792.
- Helwig Hauser, Robert S. Laramee, and Helmut Doleisch, **Topology-Based Versus Feature-Based Flow Analysis - Challenges and an Application**, in *Topo-In-Vis 2005,* pages 79-90, 2007, Springer-Verlag.
- Armin Pobitzer, Ronald Peikert, Raphael Fuchs, Benjamin Schindler, Alexander Kuhn, Holger Theisel, Kresimir Matkovic, Helwig Hauser. **On the Way Towards Topology-Based Visualization of Unsteady Flow - the State of the Art**, in EuroGraphics 2010.

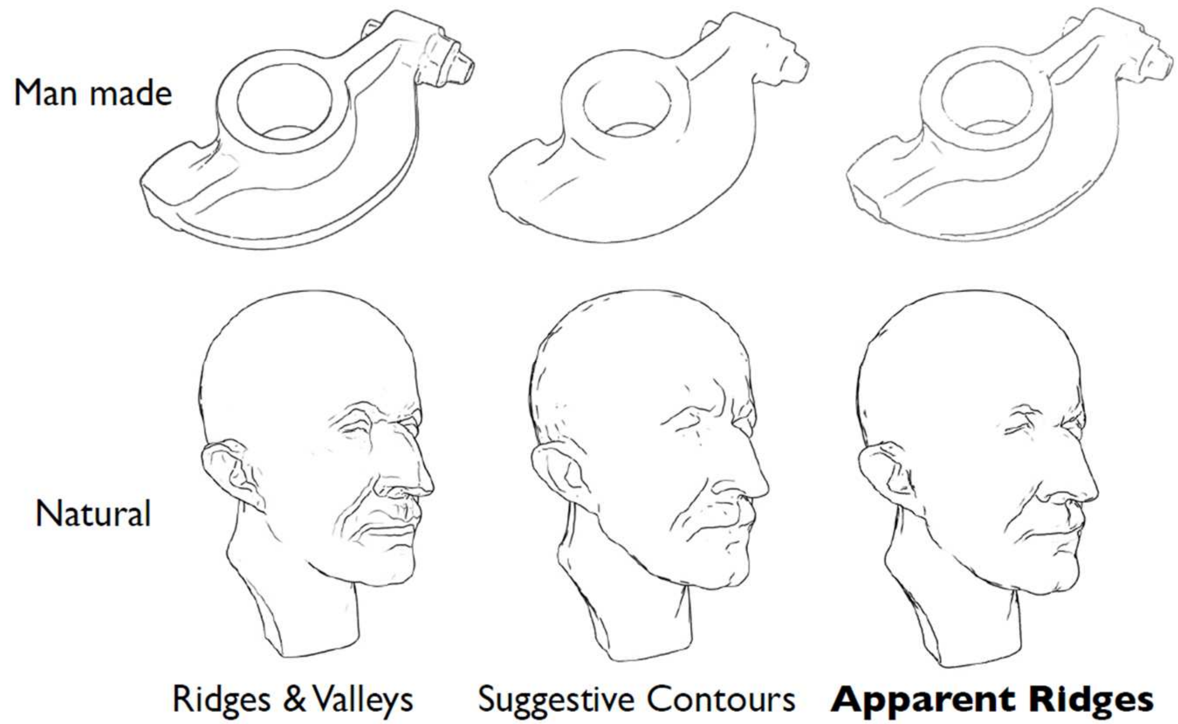
Acknowledgment

Thanks for the materials from

- Prof. Robert S. Laramee, Swansea University, UK
- Dr. Christoph Garth, University of Kaiserslautern, Germany
- Dr. Holger Theisel, University of Magdeburg, Germany.

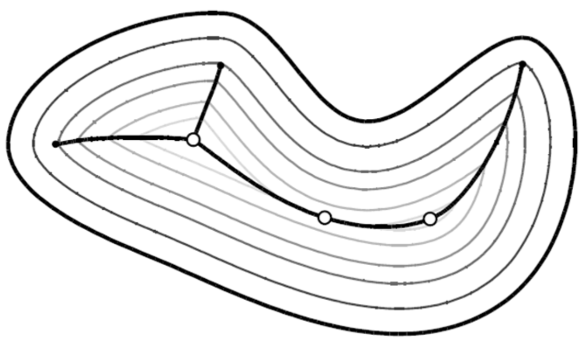
Final Project Topic Overview

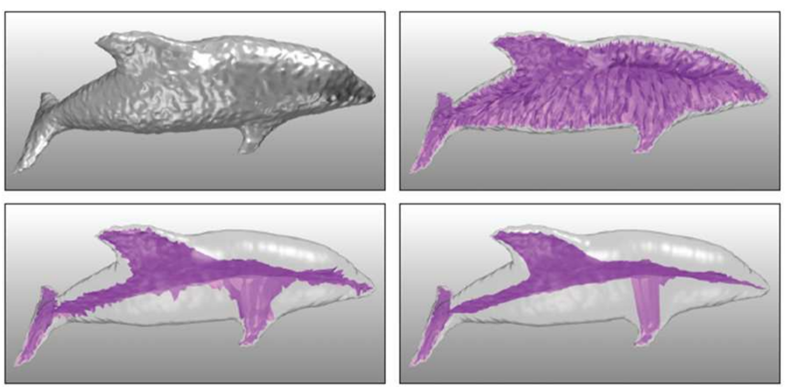
1. Curvature Tensor Estimation and Shape Illustrative Rendering

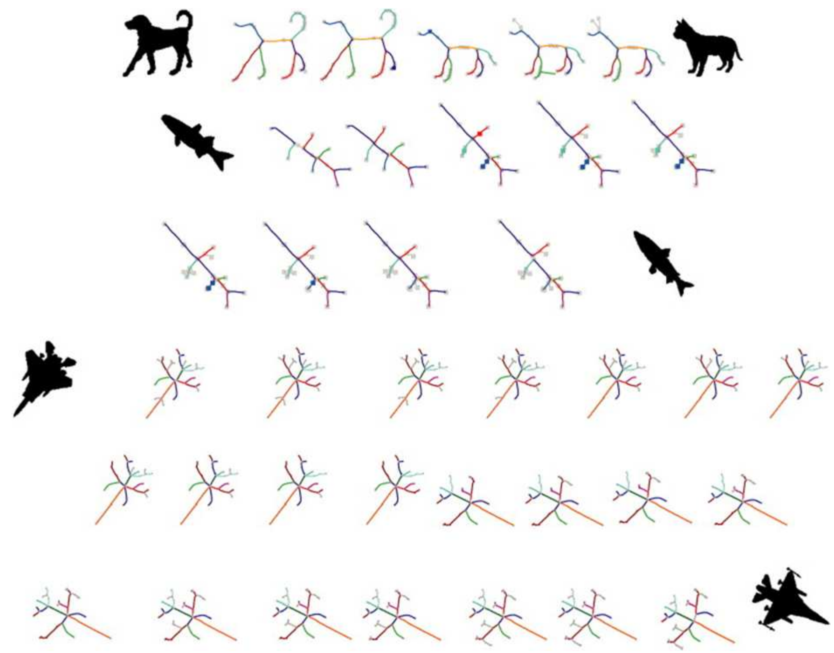


See the additional reading list of Lecture 4

2. Medial Axis Estimation For 2D and 3D Shapes

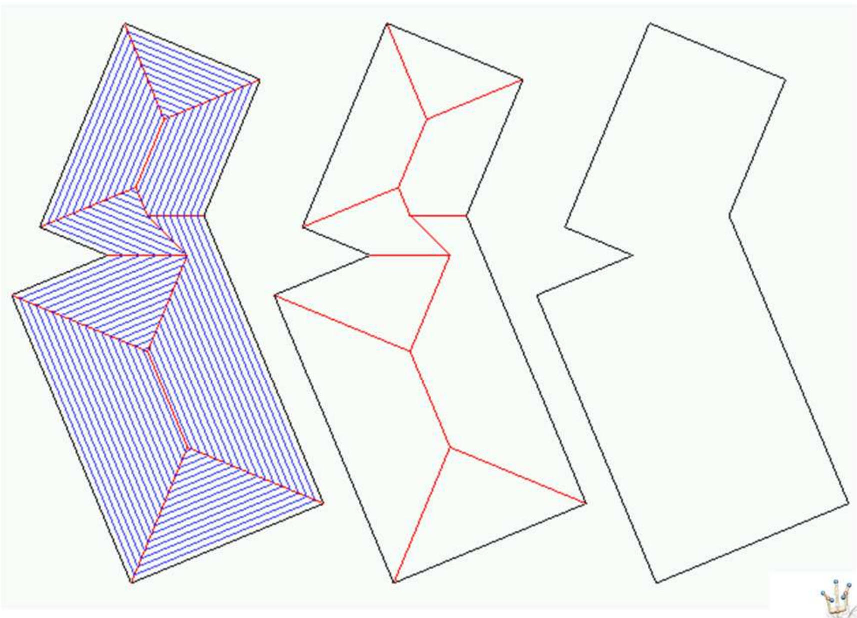


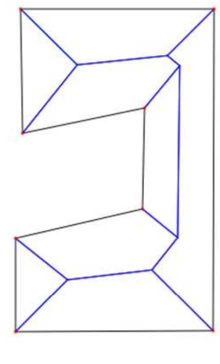


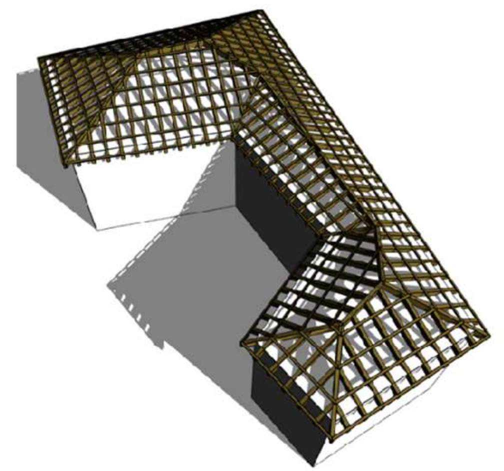


- Distance transform
- Medial axis transform
- Scale axis transform

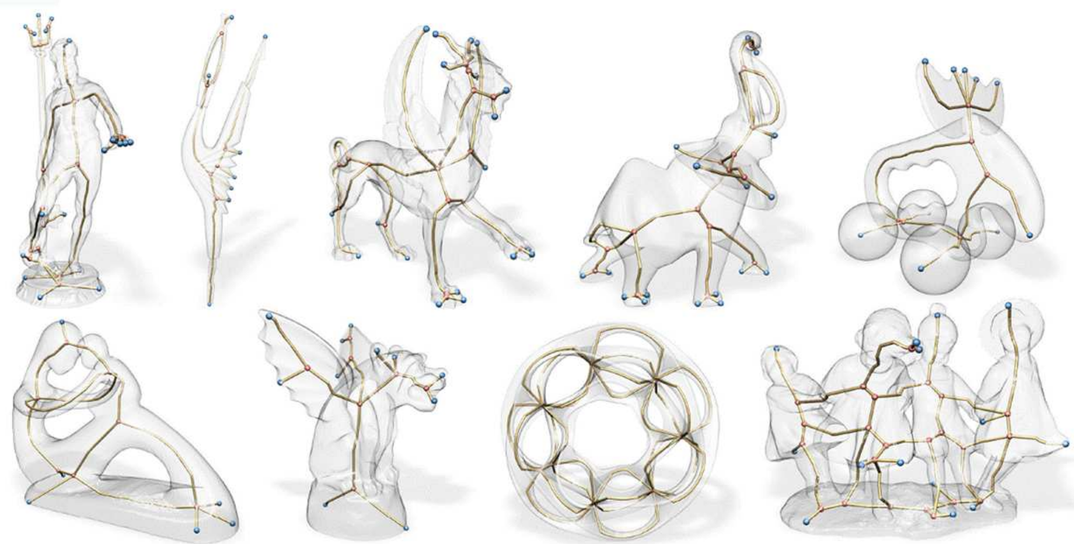
3. 1D Skeleton Extraction from 2D and 3D Shapes





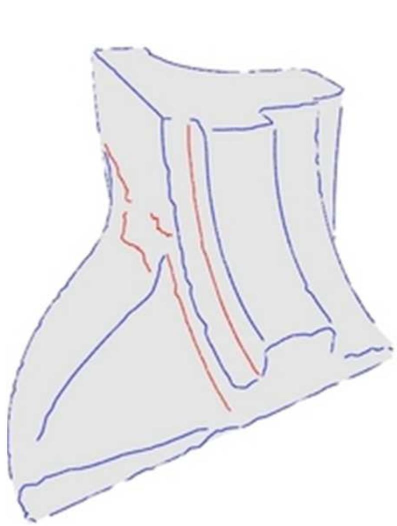


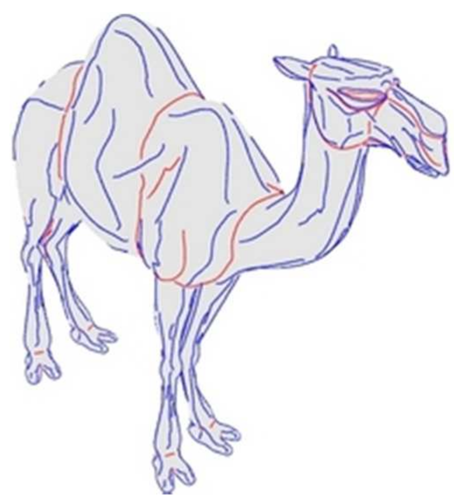
- Straight line skeleton
- Curve skeleton extraction using mesh contraction



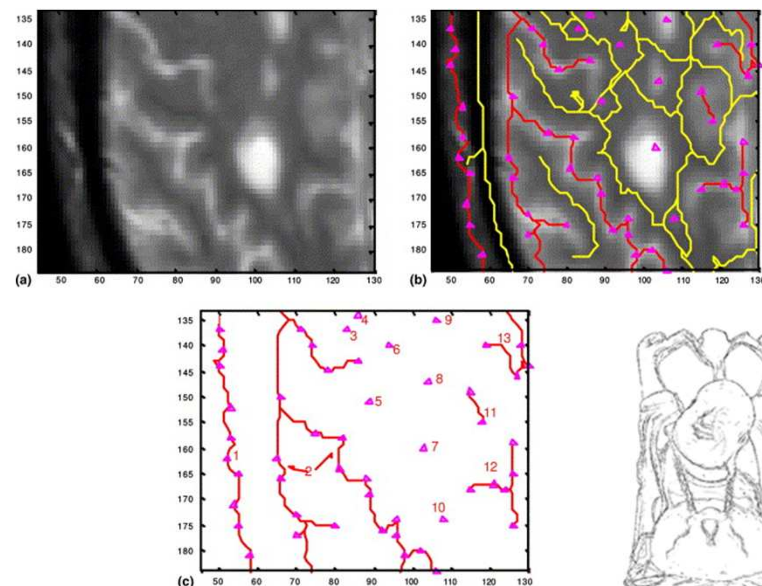
4. Ridges and Valleys Detection

- Ridges and valleys on surfaces

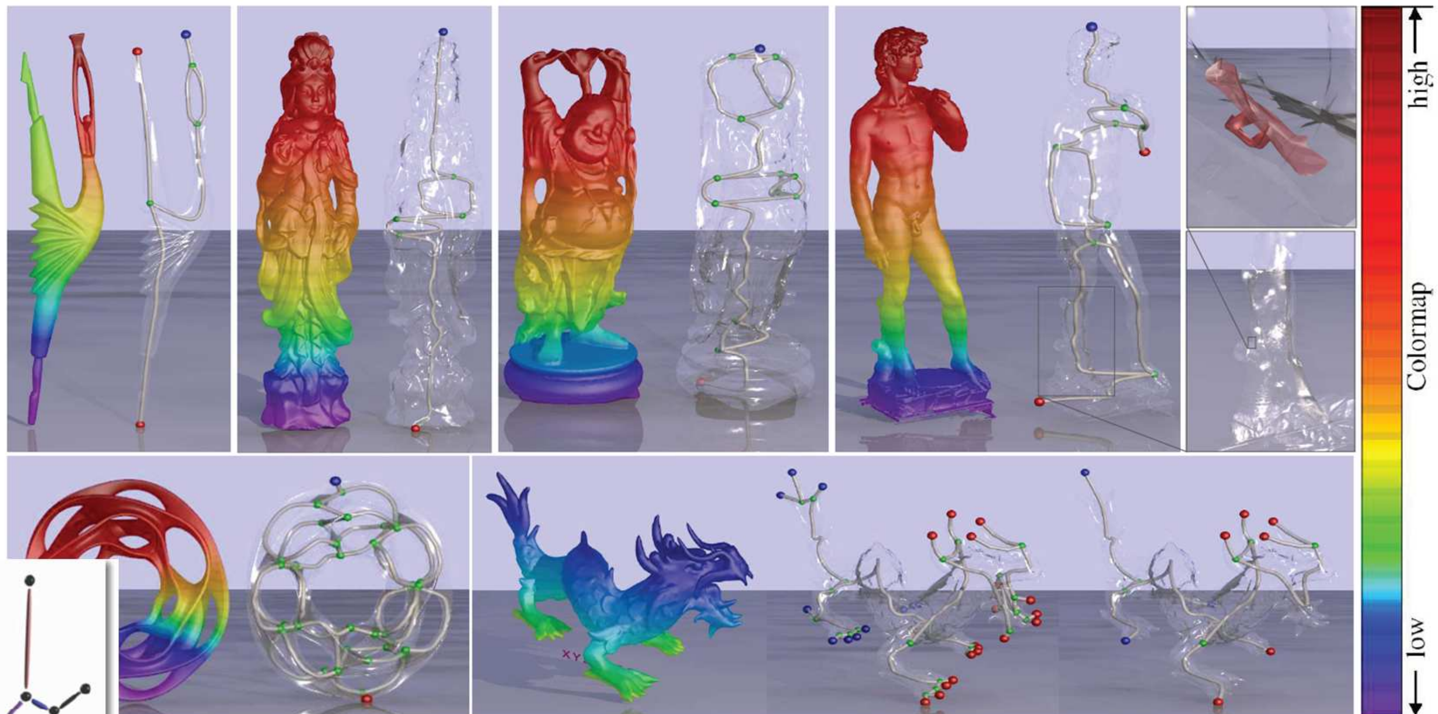


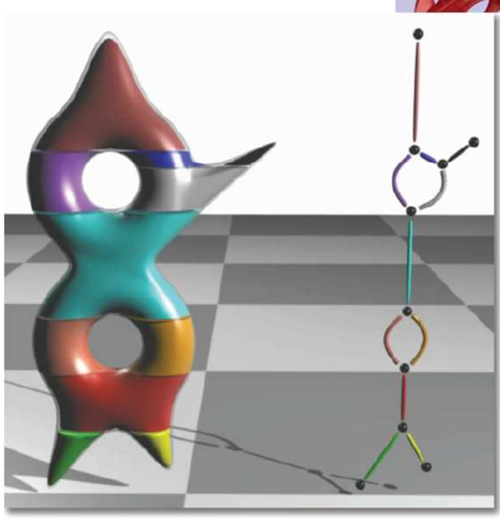


- Ridges and valleys in images
  - One example is the apparent ridges

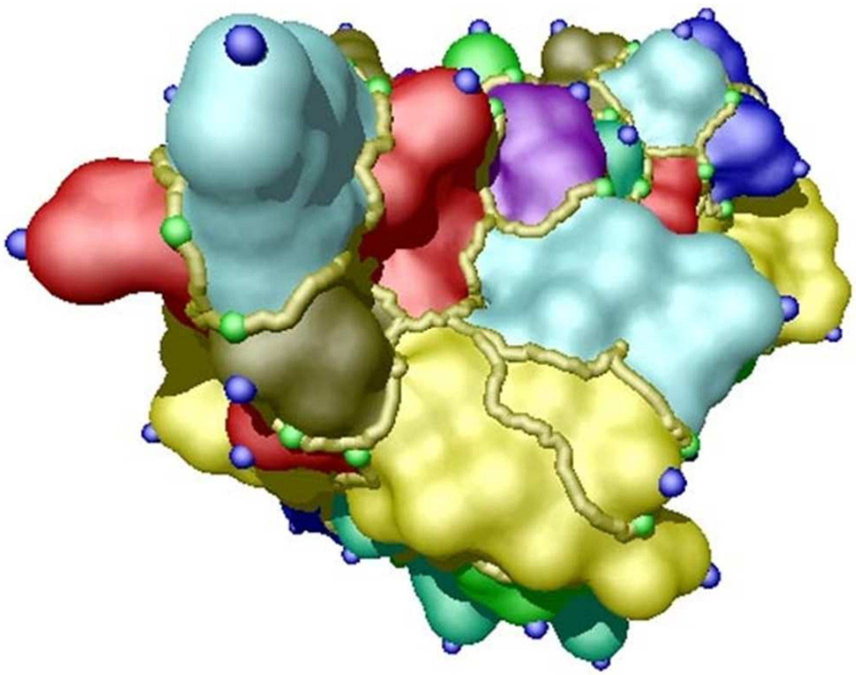


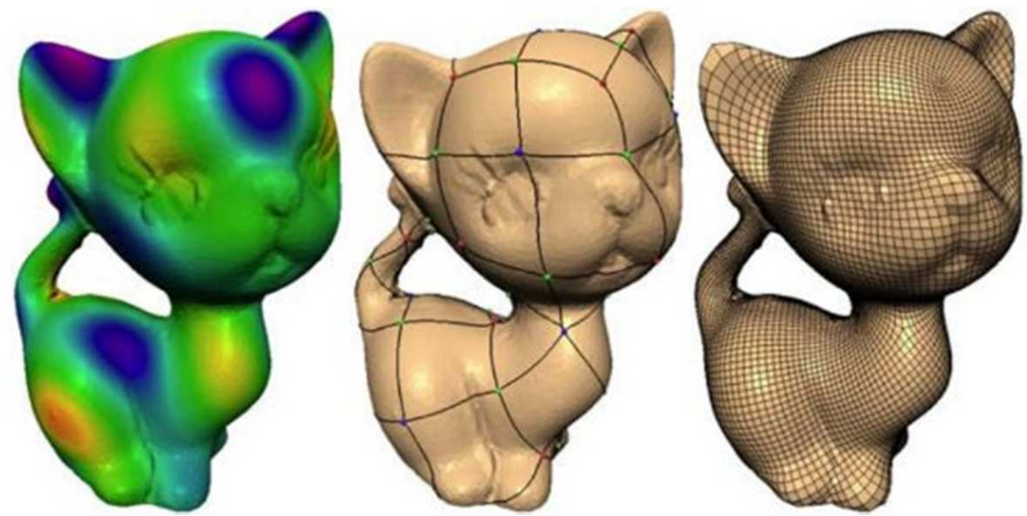
5. Compute Reeb Graphs of 3D Shapes





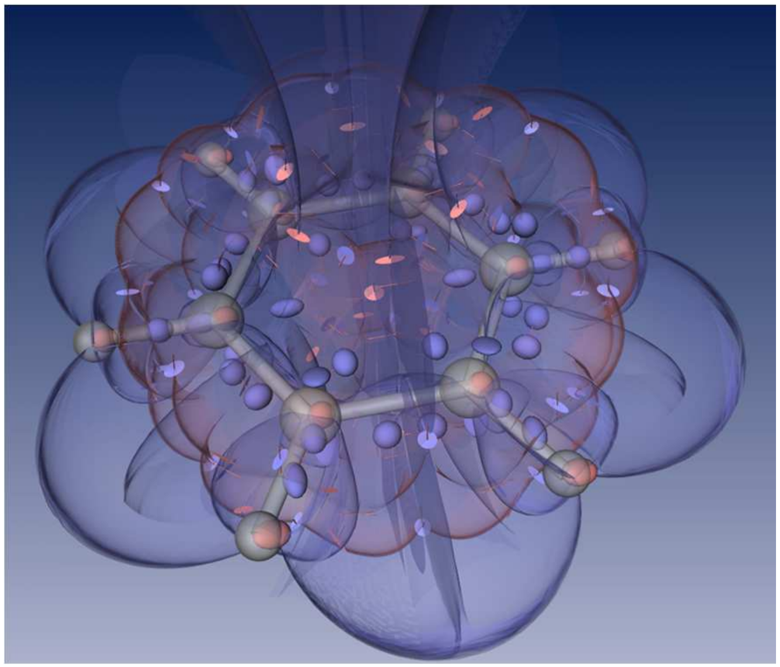
6. Compute Morse-Smale Complex

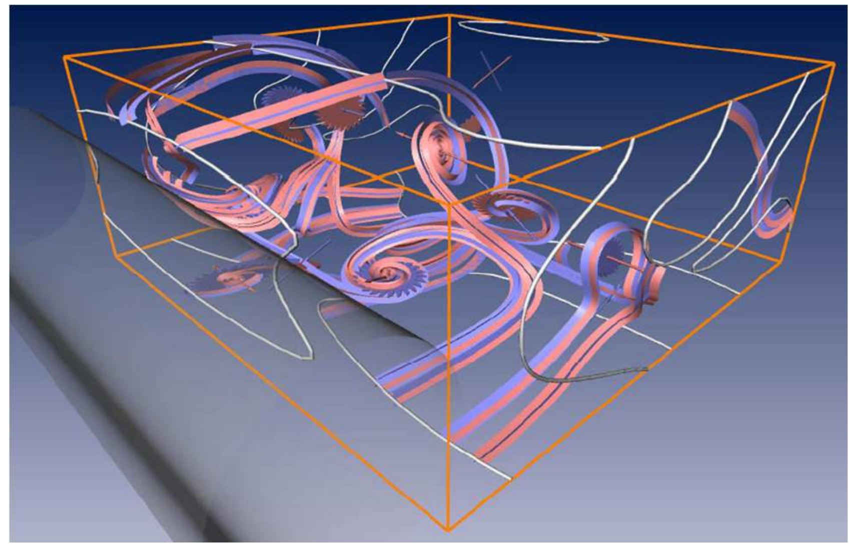




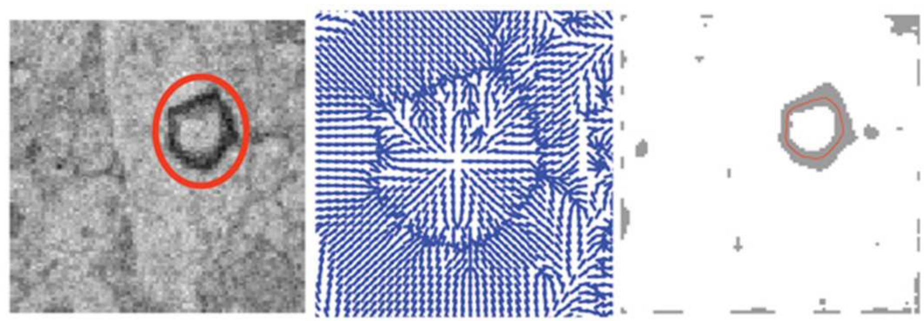
Applications?

7. Compute Differential Topology of Higher Dimensional Vector Fields



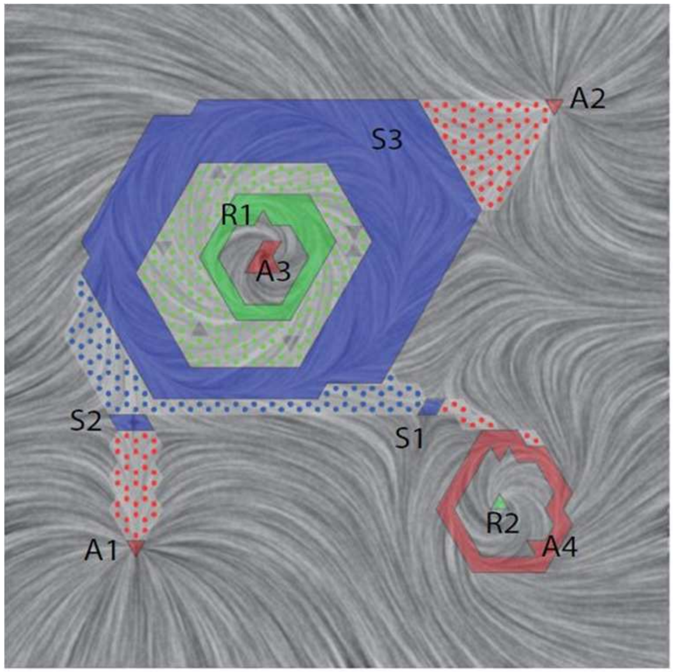


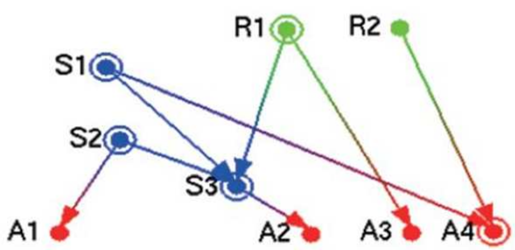
Vector field topology and its applications?

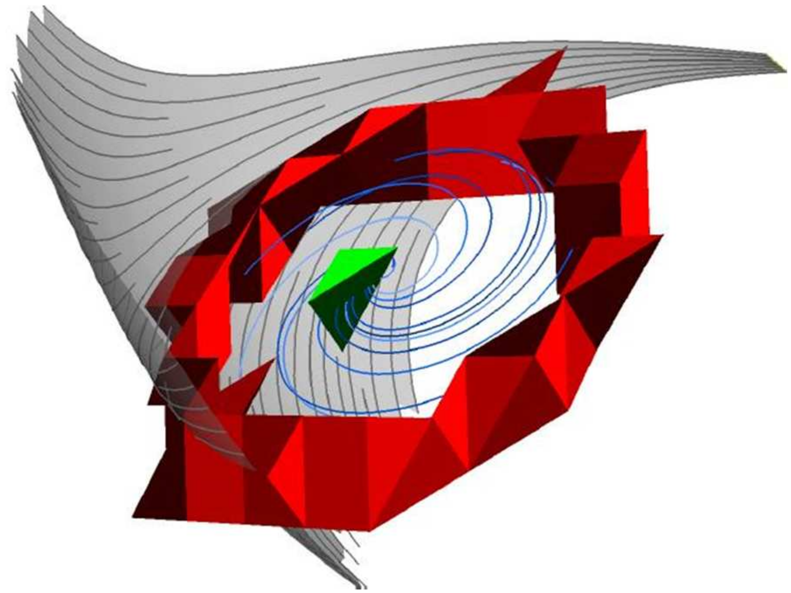


(A) (B) (C)

8. Compute Morse Decomposition

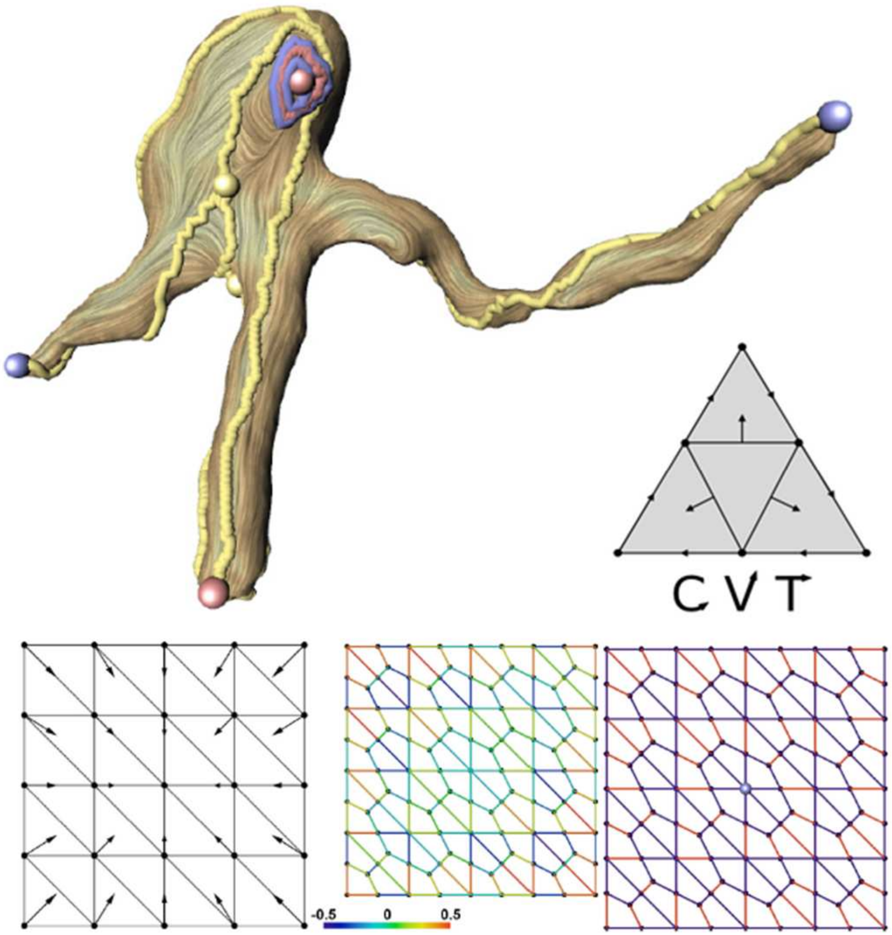


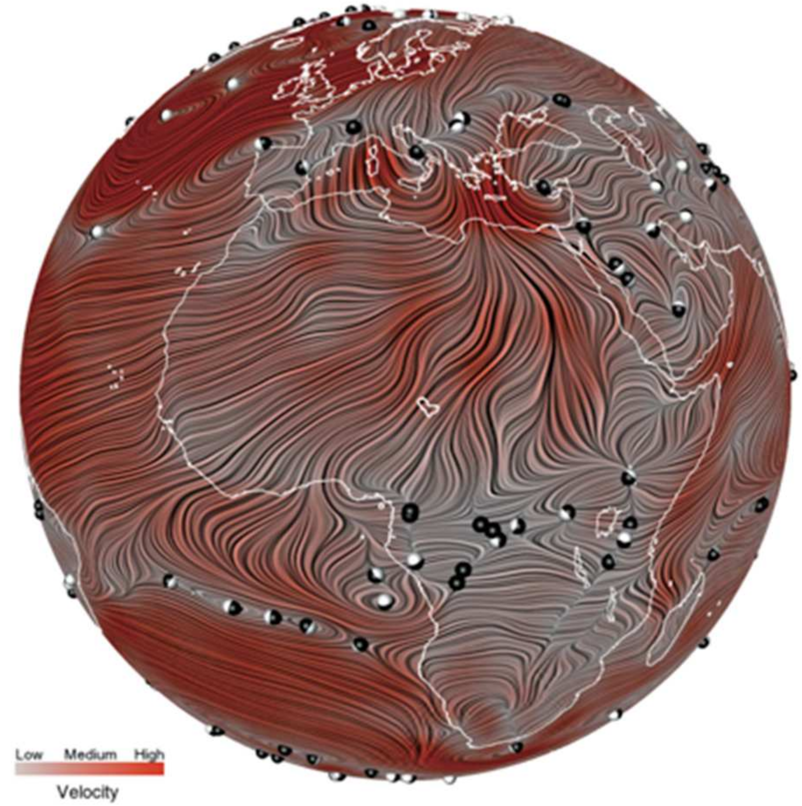




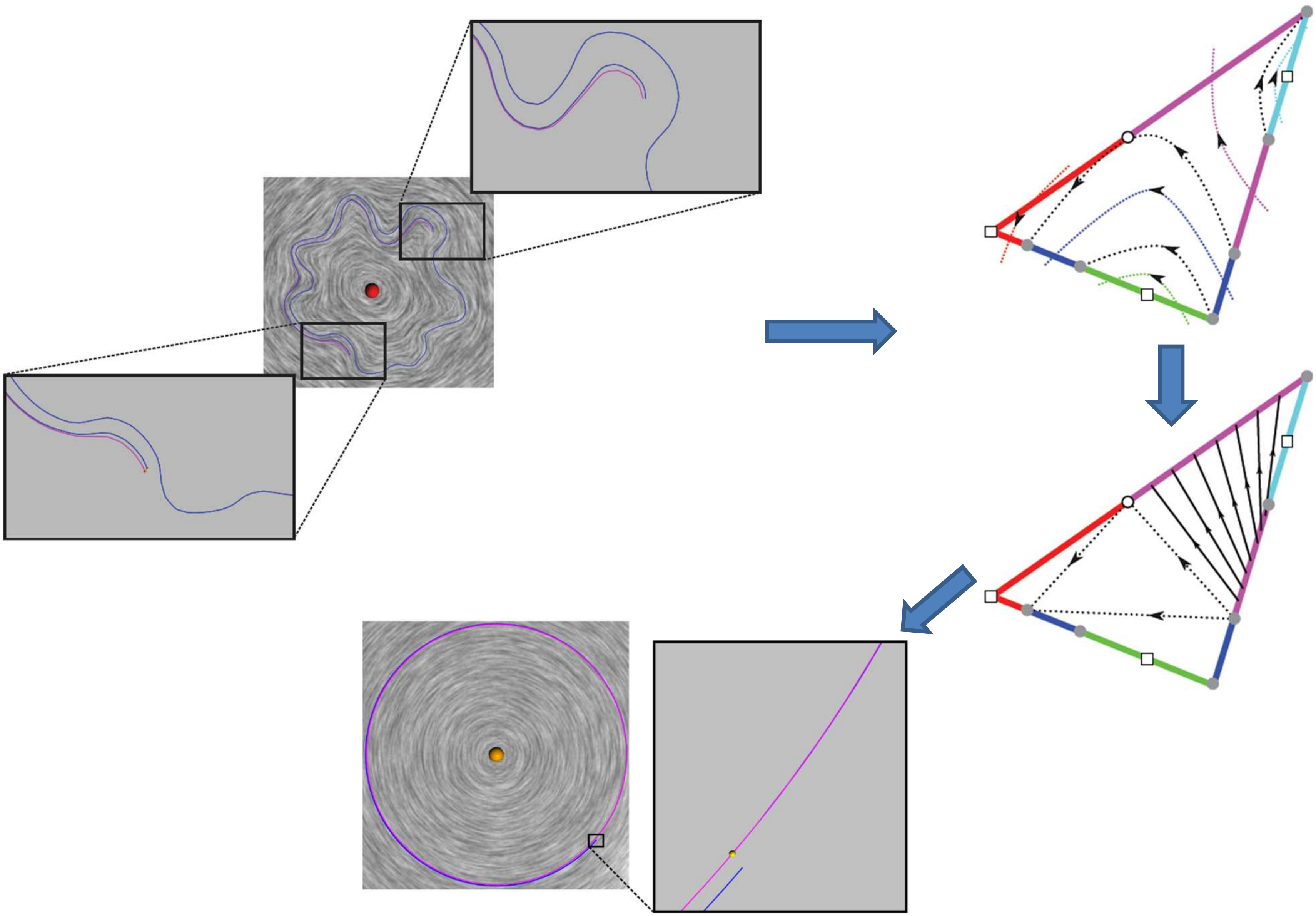
Applications?

9. Compute Combinatorial Topology

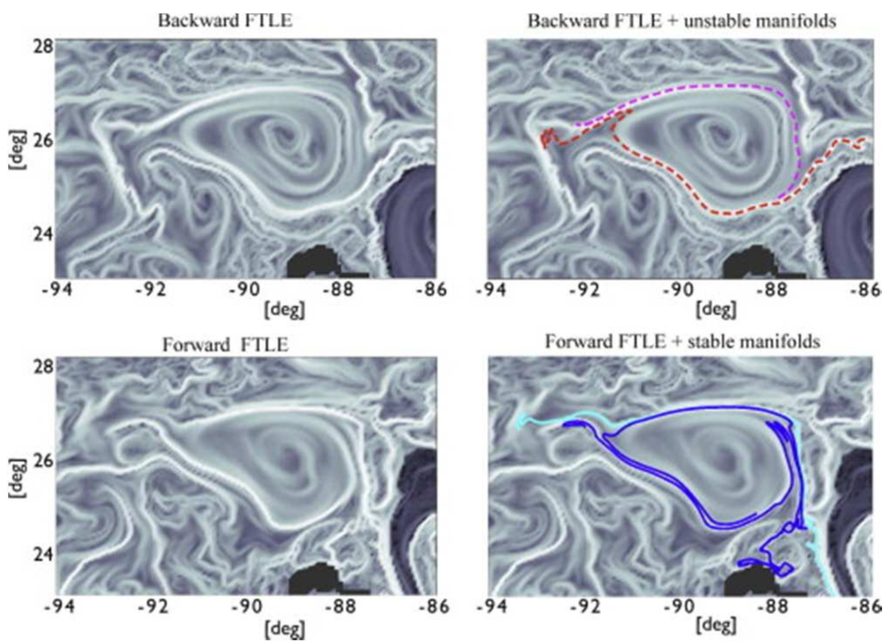


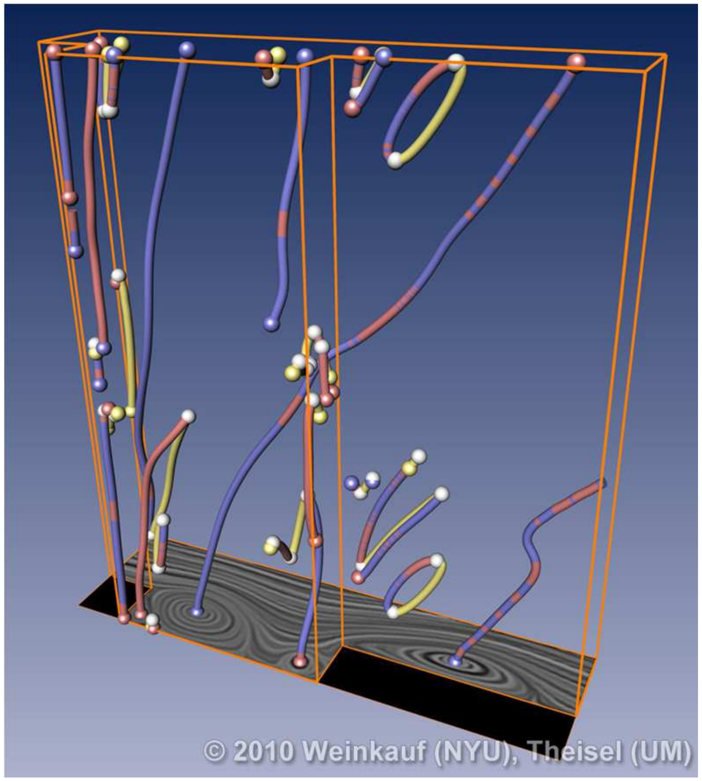


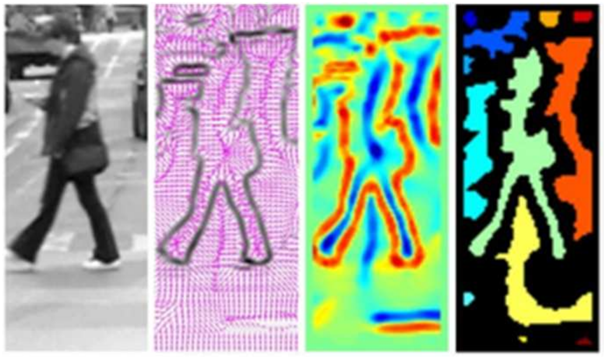
10. Edge-Map



11. Time Dependent Vector Field Analysis

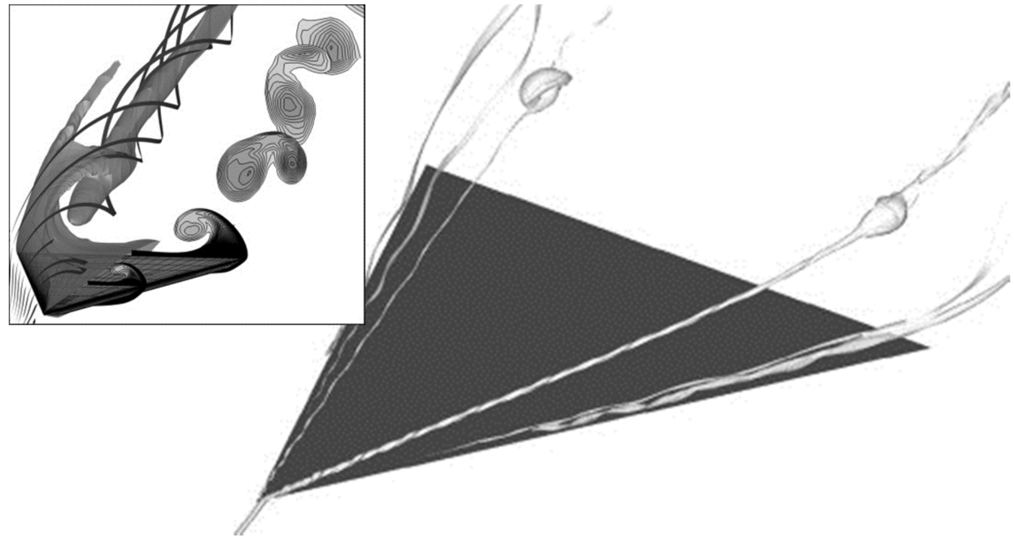


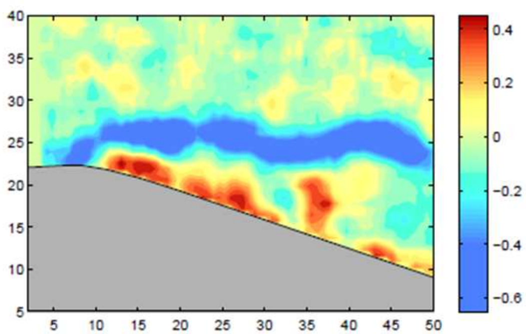


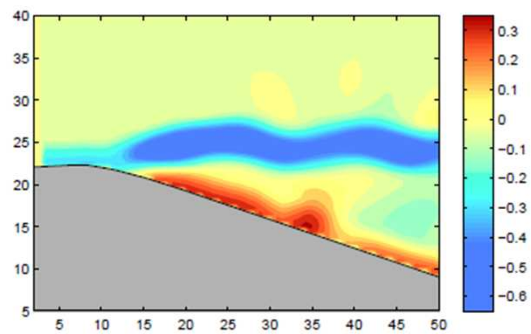


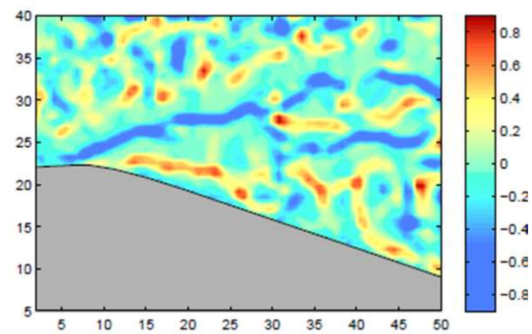
- Feature tracking
- FTLE computation
- Streak line/surface based topology

12. Vortex Detection

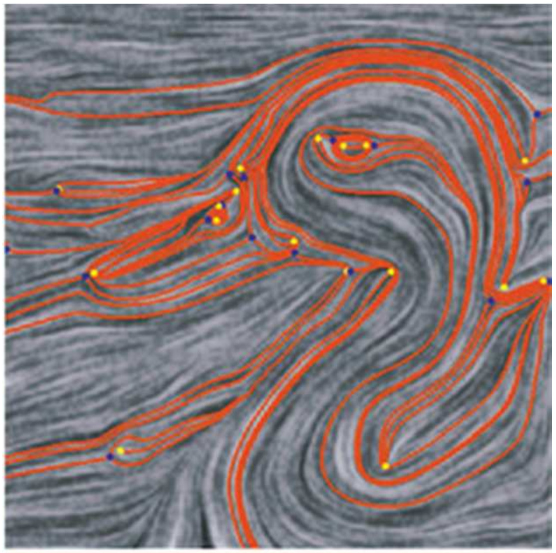


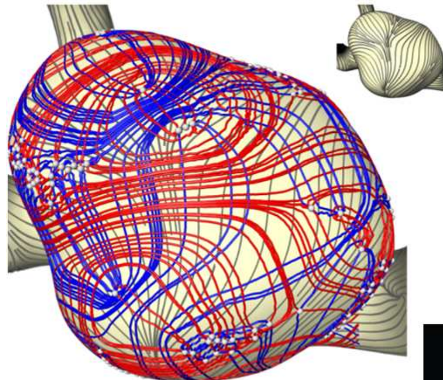


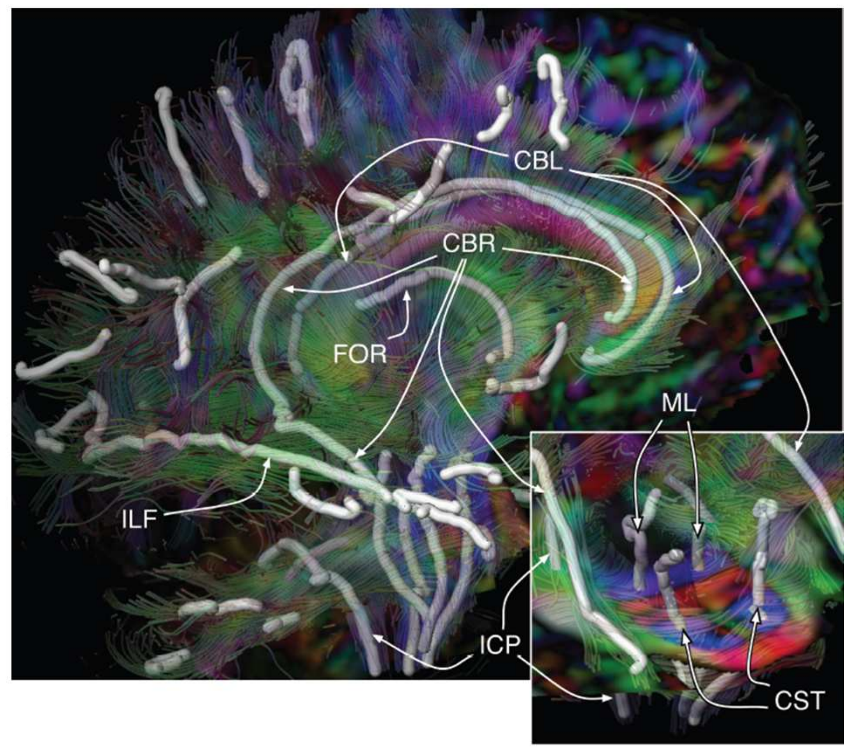




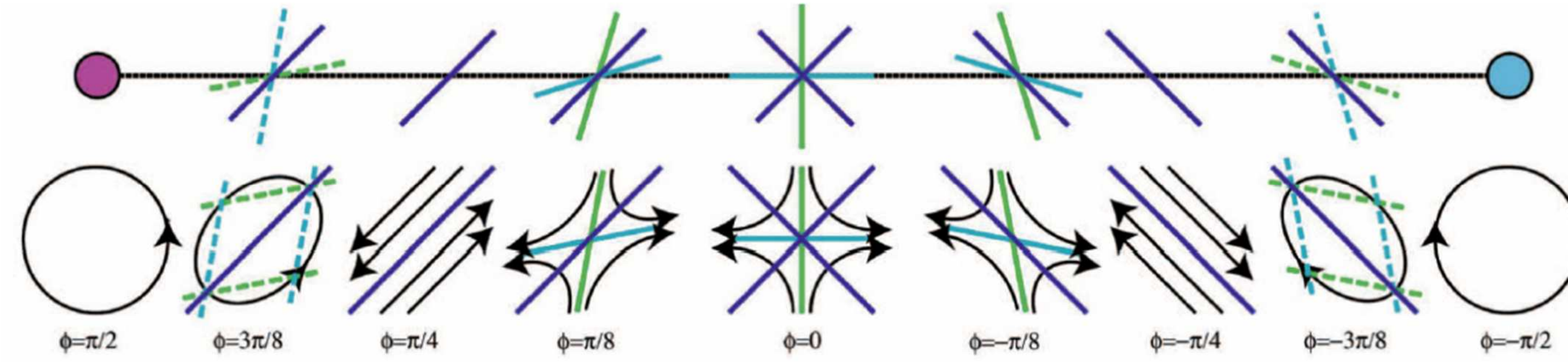
13. Symmetric Tensor Field Analysis

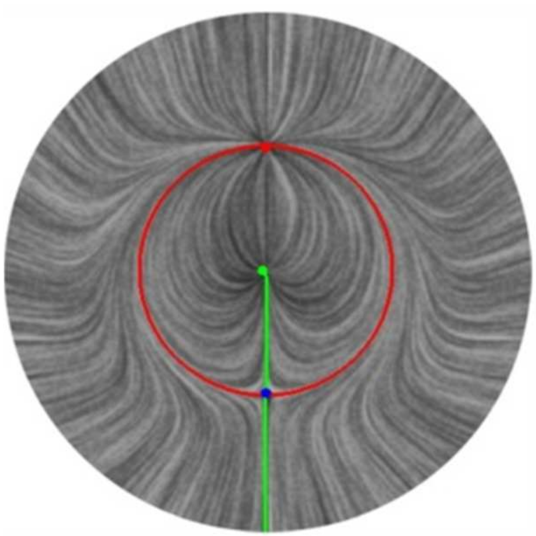


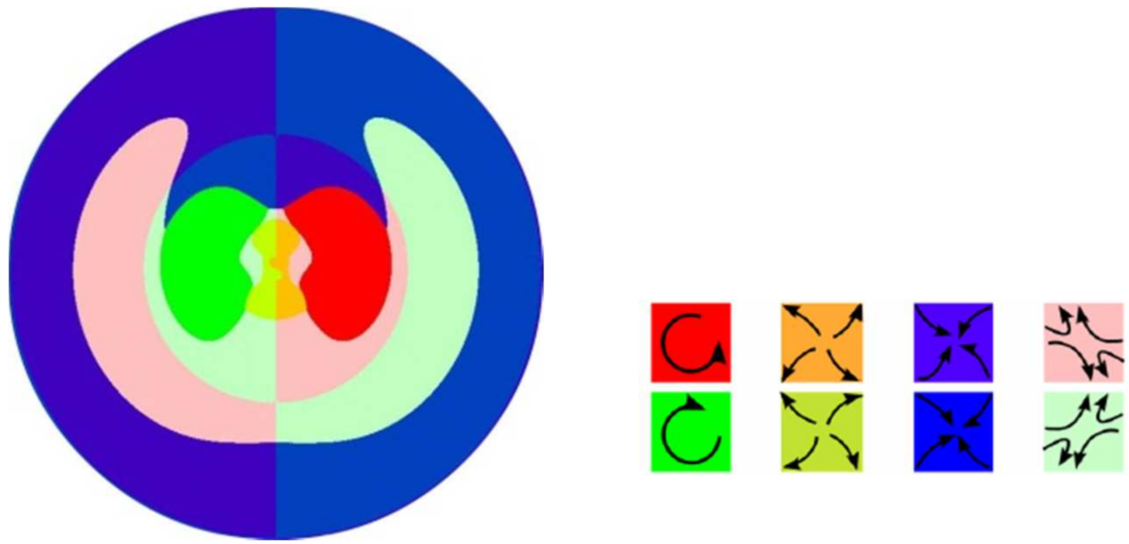




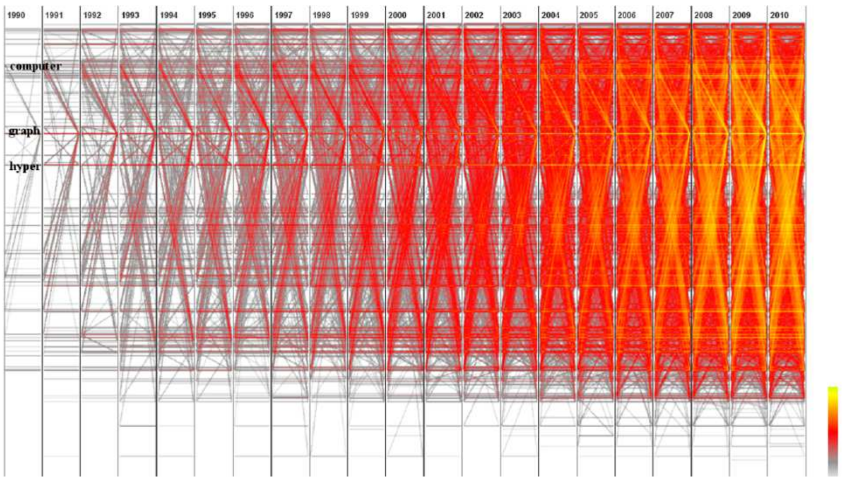
14. Asymmetric Tensor Field Analysis

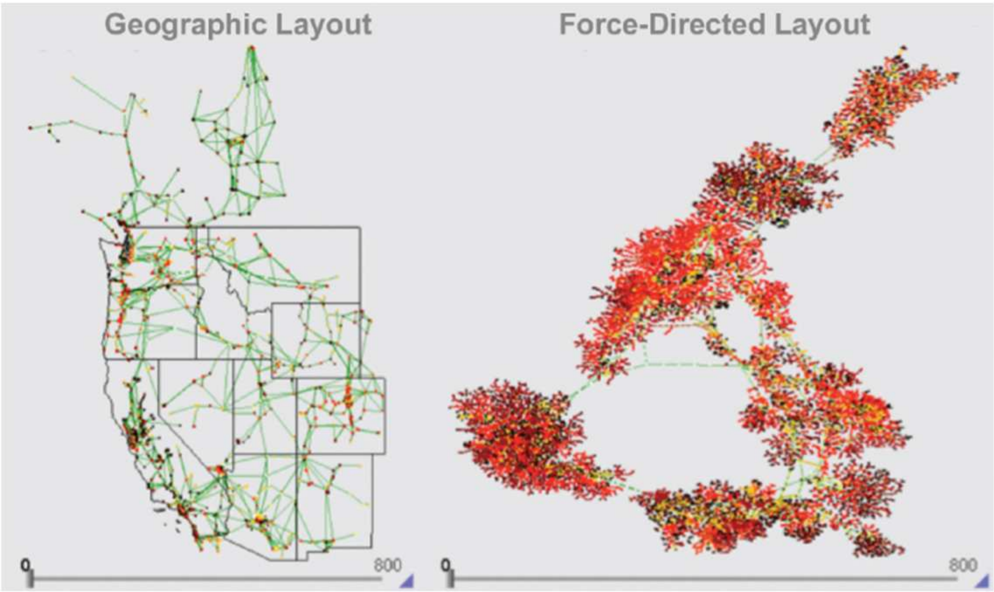


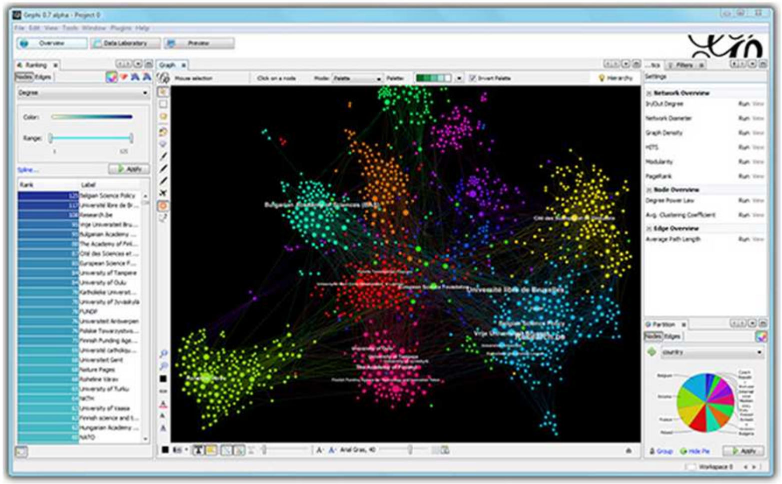




15. Large Dynamic Graph Visual Analytic



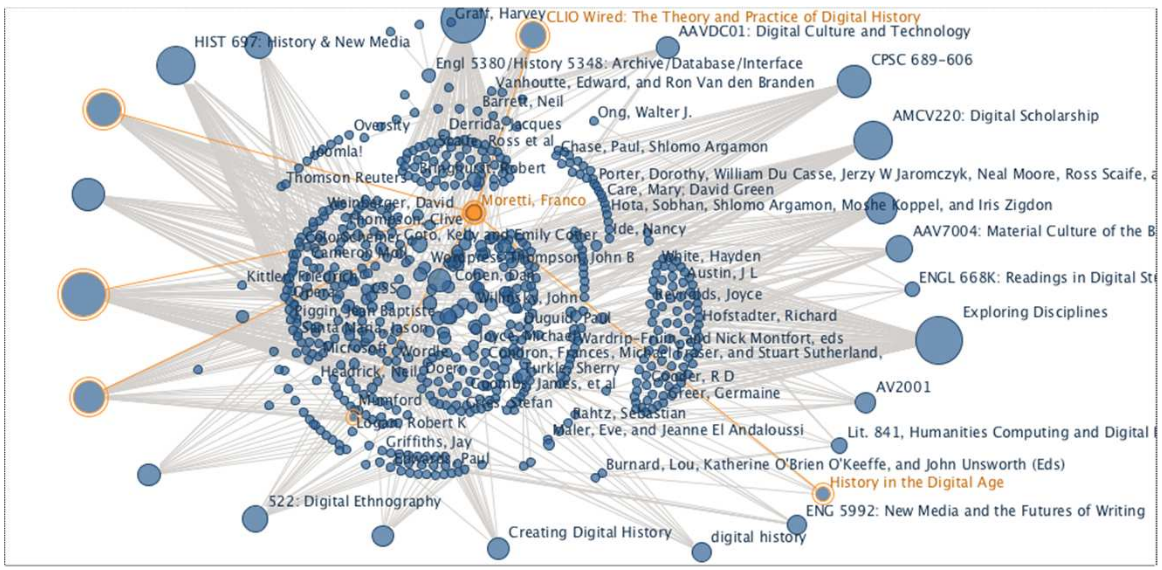


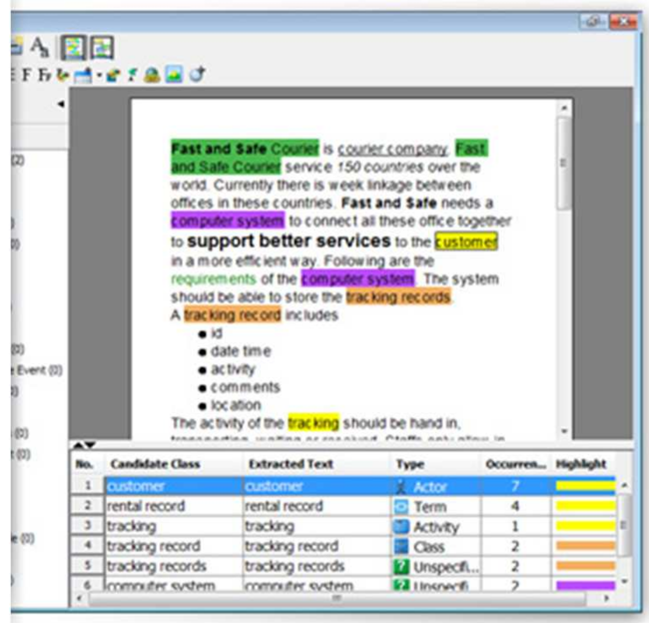


- Cluster identification
- Connectivity evaluation
- Event identification

Open source: https://gephi.org/
http://www.caida.org/tools/visualization/walrus/

16. Text Visual Analytics

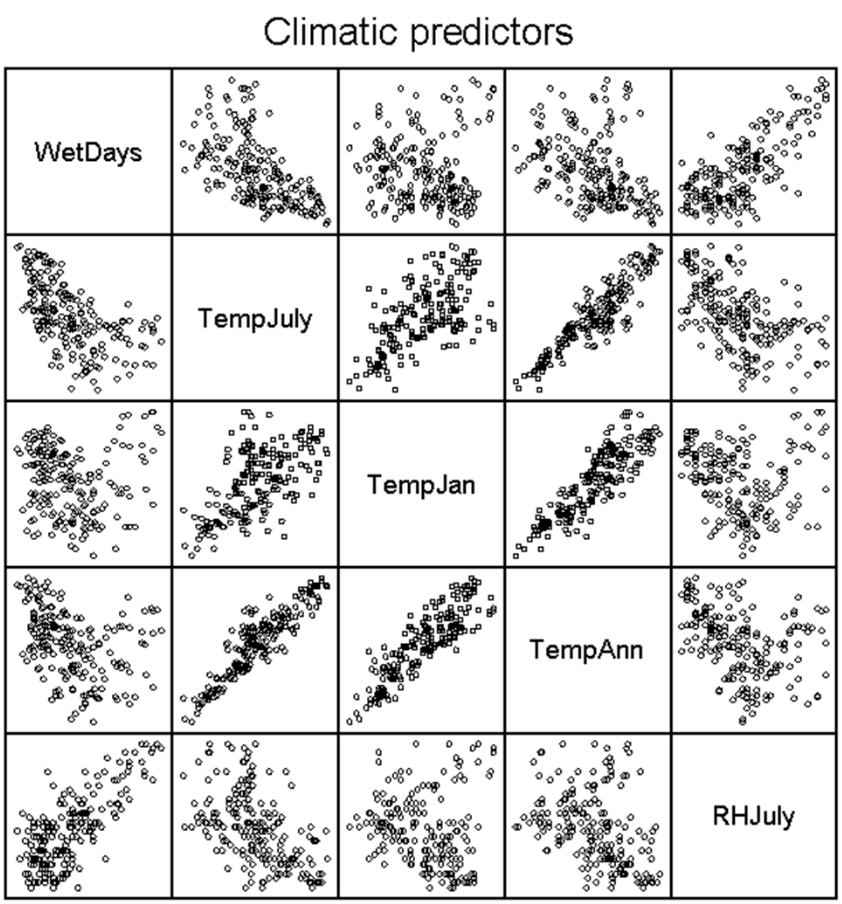




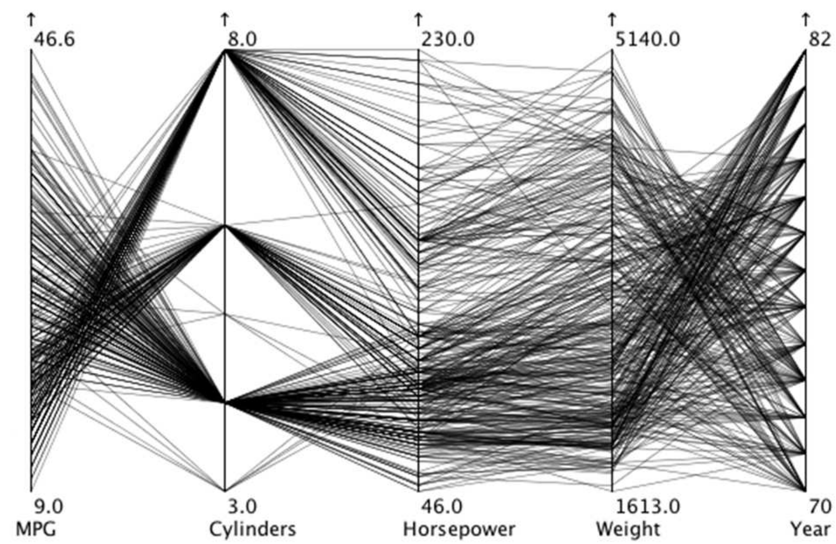
- Classifying documents
- Content extraction
- Relation identification

17. Higher-Dimensional Data Analysis

What is a person's life span given gender, annual income, living area, marriage status, medical care, smoking or not, …



What will determine/affect the temperature? Location (x, y, z), time in a day, humidity, pressure, cloudy or not, windy or not, human activities, …



What is your idea?