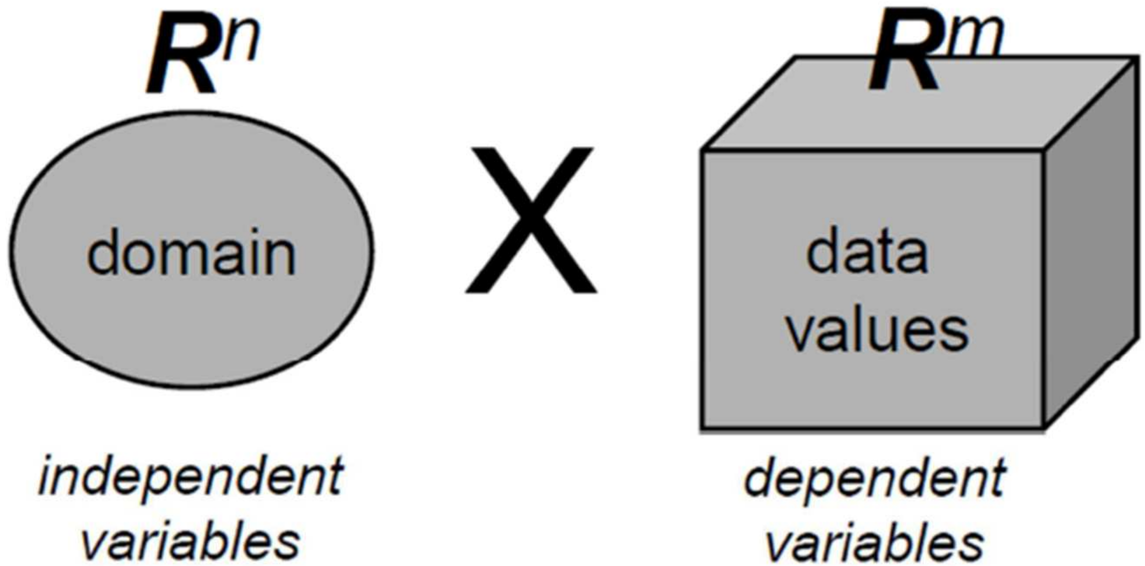


scientific data $\subseteq \boldsymbol{R}^{n+m}$

Source: VIS, University of Stuttgart

1D	scalar
2D	vector
3D	tensor
+time	

Tensor Data Analysis

A Simple Example of 2D Tensor

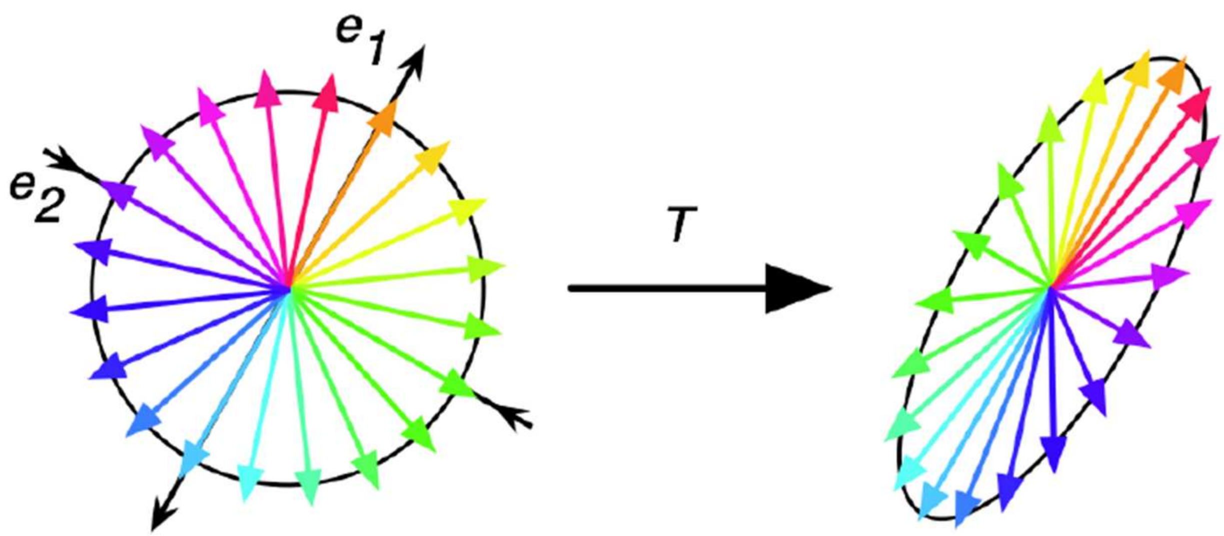
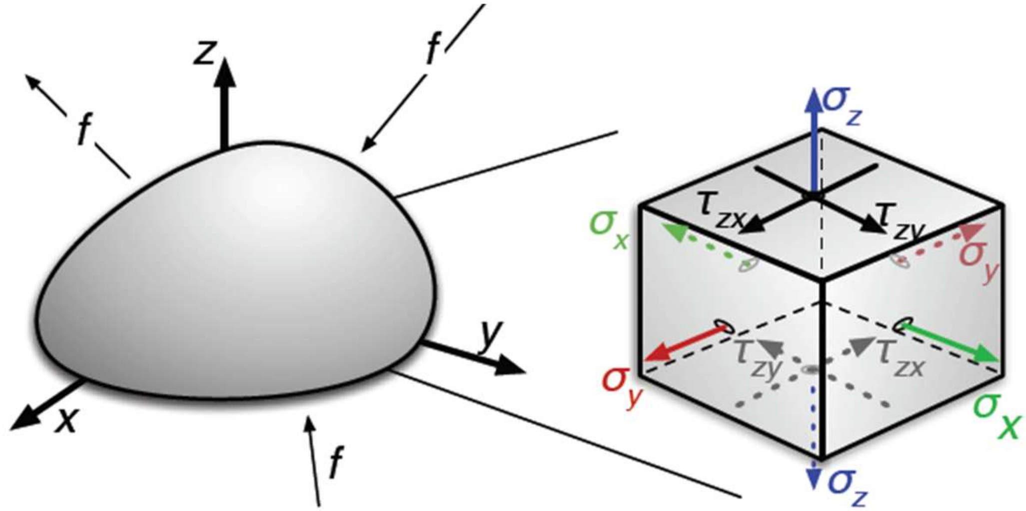


Illustration of a symmetric second-order tensor as linear operator. The tensor is uniquely determined by its action on all unit vectors, represented by the circle in the left image. The eigenvector directions are highlighted as black arrows. In this example one eigenvalue (e2) is negative. As a consequence all vectors are mirrored at the axis spanned by eigenvector e1. The eigenvectors are the directions with strongest normal deformation but no directional change.

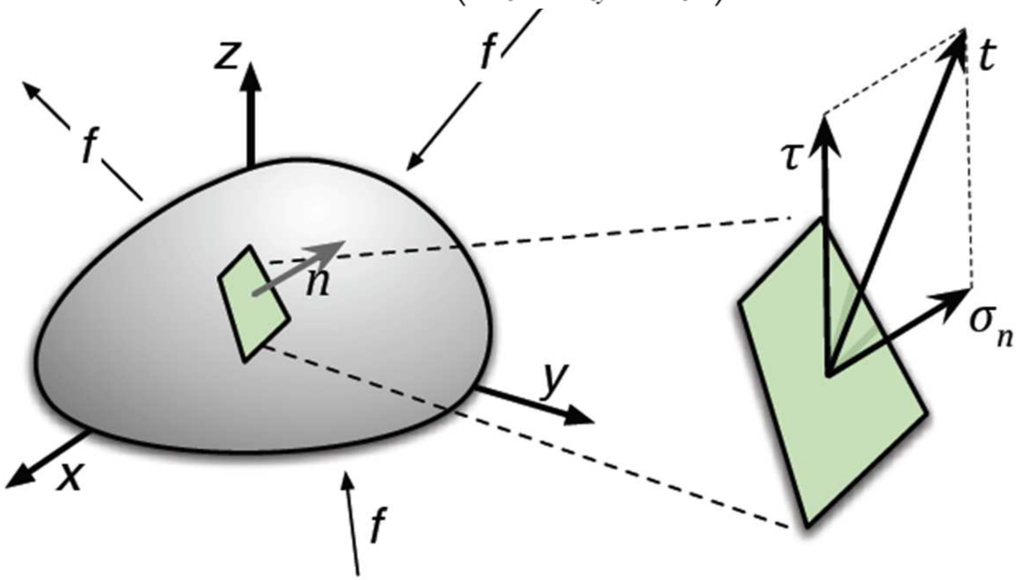
Applications

- Tensors describe entities that scalars and vectors cannot describe sufficiently, for example, the stress at a point in a continuous medium under load.
  - medicine,
  - geology,
  - astrophysics,
  - continuum mechanics
  - and many more

Tensors in Mechanical Engineering



$$t = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \cdot n = \tau_n + \sigma_n$$



Stress tensors describe internal forces or stresses that act within deformable bodies as reaction to external forces

(a) External forces f are applied to a deformable body. Reacting forces are described by a three-dimensional stress tensor that is composed of three normal stresses ***s*** and three shear stresses ***τ***.

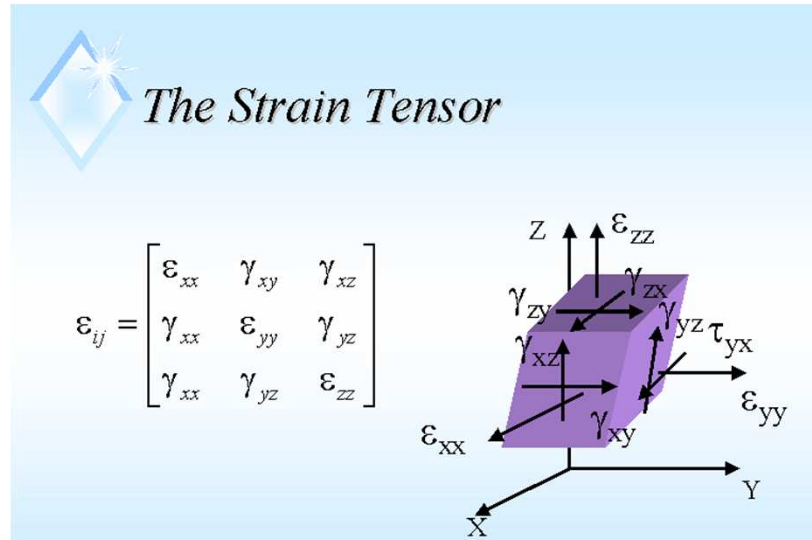
(b) Given a surface normal **n** of some cutting plane, the stress tensor maps **n** to the traction vector ***t***, which describes the internal forces that act on this plane (normal and shear stresses).

Tensors in Mechanical Engineering

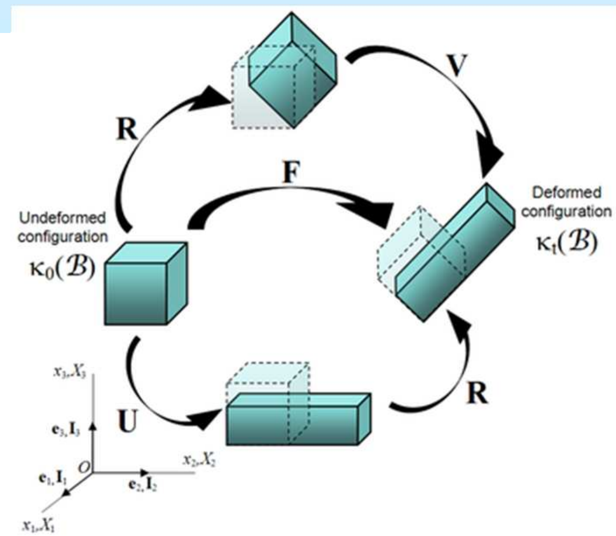
- Strain tensor - related to the deformation of a body due to stress by the material's constitutive behavior.

- Deformation gradient tensor – gradient of displacements of material points

- The strain tensor is a normalized measure based on the deformation gradient tensor



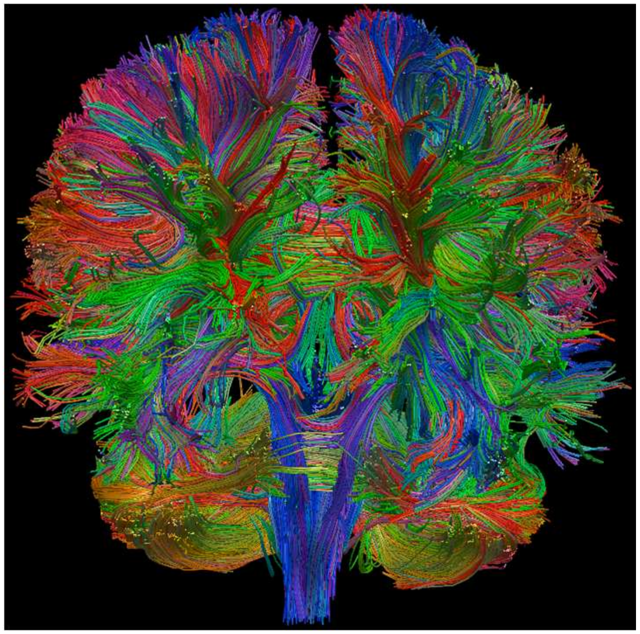
http://enpub.fulton.asu.edu/concrete/elasticity2_95/sld006.htm



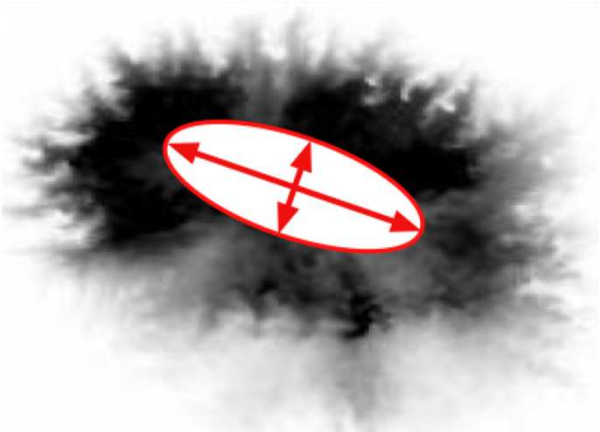
http://en.wikipedia.org/wiki/Finite_strain_theory

Diffusion Tensor Imaging (DTI)

- For medical applications, diffusion tensors describe the anisotropic diffusion behavior of water molecules in tissue.
- Here, the molecule motion is driven by the Brownian motion and not the concentration gradient.
- The tensor contains the following information about the diffusion: its strength depending on the direction and its anisotropy
- It is positive semi-definite and symmetric.

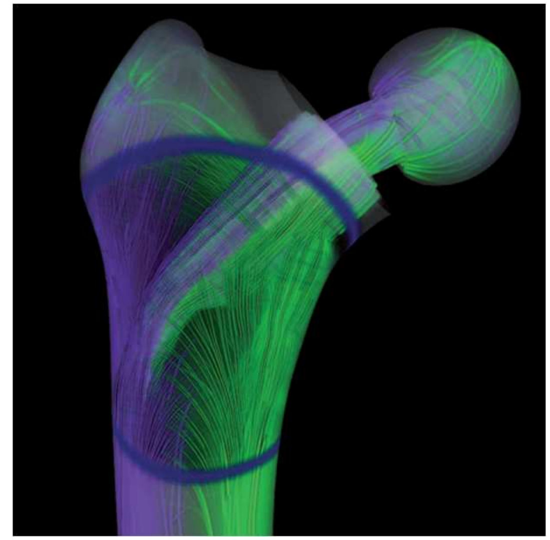


Note that in practice the positive definiteness of diffusion tensors can be violated due to measurement noise.

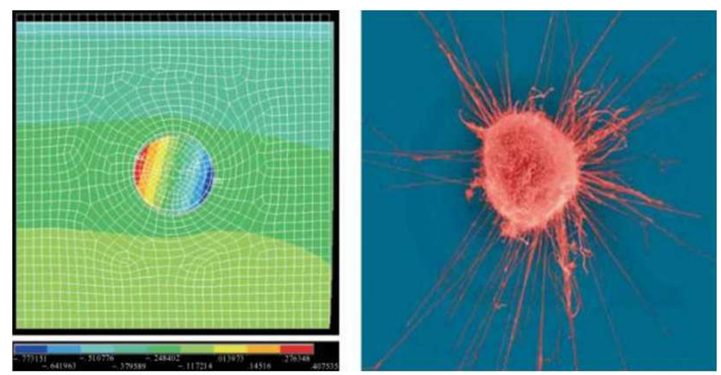


Tensors in Medicine (II)

- Diffusion tensors are not the only type of tensor that occur in the medical context.
- In the context of implant design, stress tensors result from simulations of an implant's impact on the distribution of physiological stress inside a bone.
- An application related to strain tensors is used in elastography where MRI, CT or ultrasound is used to measure elastic properties of soft tissues. Changes in the elastic properties of tissues can be an important hint to cancer or other diseases



[DICK et al. Vis09]

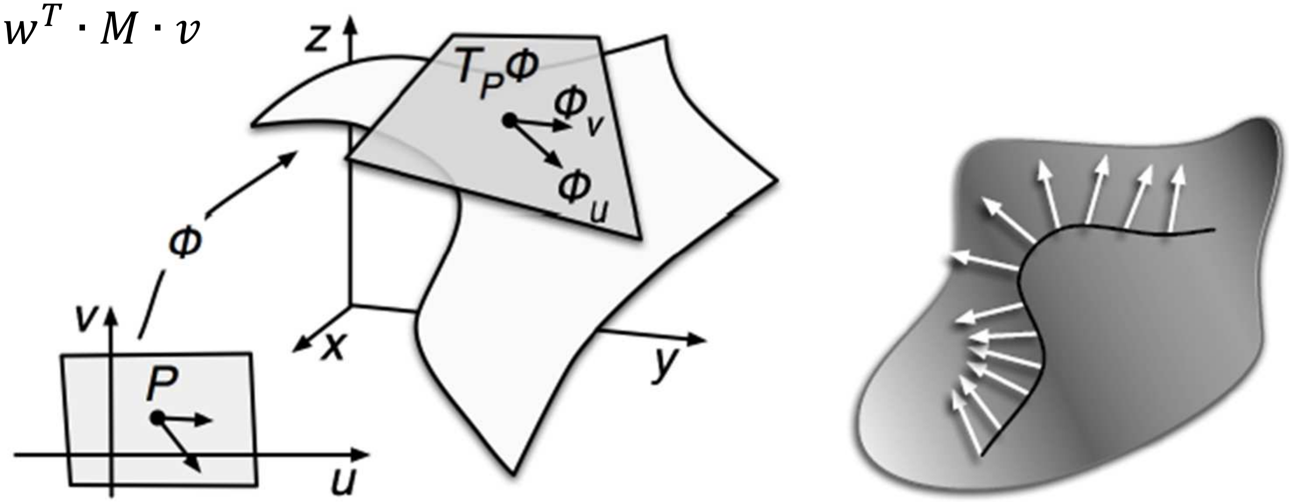


[SOSA-CABRERA et al., 2009]

Tensors in Geometry

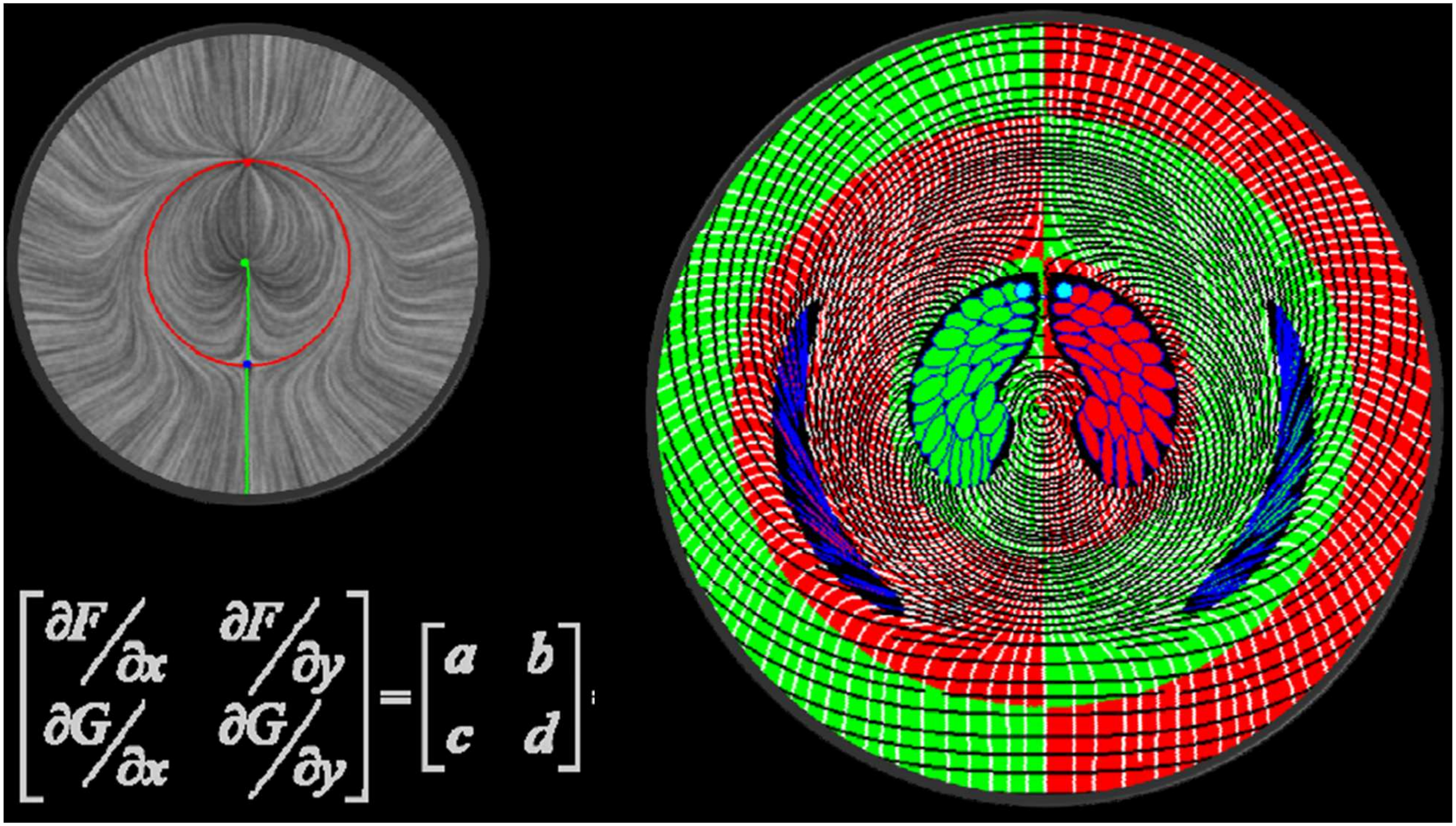
- Curvature tensors - change of surface normal in any given direction
- Metric tensors - relates a direction to distances and angles; defines how angles and the lengths of vectors are measured independently of the chosen reference frame

$$T(v, w) = w^T \cdot M \cdot v$$



(a) (b)

Gradient Tensor of Velocity Field



Tensors in Images

- Image analysis

[Zhang et al, TVCG2007]

- Computer vision

Some Math of Tensors

Definition

- A second-order tensor T is defined as a bilinear function from two copies of a vector space V into the space of real numbers

$$T: V \otimes V \rightarrow R$$

- Or: a second-order tensor T as linear operator that maps any vector v ∈V onto another vector w ∈ V

$$T: V \rightarrow V$$

- The definition of a tensor as a linear operator is prevalent in physics.

Definition

- Tensors are generally represented with respect to a specific Cartesian basis $\{b_1, \dots b_n\}$of the vector space V.
- In this case, the tensor is uniquely defined by its components and is represented as a matrix.

- Considering definition (1), we have

$$T(v, w) = w^T \cdot M \cdot v \qquad \forall v, w \in V$$

where $v = v_1 b_1 + \cdots + v_n\, b_n$, w$= w_1 b_1 + \cdots + w_n\, b_n$

- For(2), we have $T(v) = M \cdot v$

Tensor Invariance

- Tensors are independent of specific reference frames, i.e. they are invariant under coordinate transformations.

- Invariance qualifies tensors to describe physical processes independent of the coordinate system. More precisely, the tensor components change according to the transformation into another basis; the characteristics of the tensor are preserved. Consequently, tensors can be analyzed using any convenient reference frame.

- Rotational invariant
- Affine invariant

Tensor Diagonalization

- The tensor representation becomes especially simple if it can be diagonalized.
- The complete transformation of T from an arbitrary basis into the eigenvector basis, is given by

$$UTU^T = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

- The diagonal elements λ_1, λ_2, λ_3 are the eigenvalues and U is the orthogonal matrix that is composed of the eigenvectors, that is $(e_1$, e_2, $e_3)$

- The diagonalization generally is computed numerically via singular value decomposition (SVD) or principal component analysis (PCA).

Tensor Properties

- **Symmetric Tensors**. A tensor S is called symmetric if it is invariant under permutations of its arguments
$$S(v, w) = S(w, v) \quad \forall v, w \in V$$

- **Antisymmetric Tensors**. A tensor A is called antisymmetric or skew-symmetric if the sign flips when two adjacent arguments are exchanged
$$A(v, w) = -A(w, v) \quad \forall v, w \in V$$

- **Traceless Tensors**. Tensors T with zero trace, i.e. $tr(T) \ = \sum_{i=0}^{n-1} T_{ii}$, are called traceless.

Tensor Properties

- **Positive (Semi-) Definite Tensors**. A tensor T is called positive (semi-) definite if
$$T(v,v) > (\geq) 0$$
Their eigenvalues and their determinant are greater than zero.

- **Negative (Semi-) Definite Tensors**. A tensor T is called negative (semi-) definite if
$$T(v,v) < (\leq) 0$$
their determinants are smaller than (smaller than or equal to) zero.

- **Indefinite Tensors**. Each tensor that is neither positive definite nor negative definite is indefinite.

Tensor Decompositions

- **Symmetric/Antisymmetric Part**. For non-symmetric tensors T, the decomposition into a symmetric part S and an antisymmetric part A is a common practice:

$$T = S + A$$

where $S = \frac{1}{2}(T + T^T)$, $A = \frac{1}{2}(T - T^T)$

- Physically, antisymmetric part contains rotational information and the symmetric part contains information about isotropic scaling and anisotropic shear.

Tensor Decompositions

- **Isotropic/Anisotropic Part.** Symmetric tensors can be decomposed into an isotropic T_{iso} and an anisotropic (deviatoric) part D
$$T = \frac{1}{3} tr(T) I + (T - T_{iso})$$

- From a physical point of view, the isotropic part represents a direction independent transformation (e.g., a uniform scaling or uniform pressure); the deviatoric part represents the distortion.

Tensor Decompositions

- **Stretch/Rotation**. Another useful decomposition of nonsymmetric, positive-definite tensors T (e.g. deformation gradient tensors) is the polar decomposition. It decomposes the transformation represented by T in a two-stage process: a rotation R and a right stretch U or a left stretch V

$$T = R \cdot U = V \cdot R$$

- A tensor is called stretch if it is symmetric and positive definite. A tensor is called rotation if it is orthogonal with determinant equal to one.

Tensor Decompositions

- **Shape/Orientation**. Via eigen analysis symmetric tensors are separated into shape and orientation.
  - Here, shape refers to the eigenvalues and orientation to the eigenvectors.
  - Note that the orientation field is not a vector field due to the bidirectionality of eigenvectors.

Second-order Tensor Fields

- In visualization, usually not only a single tensor but a whole tensor field is of interest. This gives rise to a tensor field.

Features?

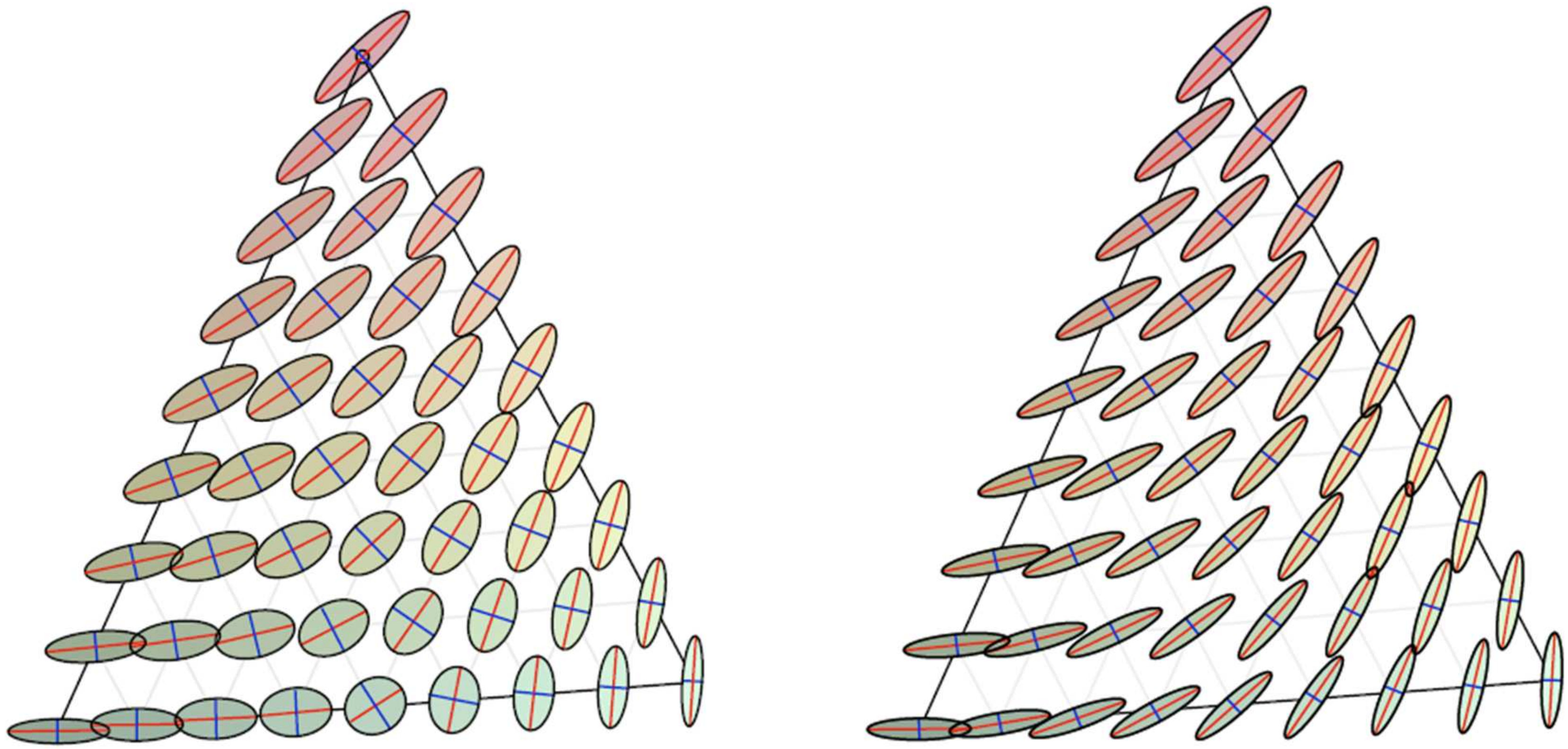
- Scalar related
  - Components
  - Determinant
  - Trace
  - Eigen-values

- Vector related
  - Eigen-vector fields

Tensor Interpolation

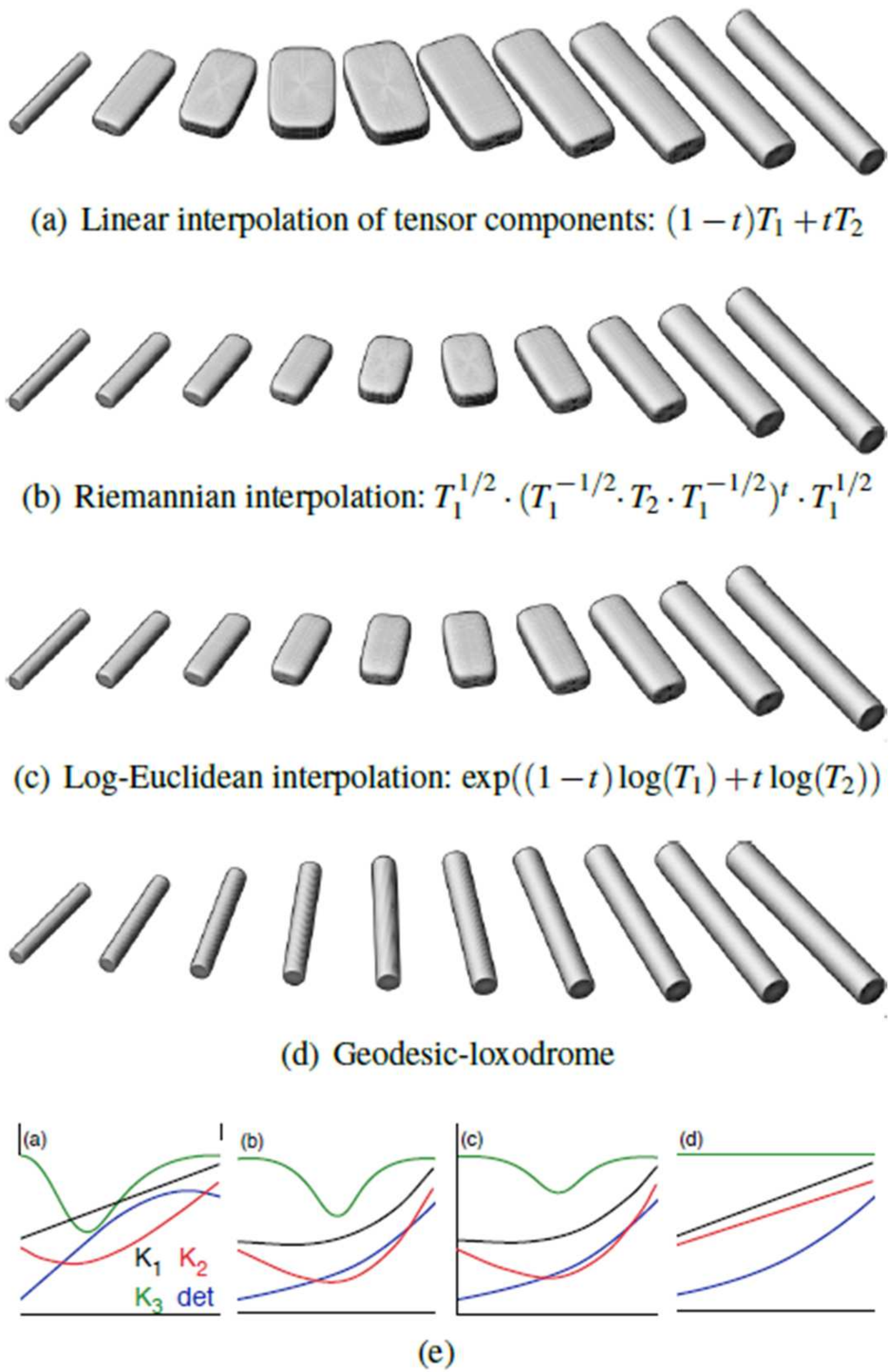
- Challenges
  - Natural representation of the original data.
    - This includes the preservation of central tensor properties (e.g., positive definiteness) and/or important scalar tensor invariants (e.g., the determinant).
  - Consistency.
    - consistent with the topology of the original data.
  - Invariance.
    - The resulting interpolation scheme needs to be invariant with respect to orthogonal changes of the reference frame.
  - Efficiency.
    - The challenge is to design an algorithm that represents a tradeoff between the above mentioned criteria and computational efficiency.

Tensor Interpolation



(a) (b)

Comparison of component-wise tensor interpolation (a) and linear interpolation of eigenvectors and eigenvalues (b). Observing the tensors depicted by ellipses, the comparison reveals that the separate interpolation of direction and shape is much more shape-preserving (b).



Interpolations between two three-dimensional positive-definite tensors T1 and T2. The interpolation results are represented using superquadrics ((a)-(d)).

The plots (e) show the behavior of four tensor invariants for the respective interpolations: det(T), K1 =tr(T), K2 = ||D|| and K3 = det(D/||D||), where D is the deviator of T and ||.|| is the Frobenius norm. Image courtesy Kindlmann et al. 2007.

Tensor Field Segmentation

- The goal of tensor segmentation algorithms is to aggregate regions that exhibit similar data characteristics to ease the analysis and interpretation of the data.

- Two classes
  - Segmentation or clustering based on certain similarity (or dis-similarity) metric
  - Topology-based

Tensor Field Segmentation

- Challenges
  - Similarity measure
    - Depending on the task or visualization goal, a first step comprises the choice of appropriate quantities (derived features or original tensor data). These in turn influence or even determine the choice of an appropriate similarity measure.
  - Simplification of complex structures
    - Topology-based segmentations may result in very complex structures, which are hard to interpret. Therefore, algorithms for simplification and tracking over time play a crucial role.
    - Topology higher than 2D is not well understood

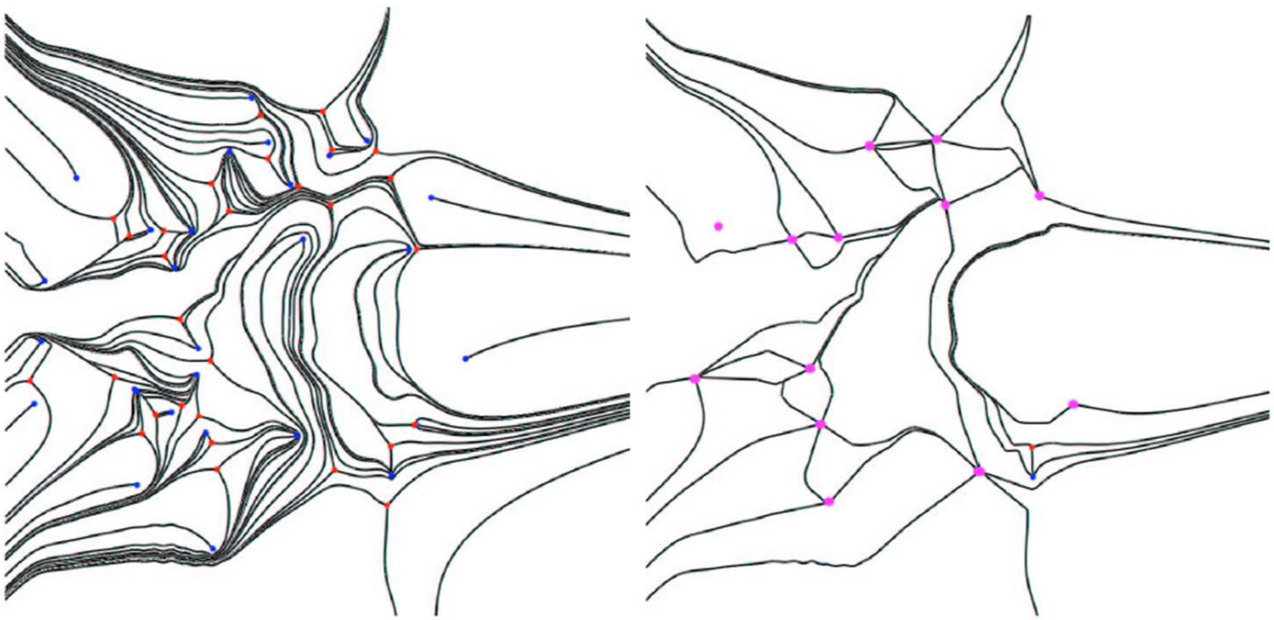
Tensor Field Segmentation

- Similarity-Measure-Based Segmentation
  - Based on tensor components, considering the tensor segmentation as a multi-channel segmentation of scalar values
  - Based on invariants or comprise the entire tensor data. Used metrics are the angular difference between principal eigenvector directions, or standard metrics considering the entire tensor, like the Euclidean or Frobenius distance

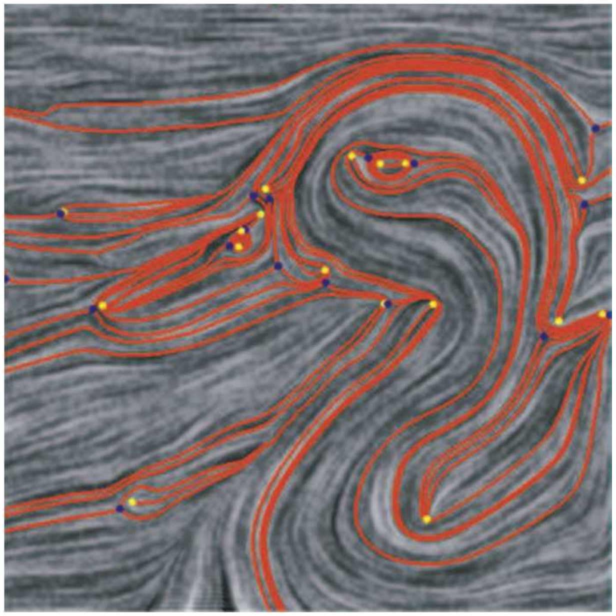
What about tensor field topology?

Tensor Field Segmentation

- Topology-Based Segmentation

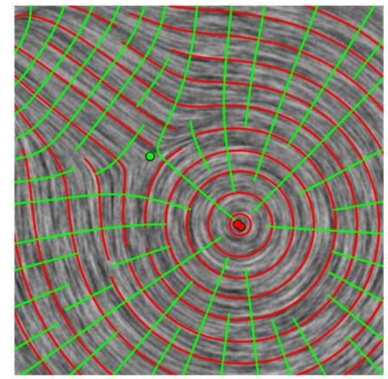


[Tricoche et al. VisSym2001]



[Zhang et al. TVCG 2007]

Hyperstreamlines



- Let **T**(**x**) be a (2nd order) symmetric tensor field
  - real eigenvalues, **<u>orthogonal eigenvectors</u>**
- Hyperstreamline: by integrating along one of the eigenvectors
- **Important: Eigenvector fields are not vector fields!**
  - eigenvectors have no magnitude and no orientation (are bidirectional)
  - the choice of the eigenvector can be made consistently as long as eigenvalues are all different
  - Hyperstreamlines can intersect only at points where two or more eigenvalues are equal, so-called **degenerate points**.

Compute One Hyperstreamline

- Choose integrator:
  - Euler
  - Runge-Kutta
- Choose step size (can be adaptive)
- Provide seed point position and determine starting direction
- Advance the front

- Note that the angle ambiguity. This is because the computation of the eigenvector at each sample point (i.e. vertex of the mesh) is independent of each other. Therefore, inconsistent directions may be chosen at neighboring vertices.
  - Additional step to remove angle ambiguity. A dot product between the current advancing direction and the eigenvector direction at current position is performed. A positive value indicates the consistent direction; otherwise, the inverse direction should be used!

Degenerate Points

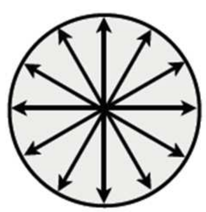
- The topology for 2nd symmetric tensor fields is extracted by identifying their **degenerate points** and their connectivity that partitions the hyperstreamlines.

- A point $\boldsymbol{p}$ is a degenerate point of the tensor field T iff the two eigenvalues of $T(\mathbf{p})$ are equal to each other.
  - There are infinite many eigenvectors at $\boldsymbol{p}$.
  - Hyperstreamlines cross each other at degenerate points

Degenerate Points

- Locations where tensor field is isotropic form singularities (also known as *umbilics*)
  - $\exists\ i \neq j,\ \lambda_i = \lambda_j$
  - tangent direction of trajectories is lost
- Different types of anisotropy

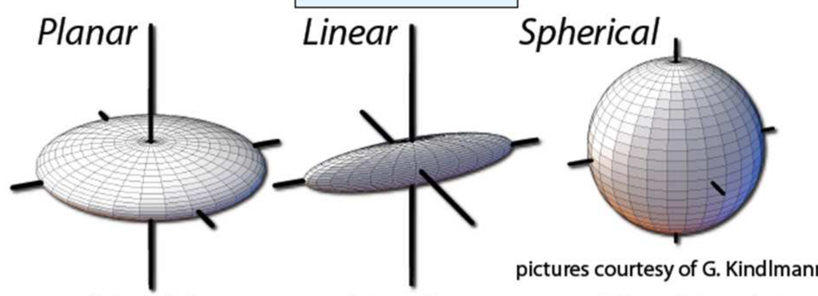
2D case



$\lambda 1 = \lambda 2$

$$\mathbf{D} = \lambda \mathbf{I_2}$$

3D case



pictures courtesy of G. Kindlmann

Planar: $\lambda 1 = \lambda 2$

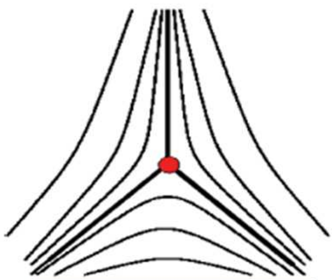
Linear: $\lambda 2 = \lambda 3$

Spherical: $\lambda 1 = \lambda 2 = \lambda 3$

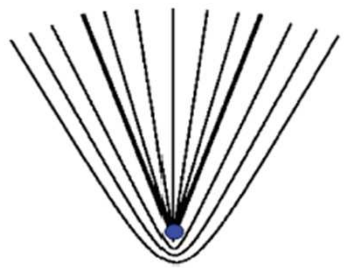
$$\mathbf{D} = \lambda \mathbf{I_3}$$

Degenerate Points in 2D

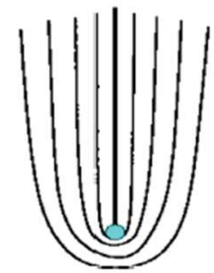
Three linear (first order) types exist



Trisector

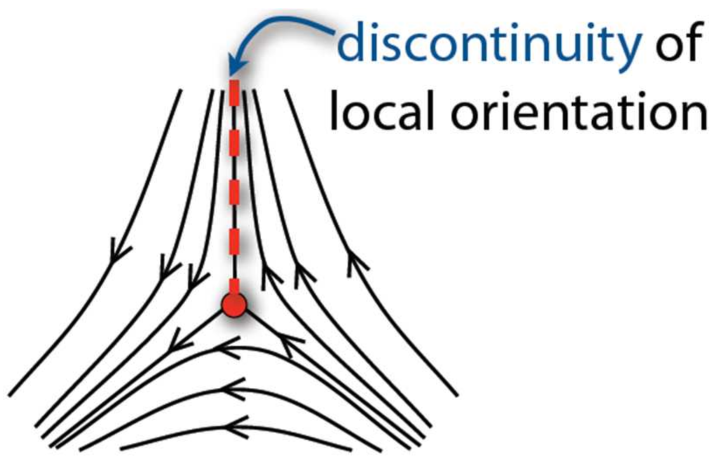


Wedge I



Wedge II

None of these patterns would be possible in vector fields!



Degenerate Points in 2D

- Find degenerate points
  - *Form* $\tilde{\mathbf{D}} := \mathbf{D} - \frac{1}{2}\mathrm{trace}(\mathbf{D})\mathbf{I}_2 \Rightarrow \tilde{\mathbf{D}} = \begin{pmatrix} \alpha & \beta \\ \beta & -\alpha \end{pmatrix}$ deviatoric
  - *Solve in each cell for* $\tilde{\mathbf{D}}(x, y) = \mathbf{0}$

$$D = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$$

$$D - \frac{1}{2} trace(D) I_2 = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} - \begin{pmatrix} \frac{T_{11} + T_{22}}{2} & 0 \\ 0 & \frac{T_{11} + T_{22}}{2} \end{pmatrix} = \cdots$$

Degenerate Points in 2D

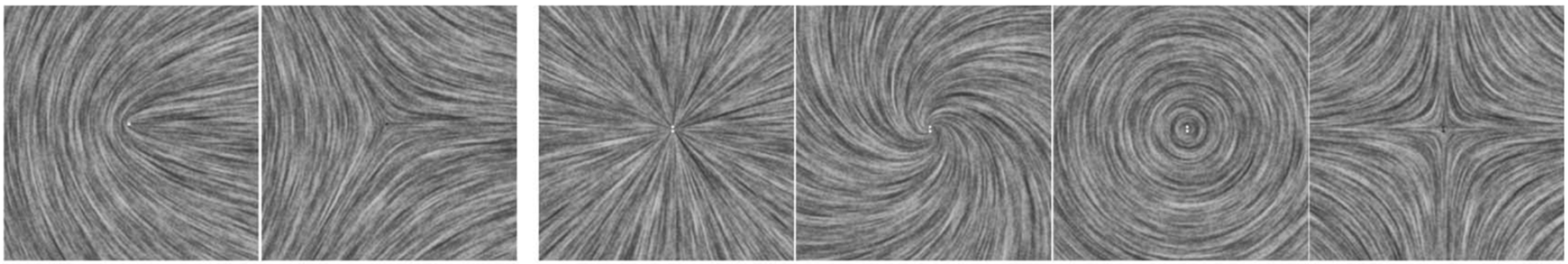
- Classifying tensor degenerate points

$$\begin{pmatrix} \alpha(x,y) & \beta(x,y) \\ \beta(x,y) & -\alpha(x,y) \end{pmatrix} = \begin{pmatrix} \alpha_1 x + \alpha_2 y & \beta_1 x + \beta_2 y \\ \beta_1 x + \beta_2 y & -(\alpha_1 x + \alpha_2 y) \end{pmatrix}$$

  - Depending on the determinant of $\begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix}$
    - \>0 wedge
    - <0 trisector
    - =0 higher-order degenerate points

A few degenerate points used in tensor field design

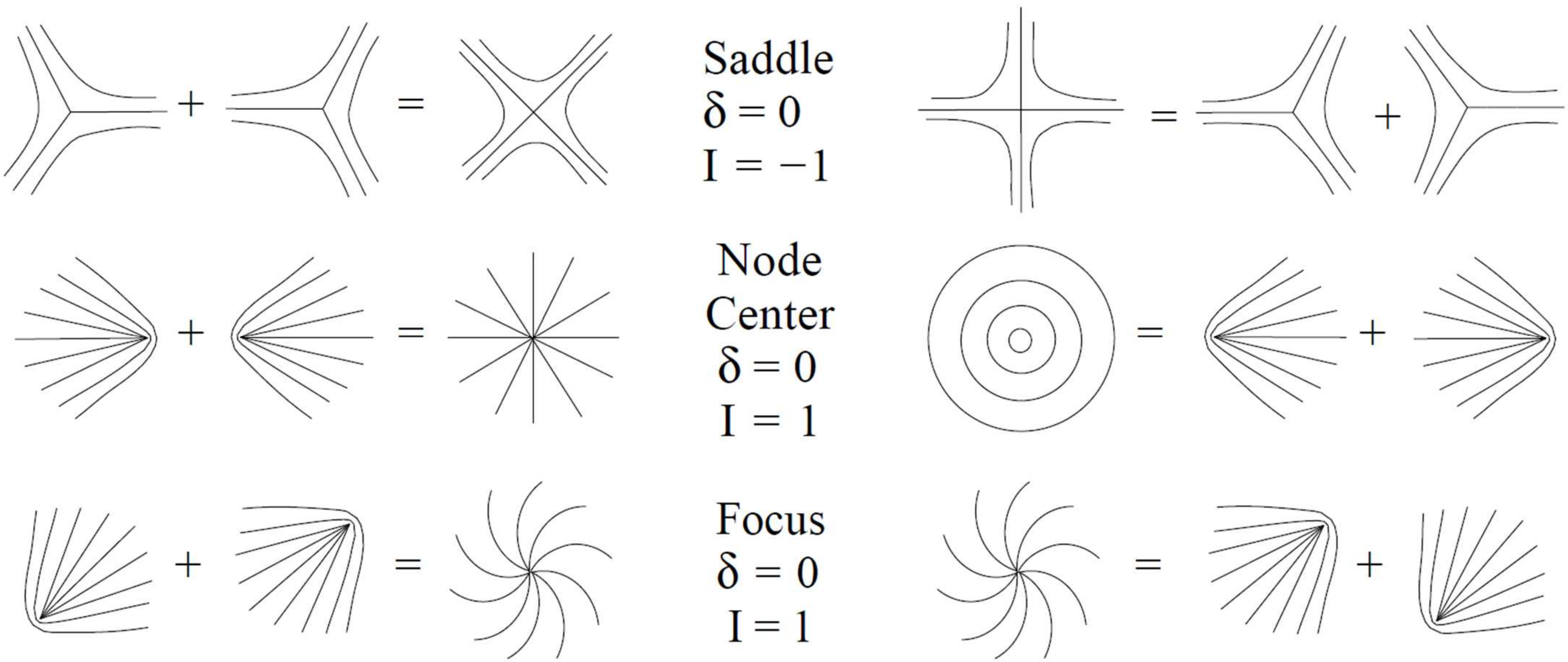
$$\begin{pmatrix} x^2 - y^2 & -2xy \\ -2xy & -(x^2 - y^2) \end{pmatrix}$$



$$\begin{pmatrix} x & y \\ y & -x \end{pmatrix} \qquad \begin{pmatrix} x & -y \\ -y & -x \end{pmatrix} \qquad \begin{pmatrix} x^2 - y^2 & 2xy \\ 2xy & -(x^2 - y^2) \end{pmatrix} \qquad \begin{pmatrix} y^2 - x^2 & -2xy \\ -2xy & -(y^2 - x^2) \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 x + \alpha_2 y & \beta_1 x + \beta_2 y \\ \beta_1 x + \beta_2 y & -(\alpha_1 x + \alpha_2 y) \end{pmatrix} \Rightarrow \begin{pmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{pmatrix}$$

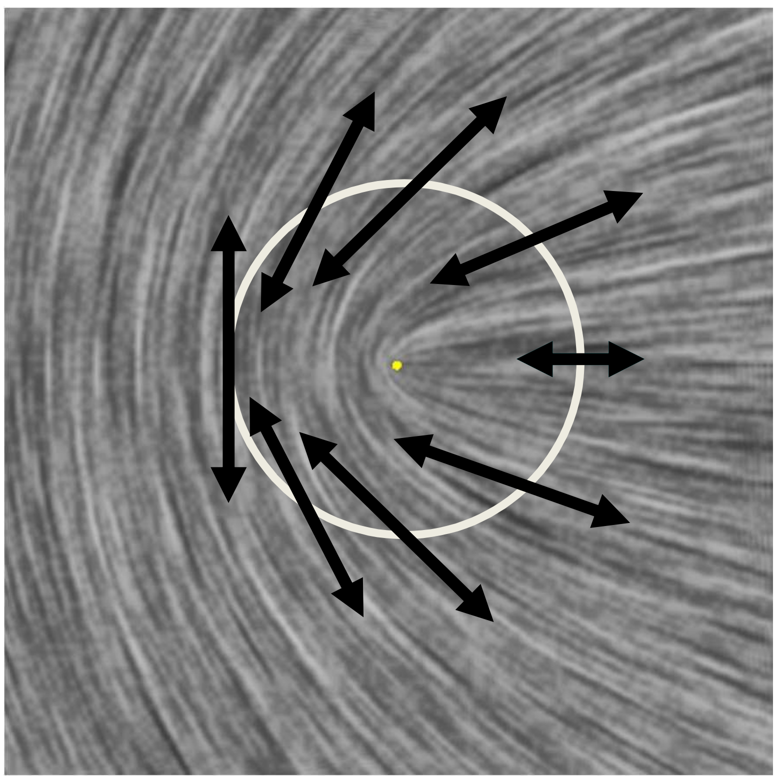
>0 wedge
<0 trisector
=0 higher-order degenerate points

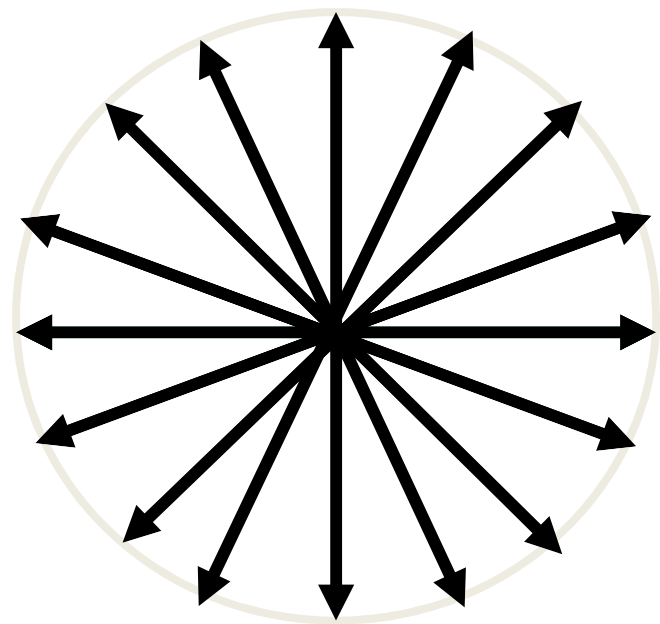


[Delmarcelle and Hesselink, 1994]

Degenerate Points in 2D

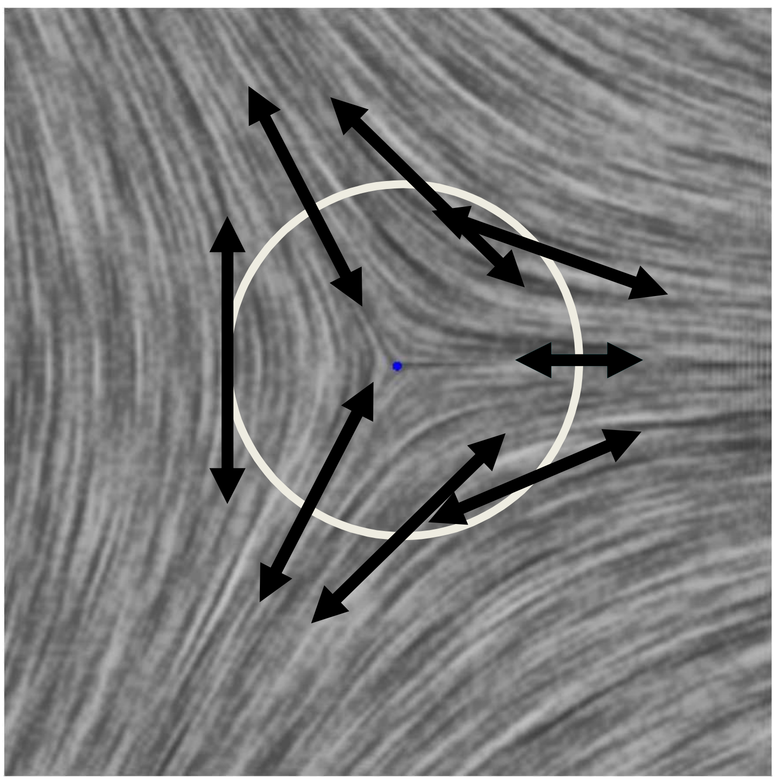
- Tensor index: wedges

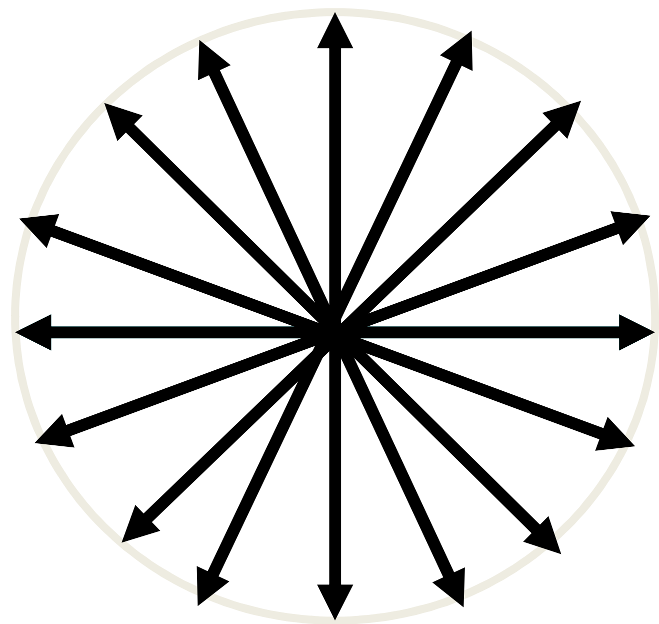




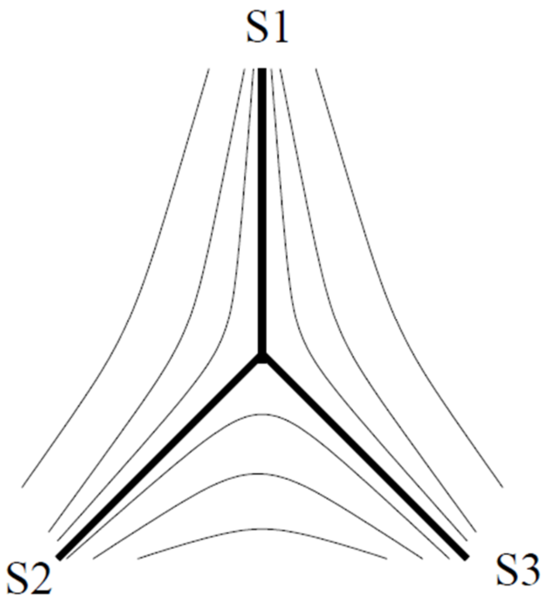
Degenerate Points in 2D

- Tensor index: trisectors

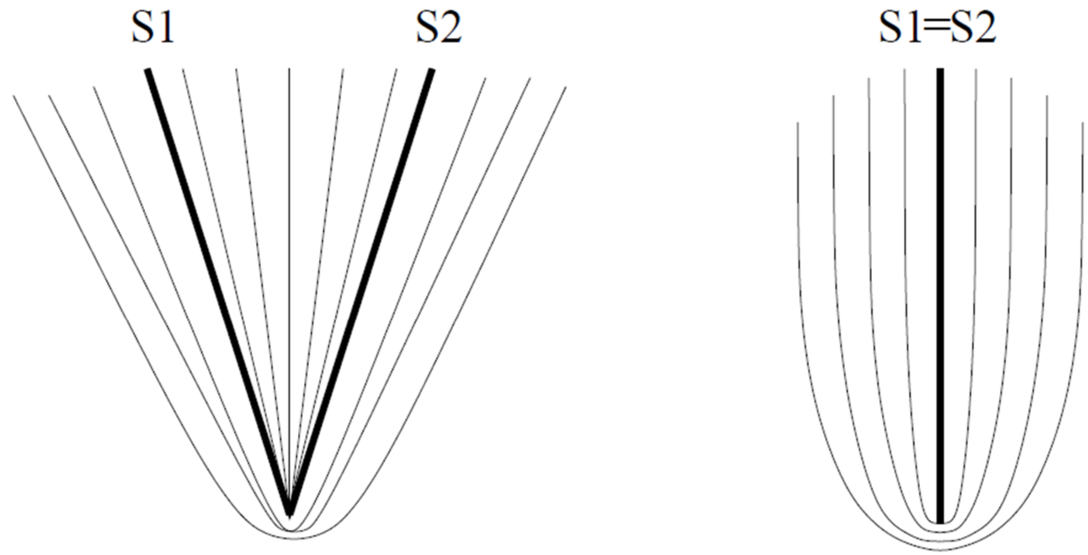




Separatrices



TRISECTOR



WEDGE POINT S

Hyperbolic sectors n_h
Parabolic sectors n_p

Index $I = 1 - \frac{n_h}{2}$

Separatrices

- Find degenerate points
  - *Form* $\tilde{\mathbf{D}} := \mathbf{D} - \frac{1}{2}\mathrm{trace}(\mathbf{D})\mathbf{I}_2 \Rightarrow \tilde{\mathbf{D}} = \begin{pmatrix} \alpha & \beta \\ \beta & -\alpha \end{pmatrix}$ deviatoric
  - *Solve in each cell for* $\tilde{\mathbf{D}}(x, y) = \mathbf{0}$
- Compute separatrices
  - *Linear analysis at each singularity* $\begin{cases} \alpha(x,y) & \approx & \alpha_1\, x + \alpha_2\, y \\ \beta(x,y) & \approx & \beta_1\, x + \beta_2\, y \end{cases}$
  - *Determine angular coordinate of separatrices*

$$\beta_2\, u^3 + (\beta_1 + 2\alpha_2)\, u^2 + (2\alpha_1 - \beta_2)\, u - \beta_1 = 0$$

$$u := \tan\theta$$

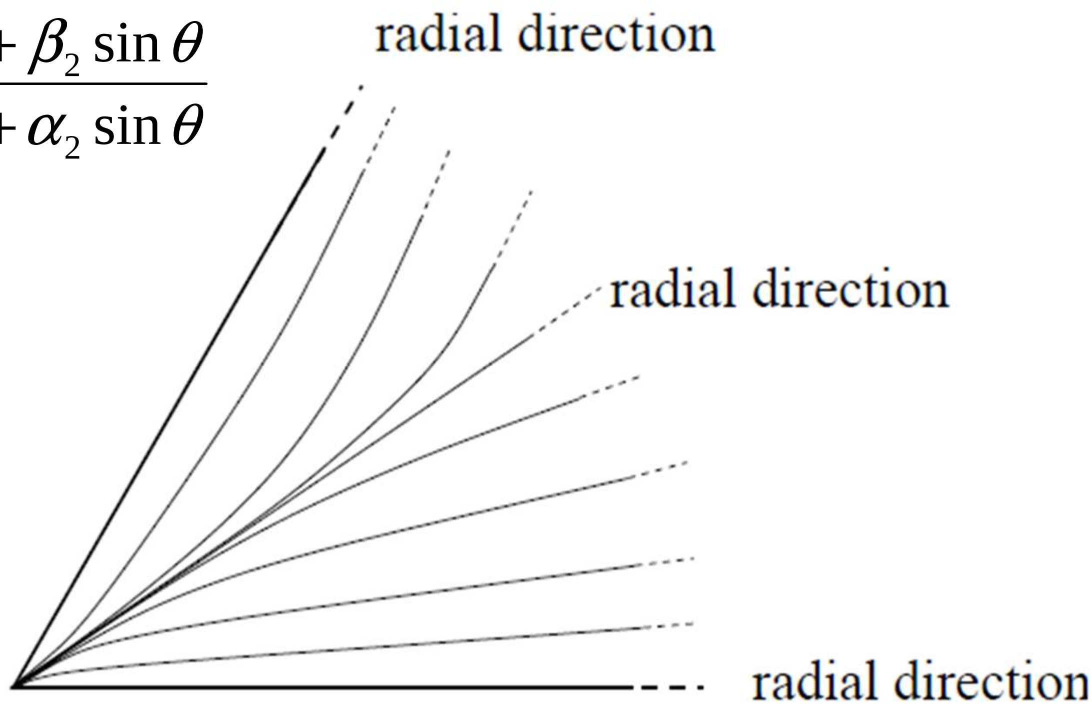
(Delmarcelle and Hesselink, 1994)

  - *Integrate separatrices (standard ODE solver with embedded orientation consistency check)*

Separatrices

- Separatrices

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{\beta_1 \cos\theta + \beta_2 \sin\theta}{\alpha_1 \cos\theta + \alpha_2 \sin\theta}$$



Topological Skeleton in 2D

We only consider the topology for 2nd symmetric tensor fields!

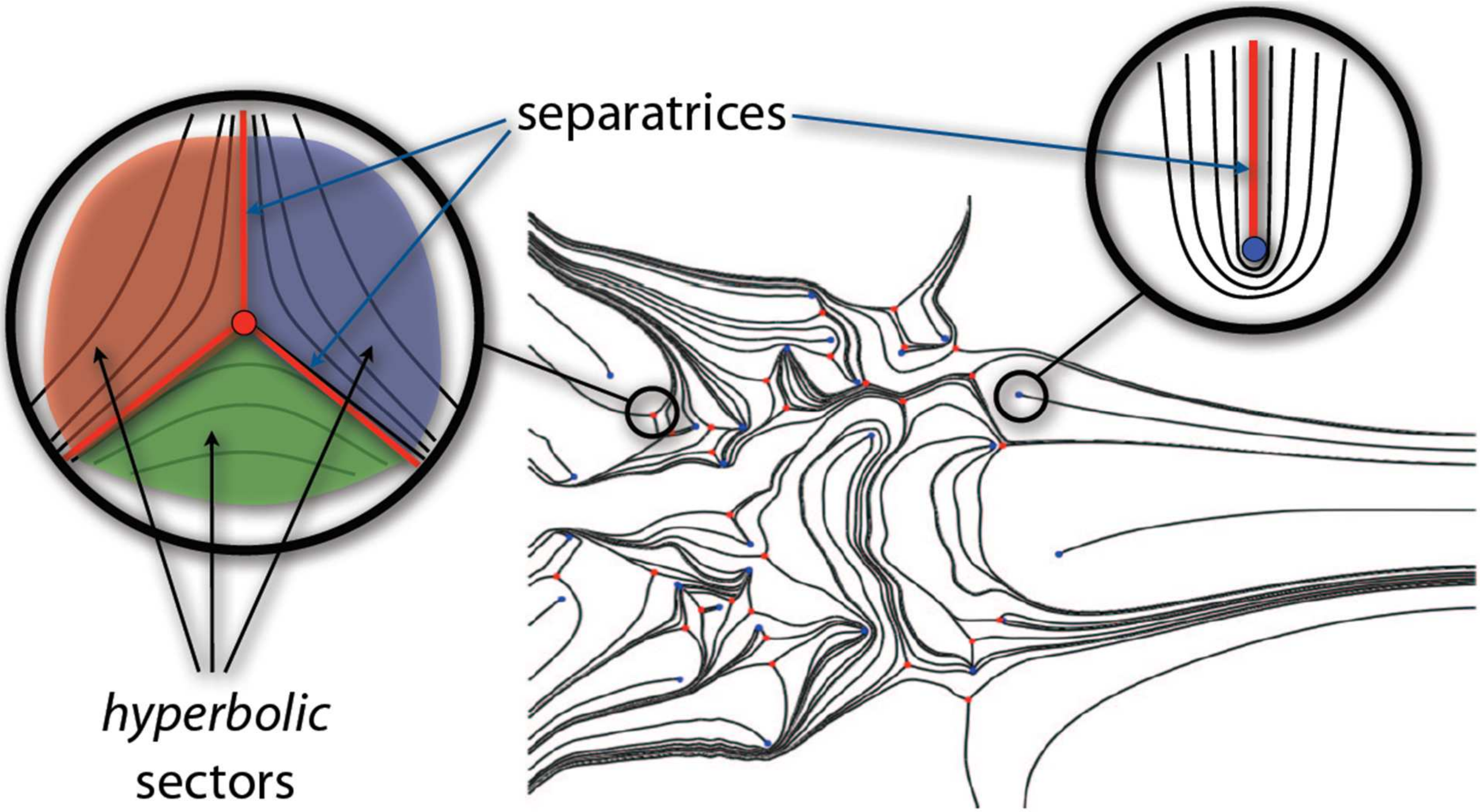
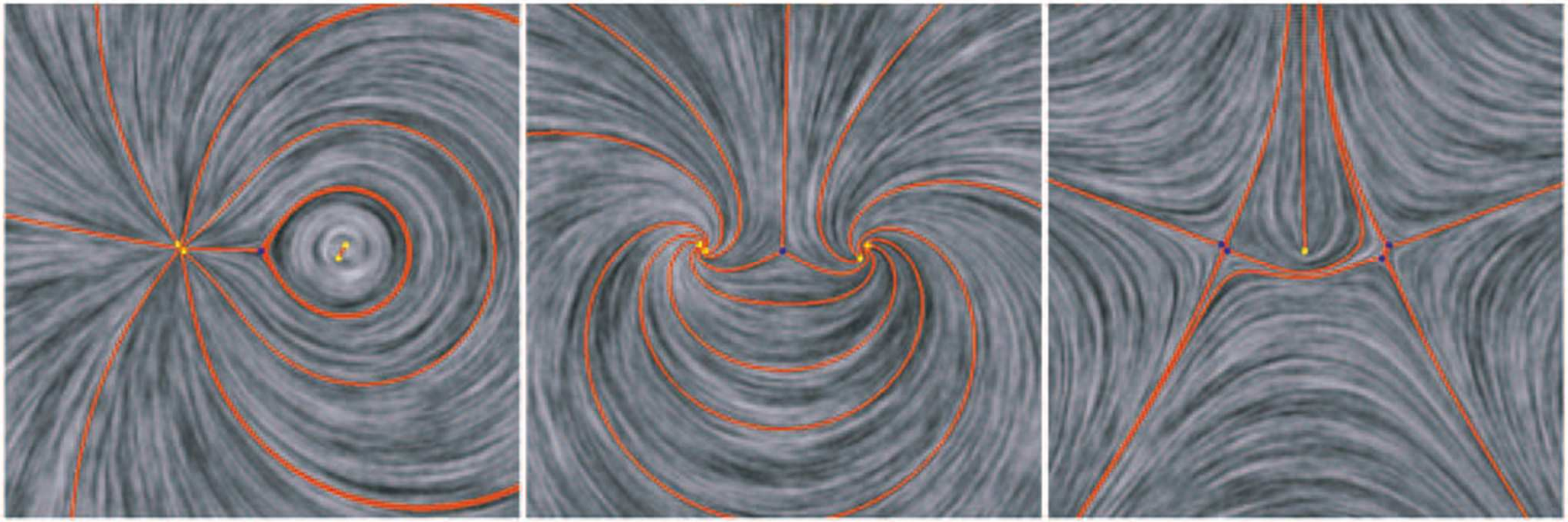


Image by Xavier Tricoche



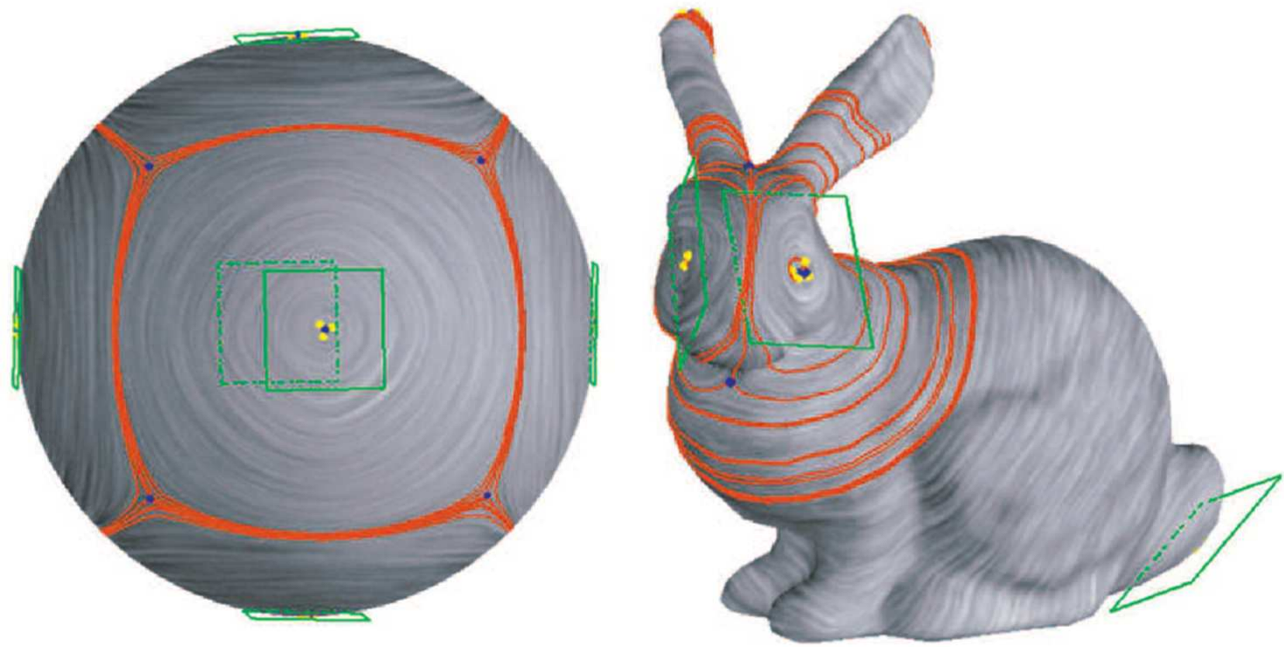
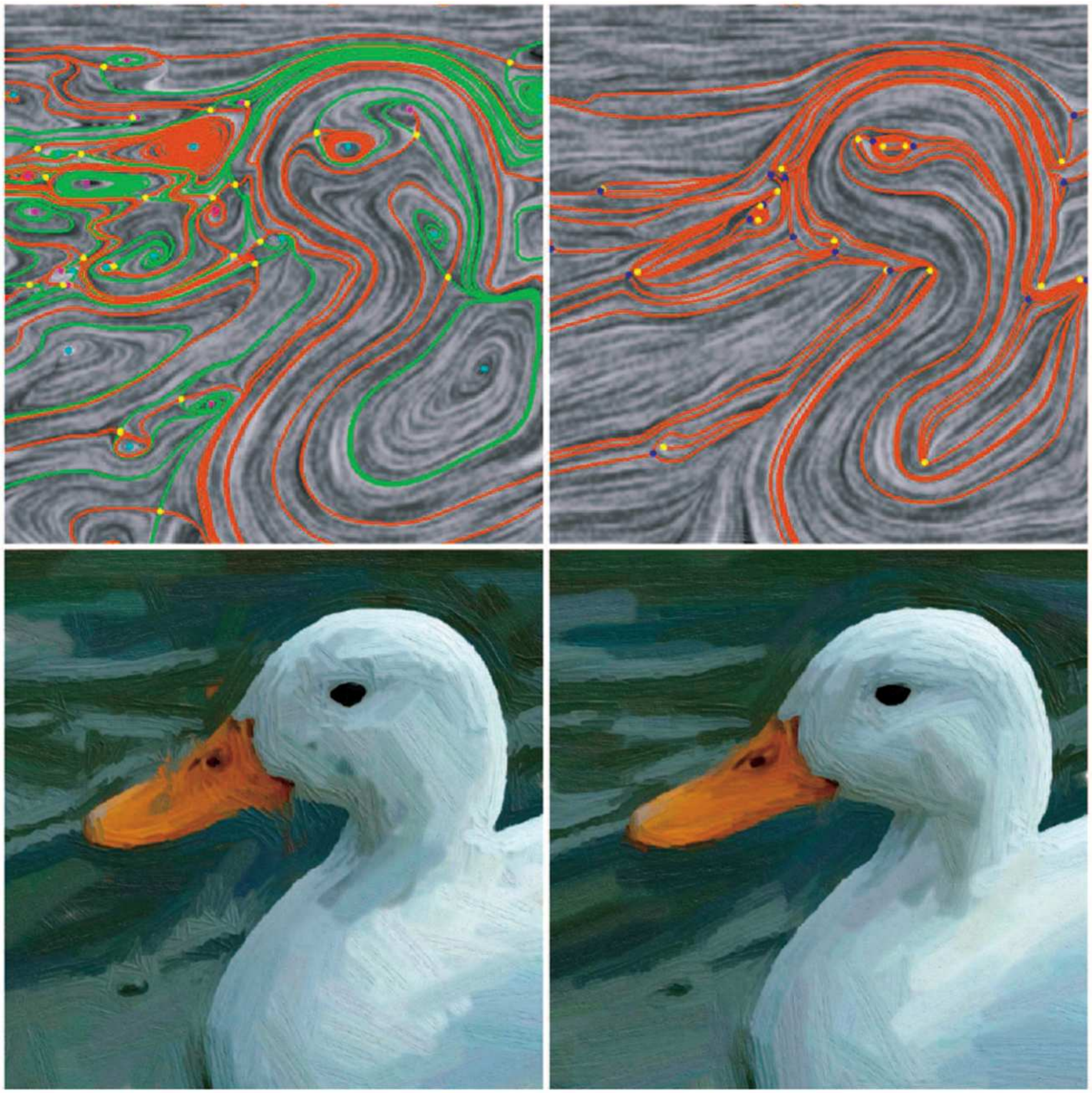


Image by Eugene Zhang

Compared With Vector Field Topology



Comparison between the vector-based image edge field (VIEF, left) and the tensor-based image edge field (TIEF, right) for painterly rendering of an image of a duck.

Notice that TIEF is much smoother than VIEF (top row), and their impact on the painterly results are clearly visible near the beak of the duck.

Image by Eugene Zhang

How about 3D topology?

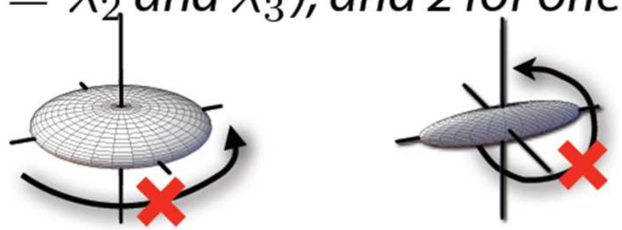
Singularities in 3D

- Loci of partial isotropy ($\lambda_1 = \lambda_2$ or $\lambda_2 = \lambda_3$) form lines, not points! Zheng and Pang, 2004

- Sketch of the proof:

  - *Generic symmetric 3D tensors have 6 degrees of freedom: 3 for shape (eigenvalues), 3 for orientation (eigenvectors)*

  - *Isotropic 3D tensors have 4 degrees of freedom: 2 for shape (say $\lambda := \lambda_1 = \lambda_2$ and λ_3), and 2 for orientation*



  - *A set of codimension 2 (=6-4) in 3D has dimension 1, i.e. forms lines*

Finding Degenerate Lines

- Characterization: $D_3(T) = (\lambda_1 - \lambda_2)^2(\lambda_2 - \lambda_3)^2(\lambda_3 - \lambda_1)^2 = 0$

  discriminant

- Equivalent to system of 7 cubic equations

$$
\begin{aligned}
f_x(T) = {} & T_{00}(T_{11}^2 - T_{22}^2) + T_{00}(T_{01}^2 - T_{02}^2) + T_{11}(T_{22}^2 - T_{00}^2) + \\
& T_{11}(T_{12}^2 - T_{01}^2) + T_{22}(T_{00}^2 - T_{11}^2) + T_{22}(T_{02}^2 - T_{12}^2) \\
f_{y1}(T) = {} & T_{12}(2(T_{12}^2 - T_{00}^2) - (T_{02}^2 + T_{01}^2) + 2(T_{11}T_{00} + T_{22}T_{00} \\
& -T_{11}T_{22})) + T_{01}T_{02}(2T_{00} - T_{22} - T_{11}) \\
f_{y2}(T) = {} & T_{02}(2(T_{02}^2 - T_{11}^2) - (T_{01}^2 + T_{12}^2) + 2(T_{22}T_{11} + T_{00}T_{11} \\
& -T_{22}T_{00})) + T_{12}T_{01}(2T_{11} - T_{00} - T_{22}) \\
f_{y3}(T) = {} & T_{01}(2(T_{01}^2 - T_{22}^2) - (T_{12}^2 + T_{02}^2) + 2(T_{00}T_{22} + T_{11}T_{22} \\
& -T_{00}T_{11})) + T_{02}T_{12}(2T_{22} - T_{11} - T_{00}) \\
f_{z1}(T) = {} & T_{12}(T_{02}^2 - T_{01}^2) + T_{01}T_{02}(T_{11} - T_{22}) \\
f_{z2}(T) = {} & T_{02}(T_{01}^2 - T_{12}^2) + T_{12}T_{01}(T_{22} - T_{00}) \\
f_{z3}(T) = {} & T_{01}(T_{12}^2 - T_{02}^2) + T_{02}T_{12}(T_{00} - T_{11})
\end{aligned}
$$

Zheng and Pang, 2004

$$
\begin{aligned}
D_3(T) = {} & f_x(T)^2 + f_{y1}(T)^2 + f_{y2}(T)^2 + f_{y3}(T)^2 + \\
& 15f_{z1}(T)^2 + 15f_{z2}(T)^2 + 15f_{z3}(T)^2
\end{aligned}
$$

- Find locations on faces of the mesh where all 7 polynomials are zero, connect them to form lines

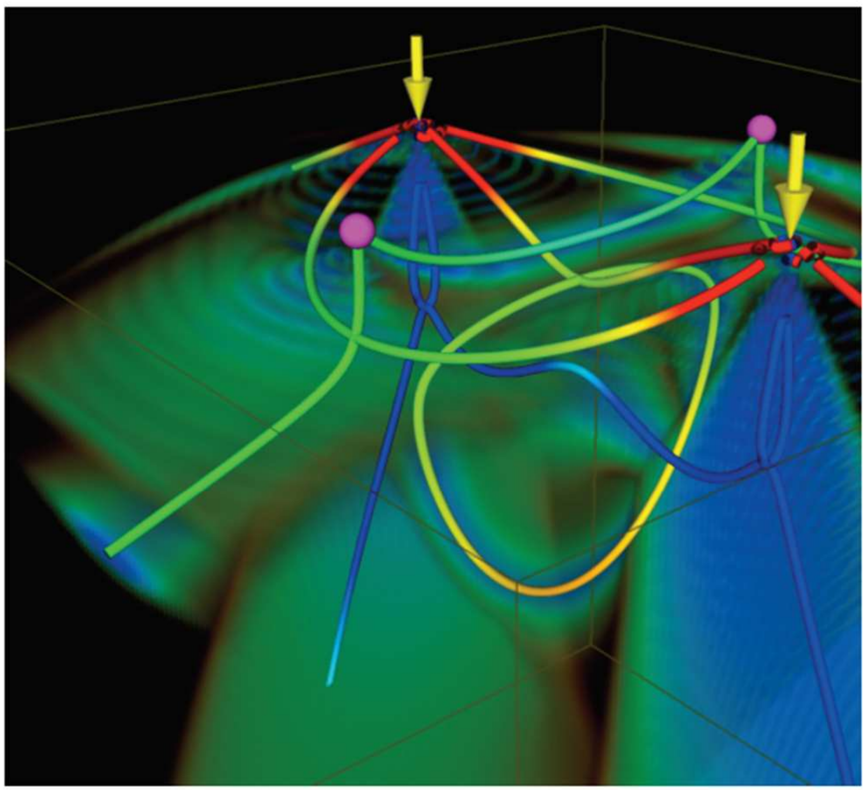
Finding Degenerate Lines

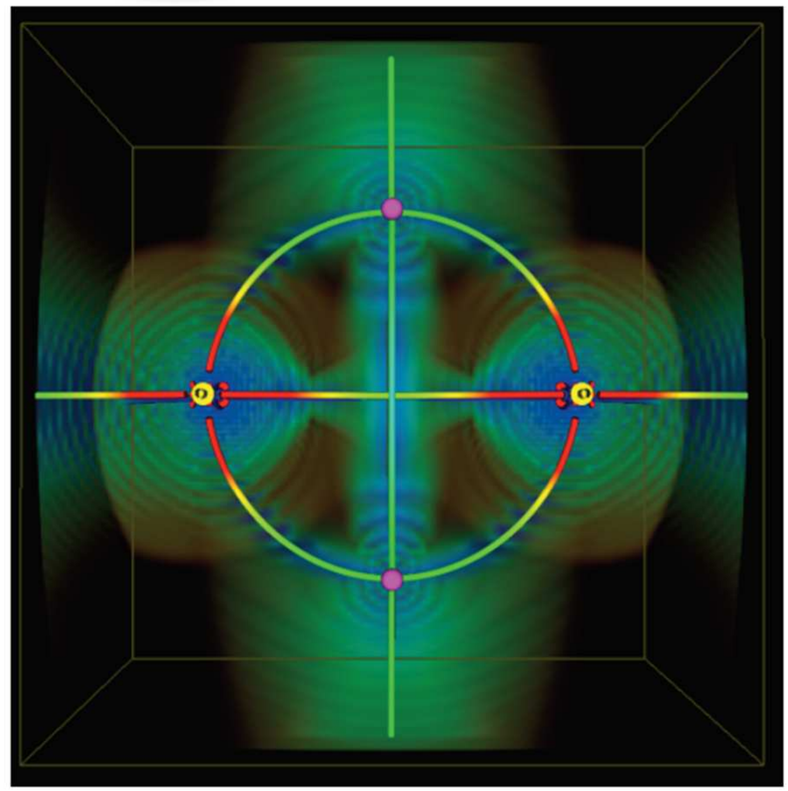
- Alternative characterization:

rank-1 tensor

$$\mathbf{D} = s\,\mathbf{I_3} \pm \mathbf{v} \cdot \mathbf{v}^T$$

Zheng, Tricoche, Pang, 2006



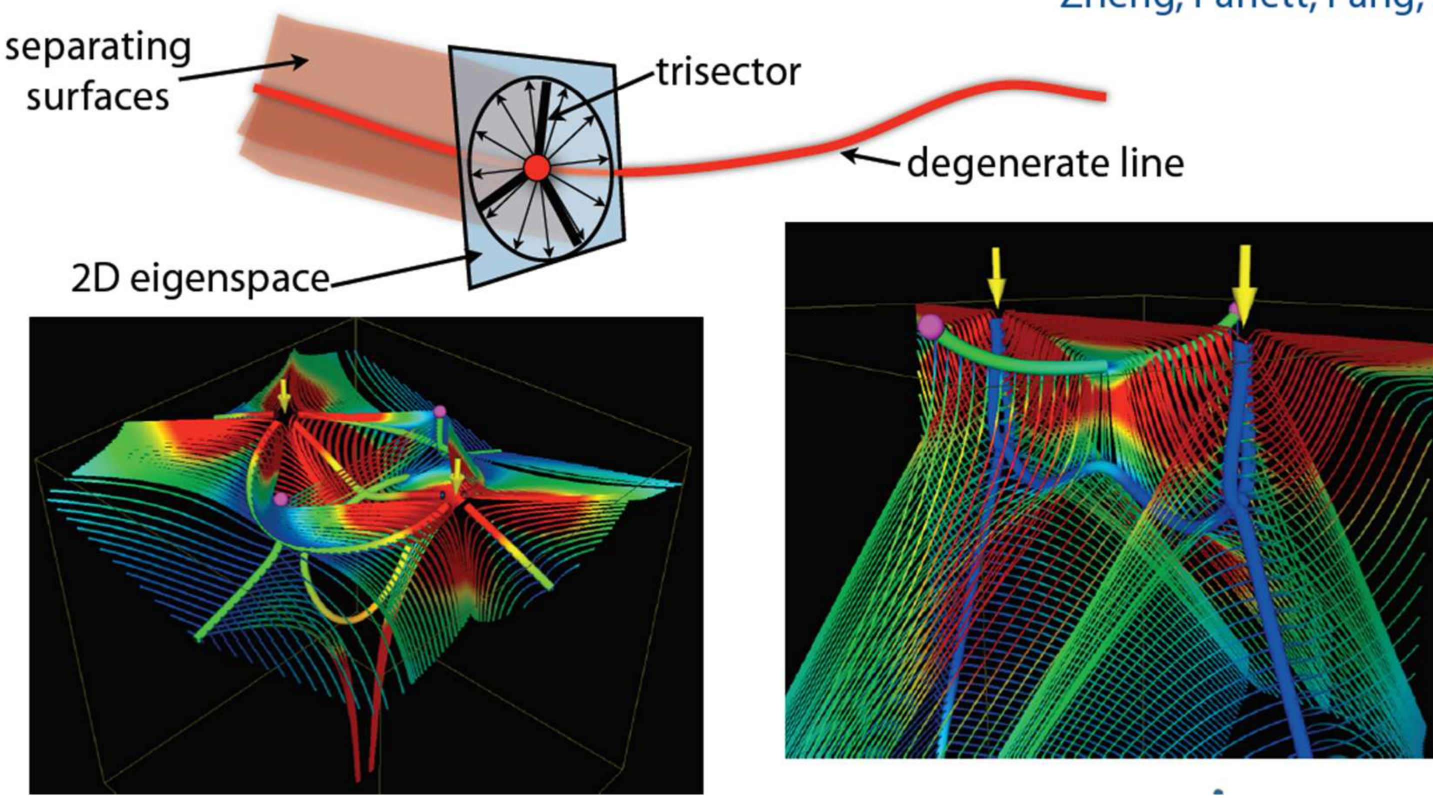


Zheng and Pang, 2004

Topological Skeleton

- Separatrices along degenerate lines obtained by analysis of the eigenvector field restriction to isotropic plane.

Zheng, Parlett, Pang, 2005



Practical Issues (I)

- Topology extraction (degenerate lines, separatrices) is difficult:
  - *involved algorithmically*
  - *computationally expensive*

$$
\begin{aligned}
f_x(T) &= T_{00}(T_{11}^2 - T_{22}^2) + T_{00}(T_{01}^2 - T_{02}^2) + T_{11}(T_{22}^2 - T_{00}^2) + \\
&\quad T_{11}(T_{12}^2 - T_{01}^2) + T_{22}(T_{00}^2 - T_{11}^2) + T_{22}(T_{02}^2 - T_{12}^2) \\
f_{y1}(T) &= T_{12}(2(T_{12}^2 - T_{00}^2) - (T_{02}^2 + T_{01}^2) + 2(T_{11}T_{00} + T_{22}T_{00} \\
&\quad -T_{11}T_{22})) + T_{01}T_{02}(2T_{00} - T_{22} - T_{11}) \\
f_{y2}(T) &= T_{02}(2(T_{02}^2 - T_{11}^2) - (T_{01}^2 + T_{12}^2) + 2(T_{22}T_{11} + T_{00}T_{11} \\
&\quad -T_{22}T_{00})) + T_{12}T_{01}(2T_{11} - T_{00} - T_{22}) \\
f_{y3}(T) &= T_{01}(2(T_{01}^2 - T_{22}^2) - (T_{12}^2 + T_{02}^2) + 2(T_{00}T_{22} + T_{11}T_{22} \\
&\quad -T_{00}T_{11})) + T_{02}T_{12}(2T_{22} - T_{11} - T_{00}) \\
f_{z1}(T) &= T_{12}(T_{02}^2 - T_{01}^2) + T_{01}T_{02}(T_{11} - T_{22}) \\
f_{z2}(T) &= T_{02}(T_{01}^2 - T_{12}^2) + T_{12}T_{01}(T_{22} - T_{00}) \\
f_{z3}(T) &= T_{01}(T_{12}^2 - T_{02}^2) + T_{02}T_{12}(T_{00} - T_{11})
\end{aligned}
$$

$$
\begin{aligned}
D_3(T) = f_x(T)^2 + f_{y1}(T)^2 + f_{y2}(T)^2 + f_{y3}(T)^2 + \\
15 f_{z1}(T)^2 + 15 f_{z2}(T)^2 + 15 f_{z3}(T)^2
\end{aligned}
$$

Time-dependent tensor field topology?

General tensor field topology?

Higher-order tensor field topology?

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- Thanks material from
- Prof. Eugene Zhang, Oregon State University
- Prof. Xavier Tricoche, Purdue University

- And a survey paper
  - Kratz, A.; Auer, C.; Stommel, M.; Hotz, I. *Visualization and Analysis of Second-Order Tensors: Moving Beyond the Symmetric Positive-Definite Case.* Computer Graphics Forum - State of the Art Reports