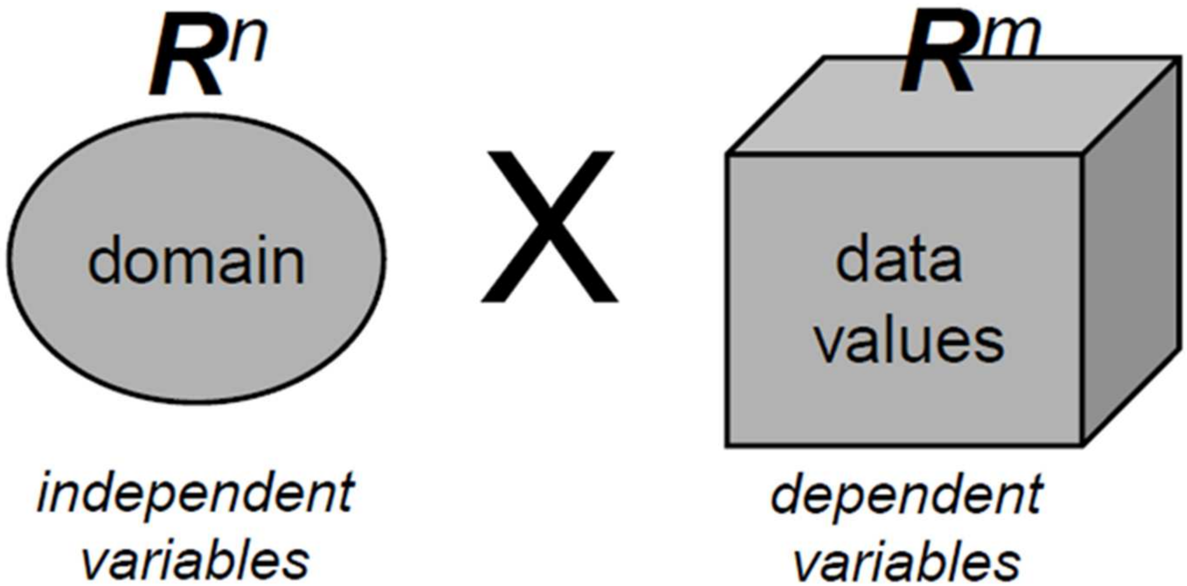
Data we are discussing in the class



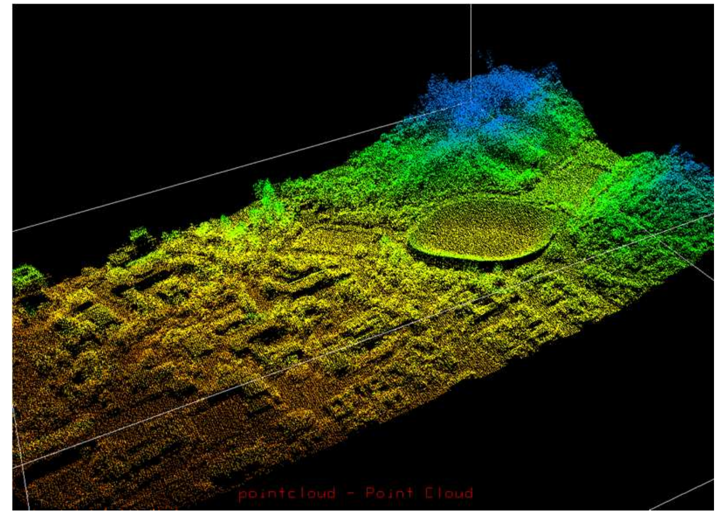
Source: VIS, University of Stuttgart

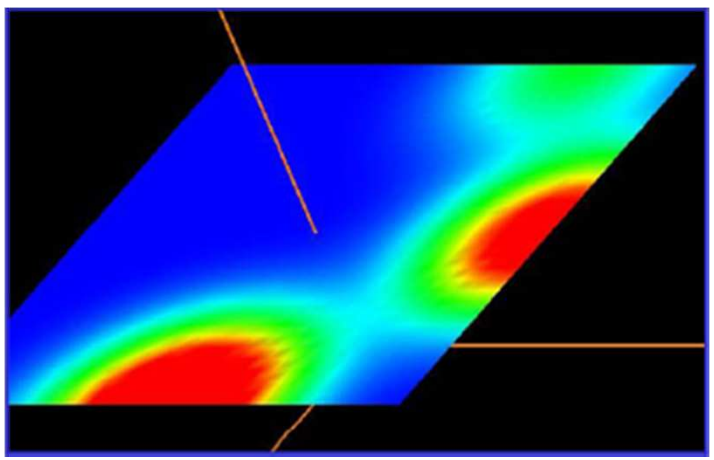
Scientific data	3**D**+time (*n*<4)	Scalar/vector/tensor
Information data	*n***D** (*n*>3)	Heterogeneous

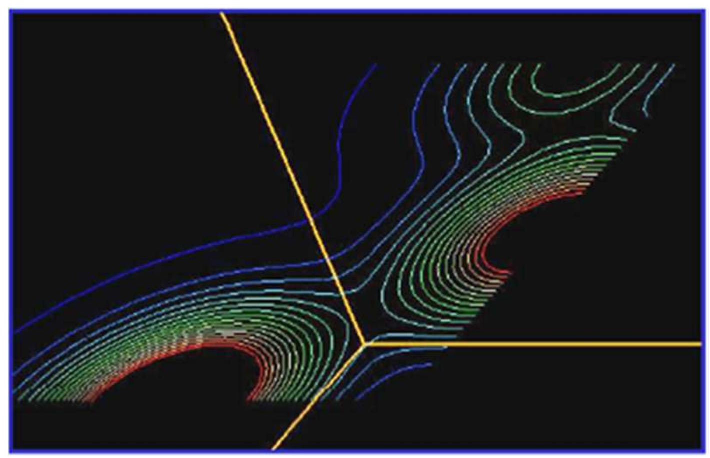
Scalar Data Analysis

Scalar Fields

- The approximation of certain scalar function in physical space *f(x,y,z)*.
- Discrete representation.
- Visualization primitives: colors, transparency, iso-contours (2D), iso-surfaces (3D), 3D textures.

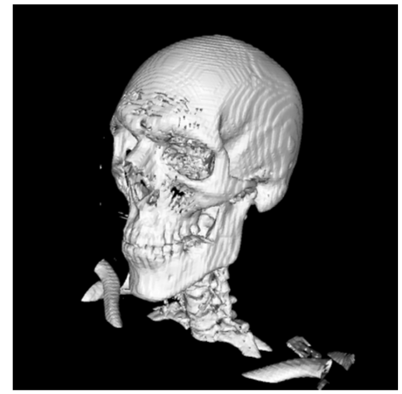




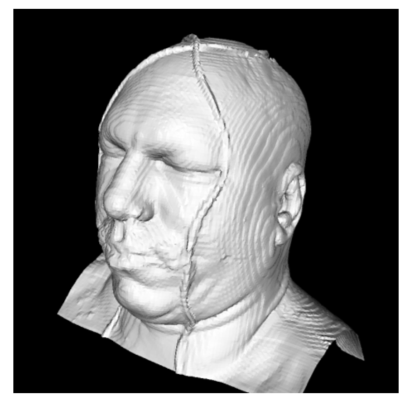


3D Surface

- 3D surfaces can be considered some sub-sets corresponding to certain iso-values of some scalar functions (e.g. a distance field).



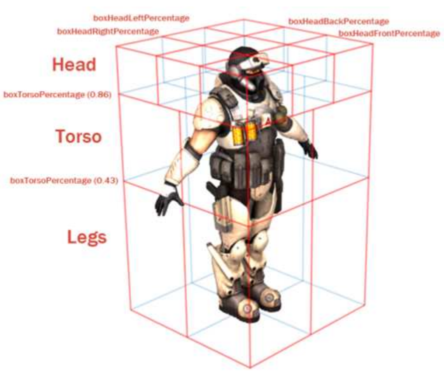
- Therefore, analyzing the characteristics of the surfaces can help understand the scalar function they represent.

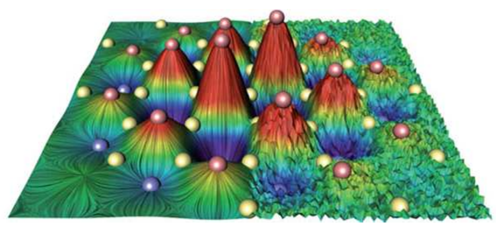


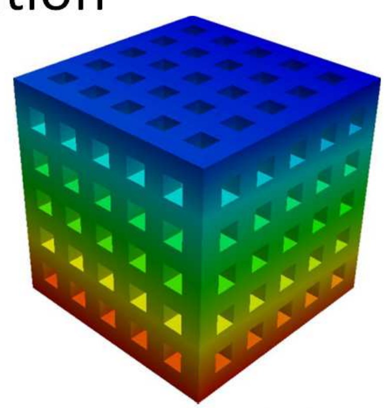
Source: http://www.cs.utah.edu/~angel/vis/project3/

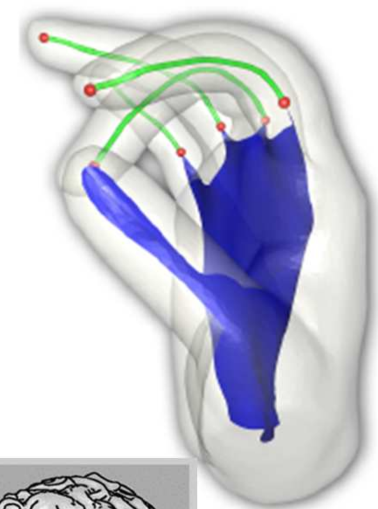
Shape Analysis

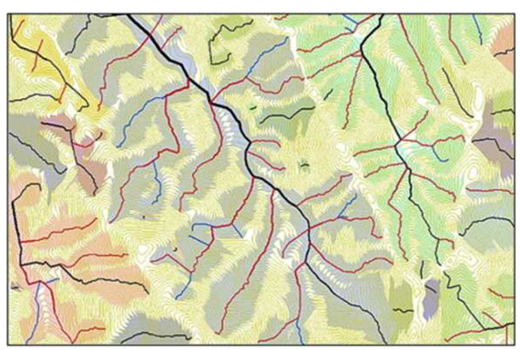
- What features do people care about?
  - Computing orientation and range of the shape
  - Computing extrema (protrusion)
  - Finding handles and holes
  - Finding ridges and valleys or other feature lines
  - Computing skeletal representation

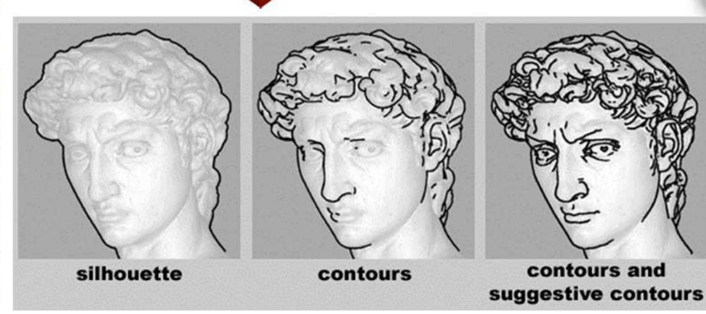












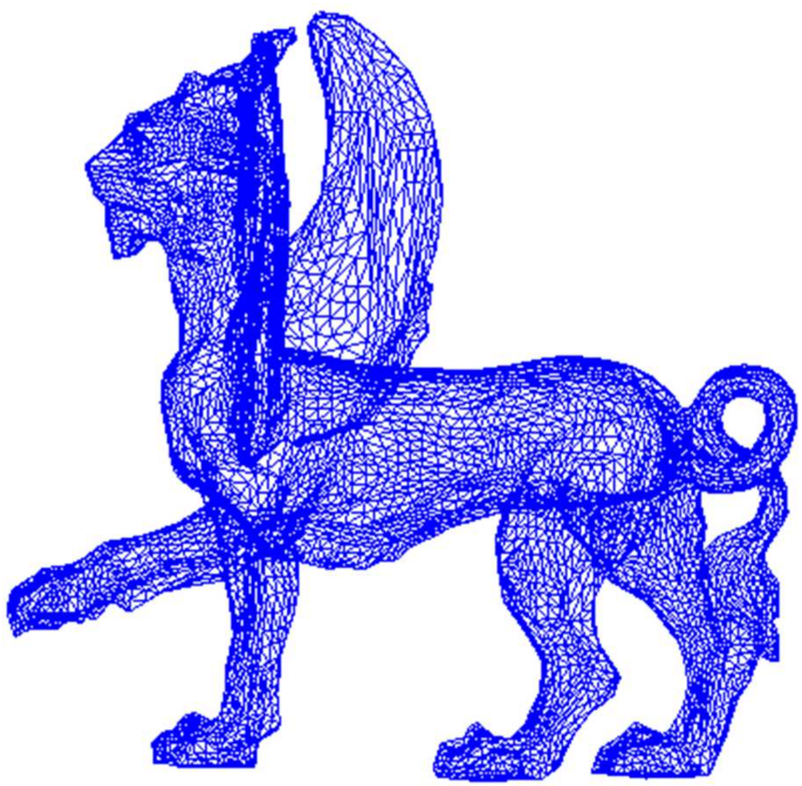
Sub-Topics

- Compute bounding box
- Compute Euler Characteristic
- Estimate surface curvature
- Line description for conveying surface shape
- Morse function and surface topology--Reeb graph
- Scalar field topology--Morse-Smale complex

Bounding Box

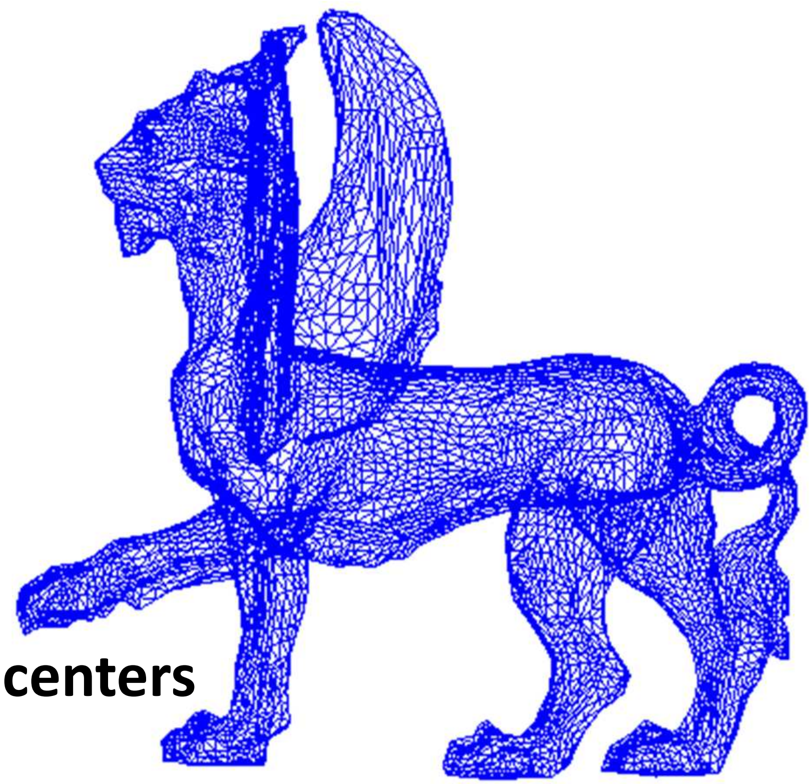
Shape Descriptors

- We need centers and variances of all points on the surface.



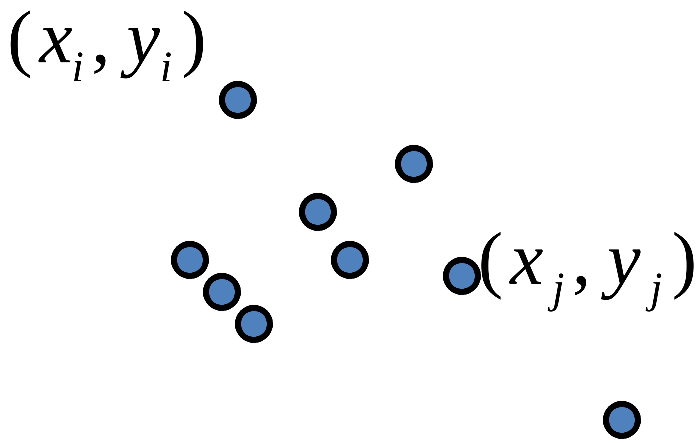
Shape Descriptors

- We need centers and variances of all points on the surface.



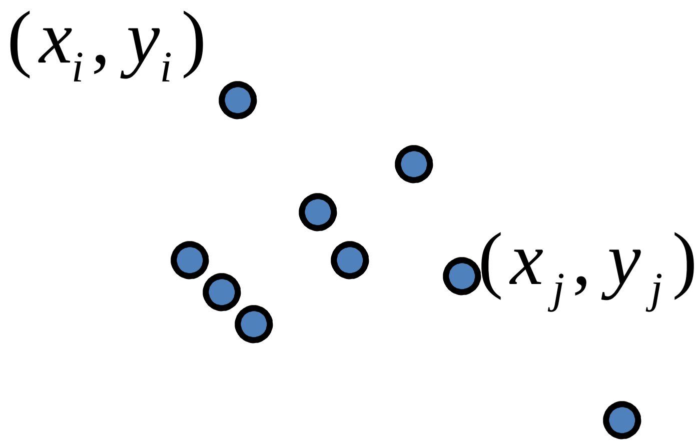
There are different definitions of centers

Center of Mass



Center of Mass

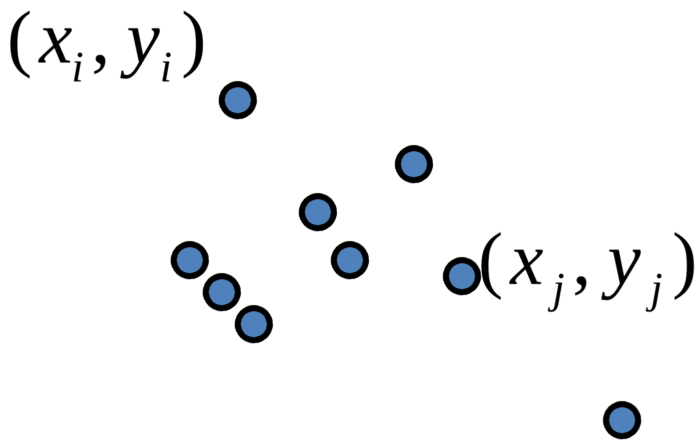
- Center of Mass



$$\left(\frac{\sum_{i=1}^{N} x_i}{N}, \frac{\sum_{i=1}^{N} y_i}{N}\right)$$

Geometric Center

- Geometric center



$$\left(\frac{\min \; x_i + \max \; x_i}{2}, \frac{\min \; y_i + \max \; y_i}{2}\right)$$

Centers

- Geometric center

$$(\frac{\min x_i + \max x_i}{2}, \frac{\min y_i + \max y_i}{2})$$

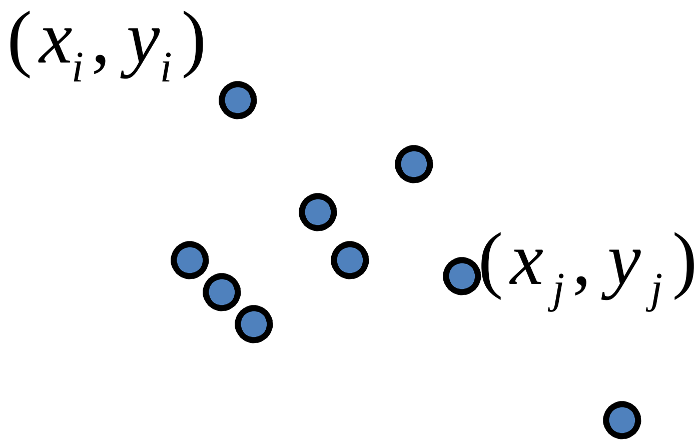
- Center of Mass

$$(\frac{\sum_{i=1}^{N} x_i}{N}, \frac{\sum_{i=1}^{N} y_i}{N})$$

- Which one is better and why?

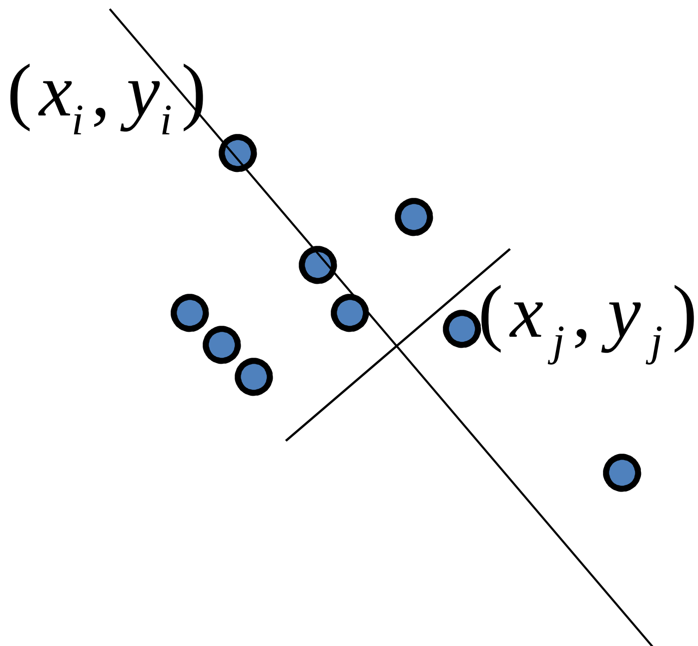
Variance

- What are the dominant directions?



Variance

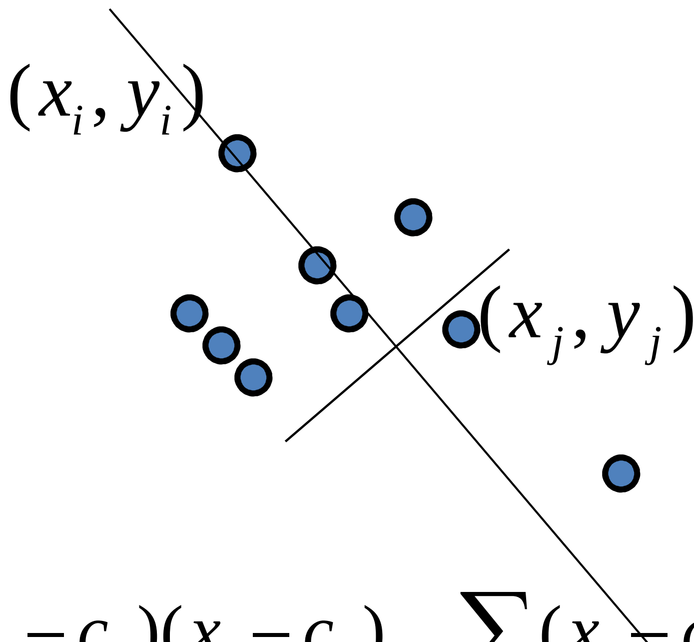
- What are the dominant directions?



- How are they defined?
- Principle components analysis

Variance

- What are the dominant directions?

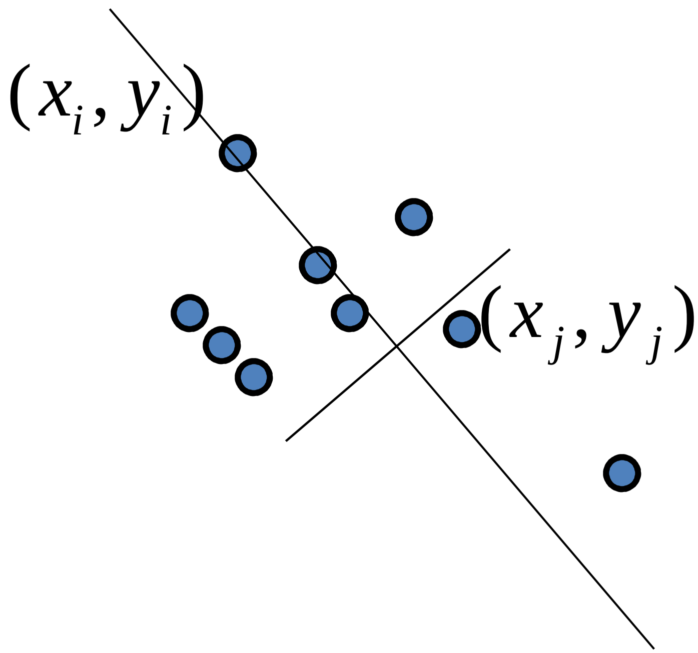


$$V = \begin{pmatrix} \sum_i (x_i - c_x)(x_i - c_x) & \sum_i (x_i - c_x)(y_i - c_y) \\ \sum_i (x_i - c_x)(y_i - c_y) & \sum_i (y_i - c_y)(y_i - c_y) \end{pmatrix}$$

Covariance matrix

Variance

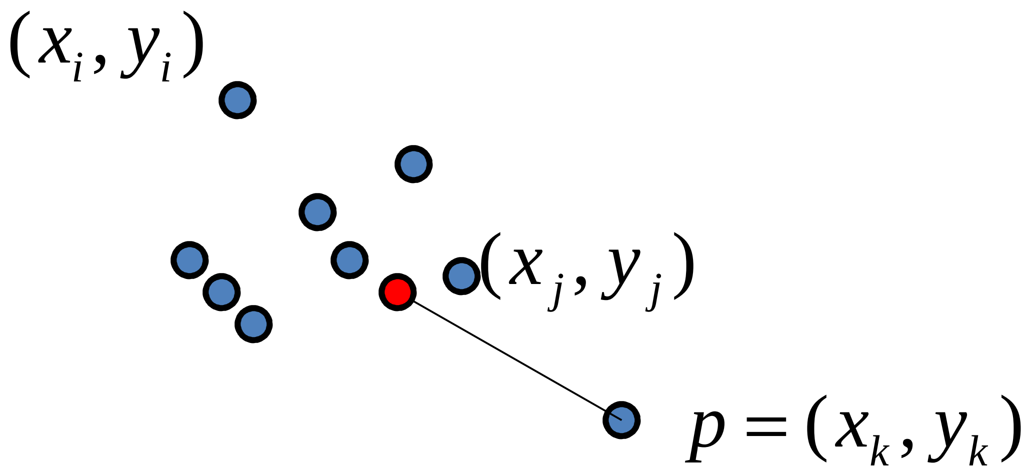
- What are the dominant directions?



- Major, minor eigenvectors give the directions

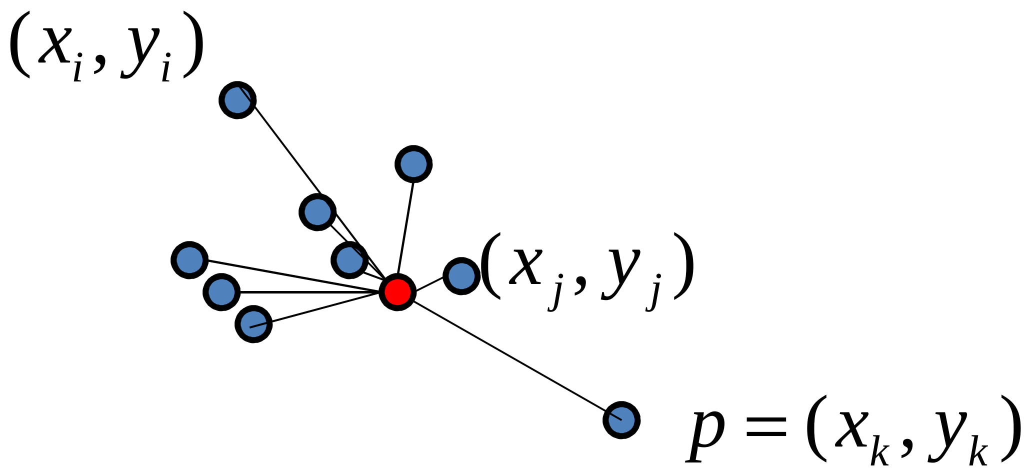
Variance

$$\begin{bmatrix} (x_k - c_x)^2 & (x_k - c_x)(y_k - c_y) \\ (x_k - c_x)(y_k - c_y) & (y_k - c_y)^2 \end{bmatrix} = \begin{bmatrix} x_k - c_x \\ y_k - c_y \end{bmatrix} \begin{bmatrix} x_k - c_x & y_k - c_y \end{bmatrix}$$



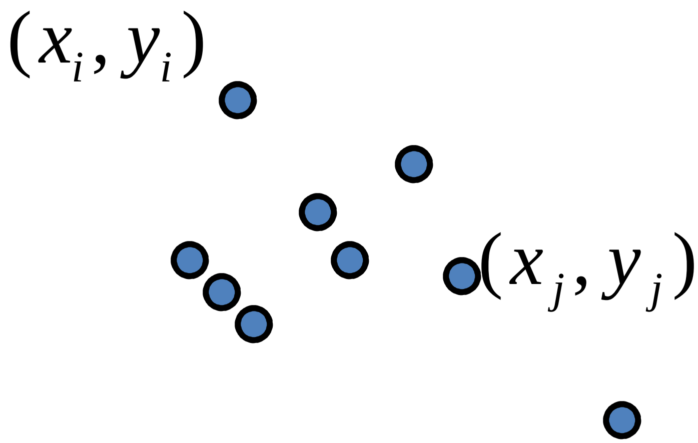
Variance

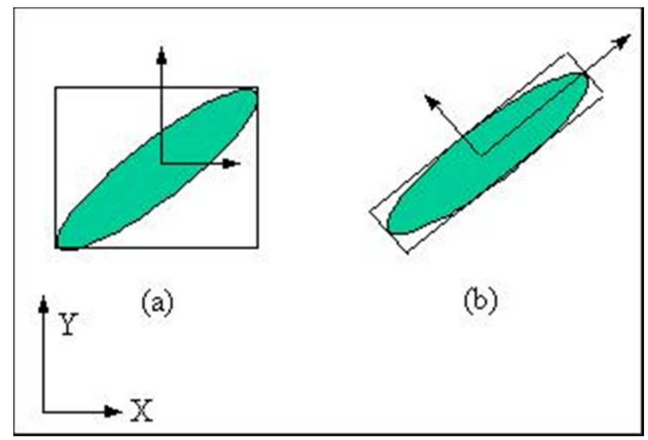
$$\begin{bmatrix} (x_k - c_x)^2 & (x_k - c_x)(y_k - c_y) \\ (x_k - c_x)(y_k - c_y) & (y_k - c_y)^2 \end{bmatrix} = \begin{bmatrix} x_k - c_x \\ y_k - c_y \end{bmatrix} \begin{bmatrix} x_k - c_x & y_k - c_y \end{bmatrix}$$



- It is a voting scheme

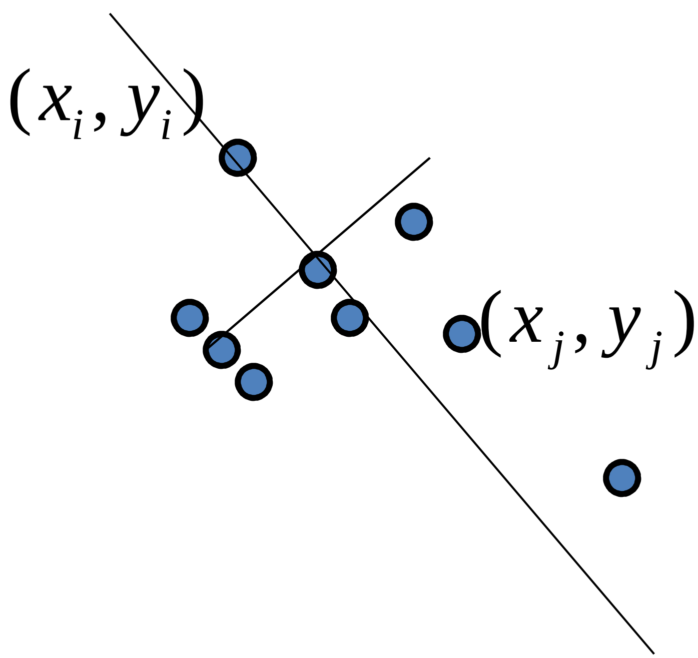
Bounding Boxes



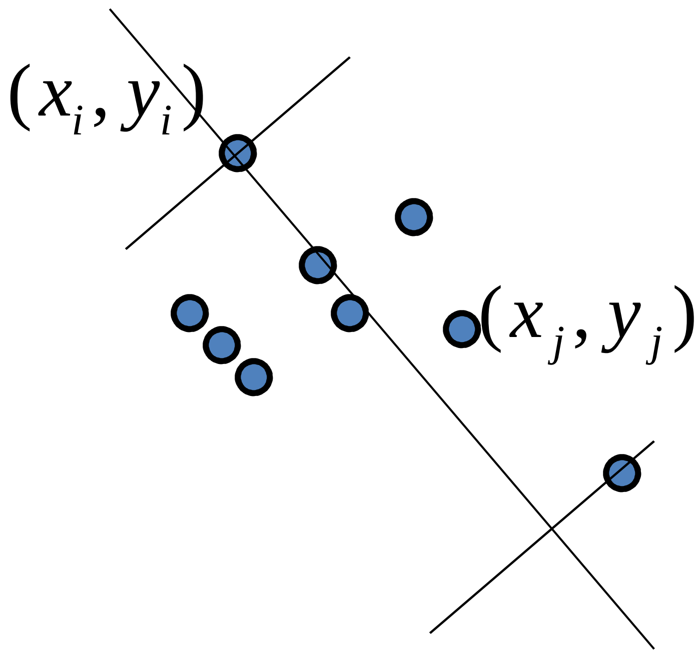


- Axis-aligned bounding box
- Oriented bounding box

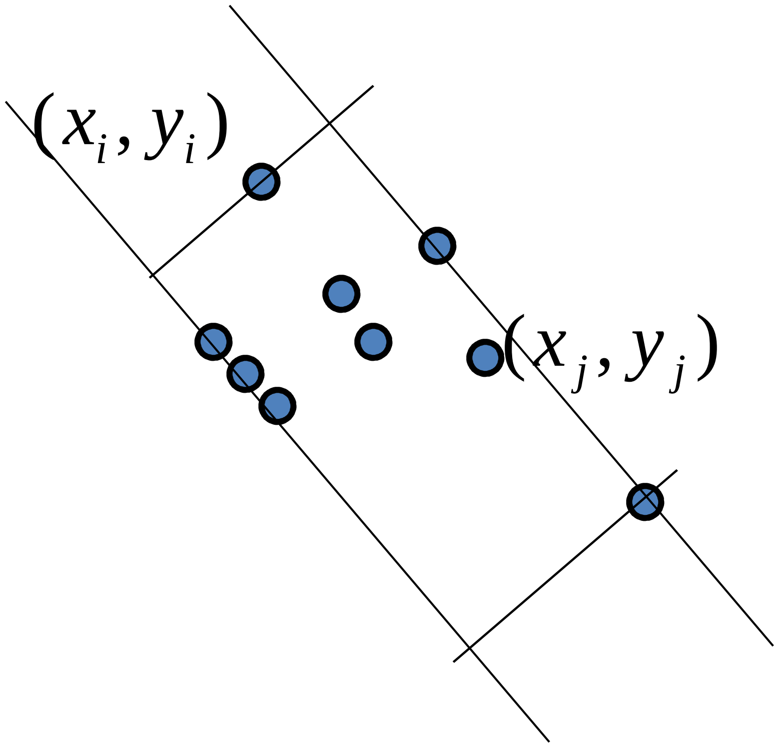
Oriented Bounding Boxes



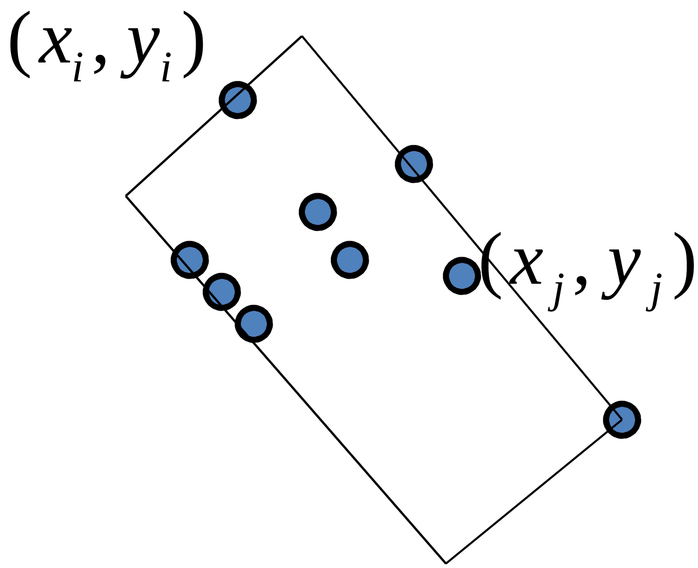
Oriented Bounding Boxes



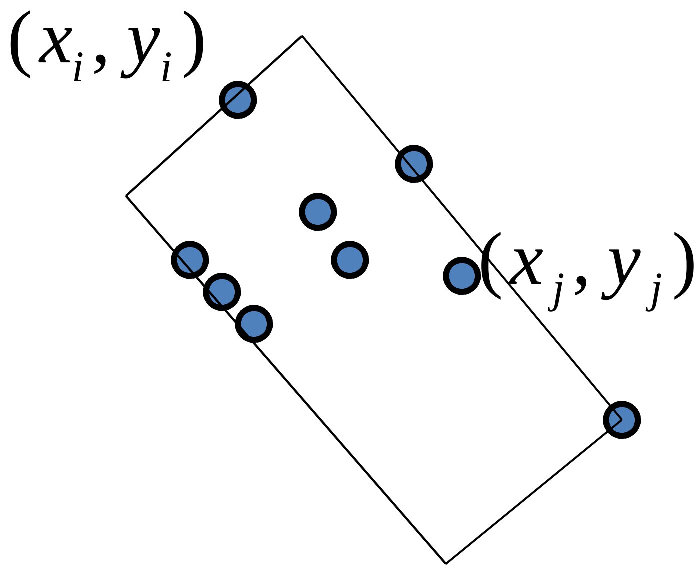
Oriented Bounding Boxes



Oriented Bounding Boxes



Oriented Bounding Boxes

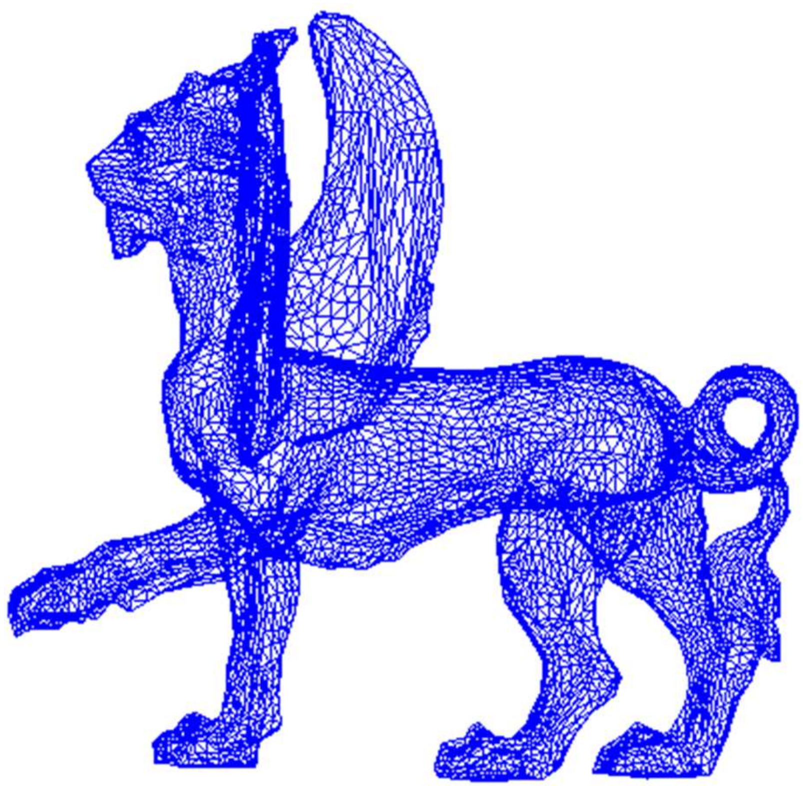


- How do you extend this to 3D?

Compute Euler Characteristics

Surface Representation

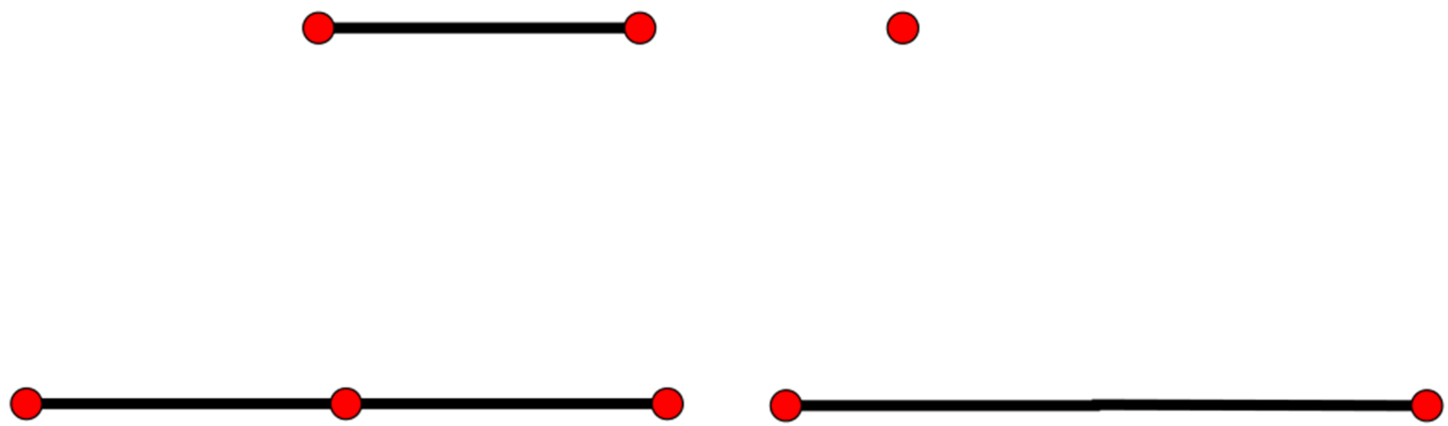
- Consider a discrete polygonal representation
  - V – number of vertices
  - E – number of edges
  - F – number of faces



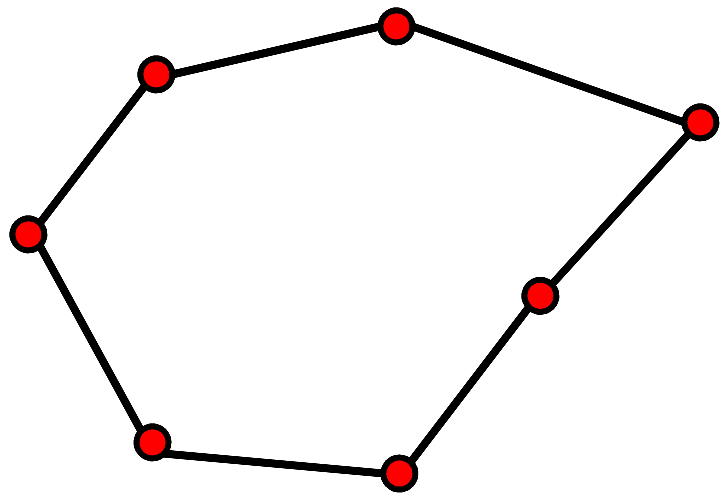
Euler characteristics L = V-E+F

Euler characteristic is a topological invariant, a number that describes a topological space's shape or structure regardless of the way it is bent. (Wiki)

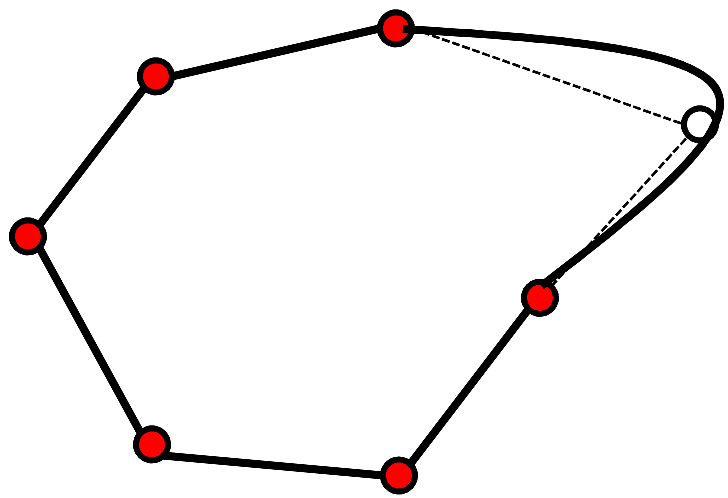
Elementary Collapse on Edges



Elementary Collapse on Edges



Elementary Collapse on Edges



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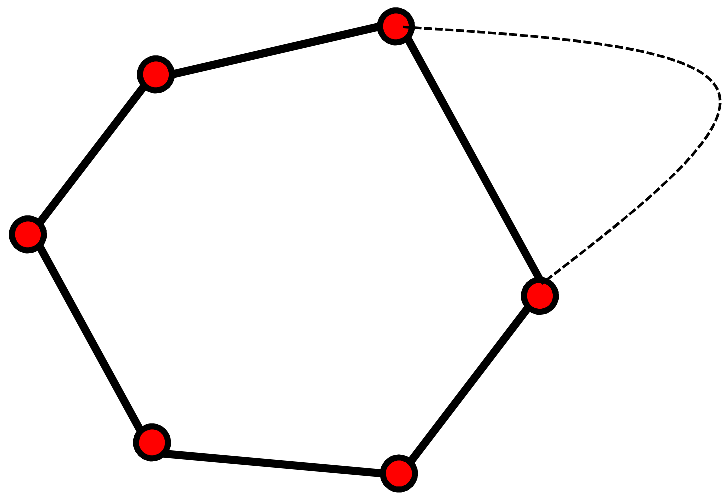

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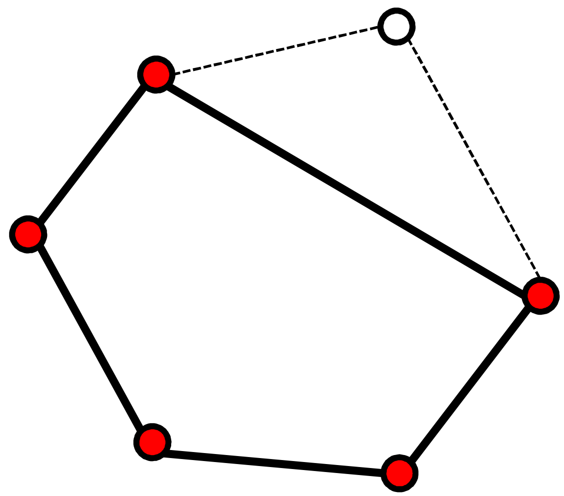
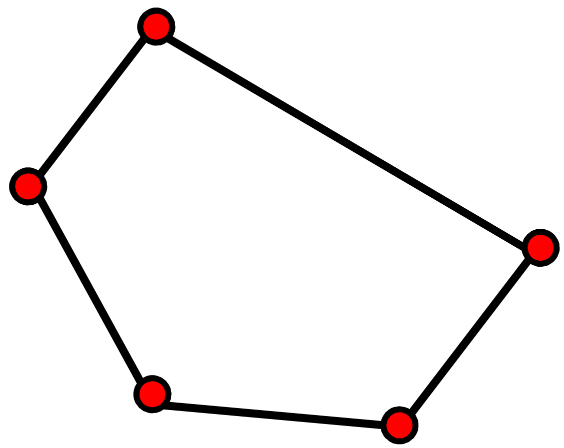
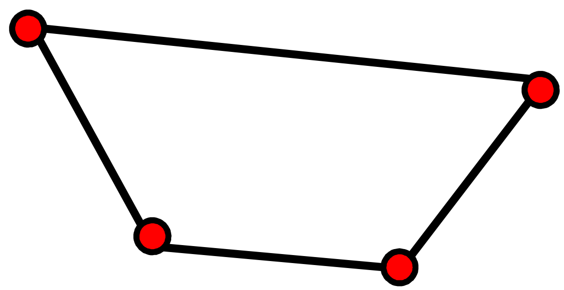


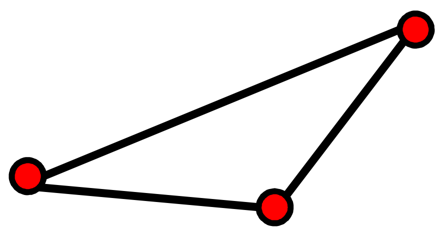
Elementary Collapse on Edges



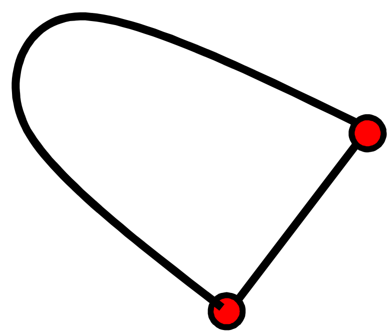
Elementary Collapse on Edges



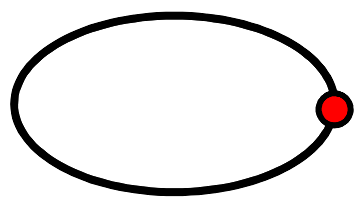
Elementary Collapse on Edges



Elementary Collapse on Edges



Elementary Collapse on Edges



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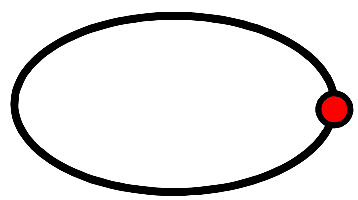
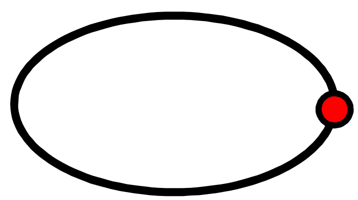


V=E for a closed and simple planar curve.

Elementary Collapse on Edges

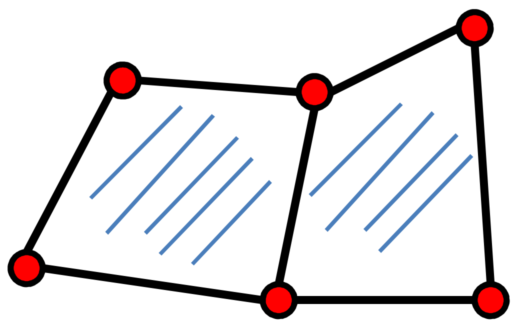


V=E for a closed and simple planar curve.

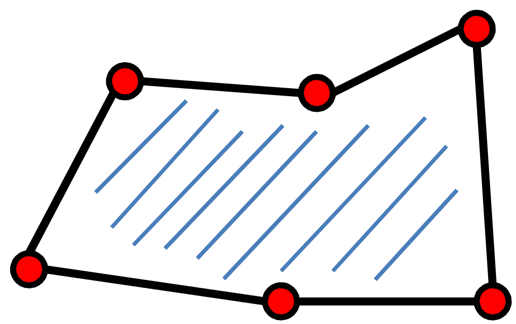
What about 3D surfaces ?

Need to consider the merging of the faces!

Elementary Collapse for Faces

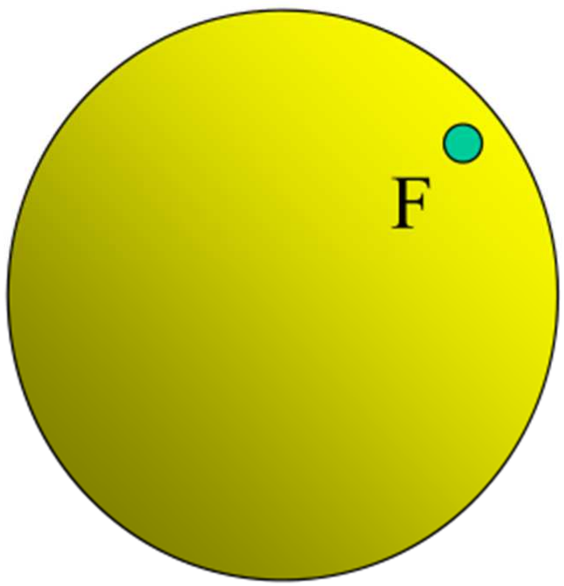


V=6
E=7
F=2
L= ?

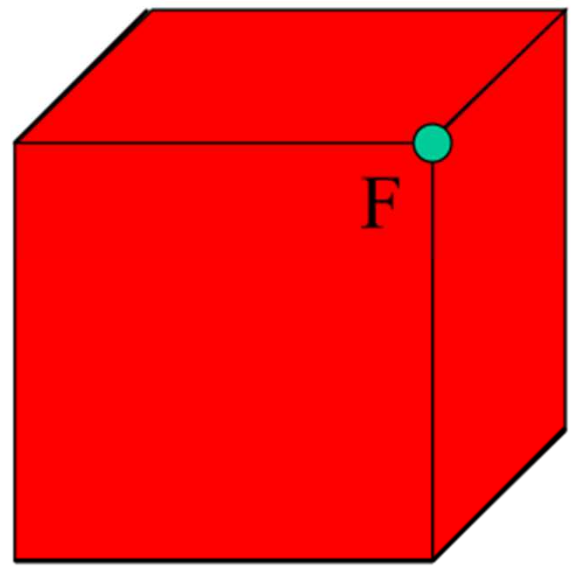


V=6
E=6
F=1
L= ?

Euler Characteristic of a Cube

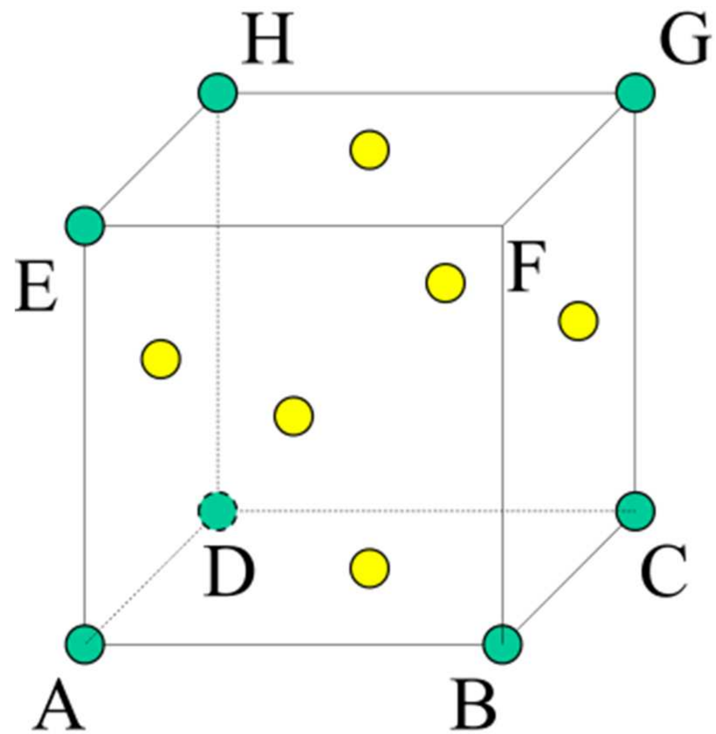


V=1
E=0
F=1
L= ?

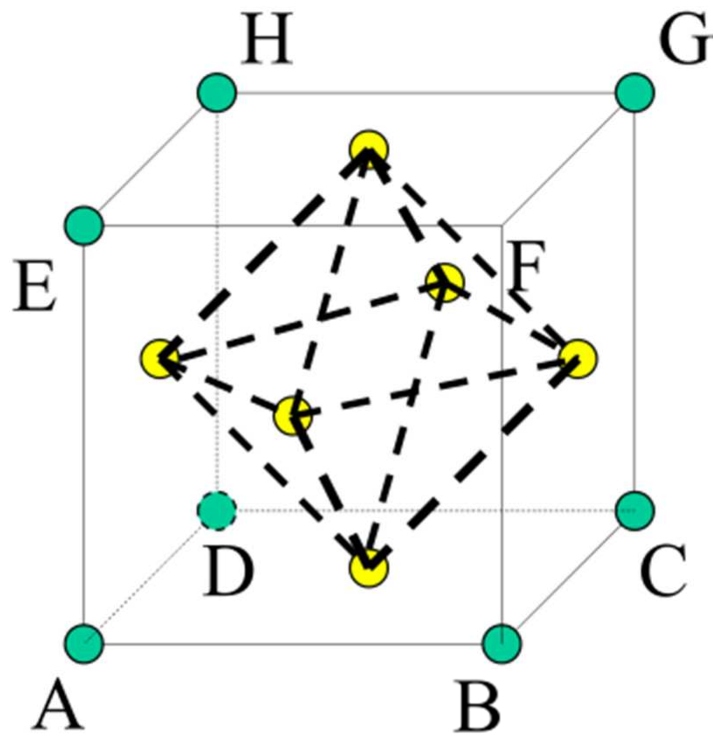


V=8
E=12
F=6
L= ?

Dual of a Hexahedron

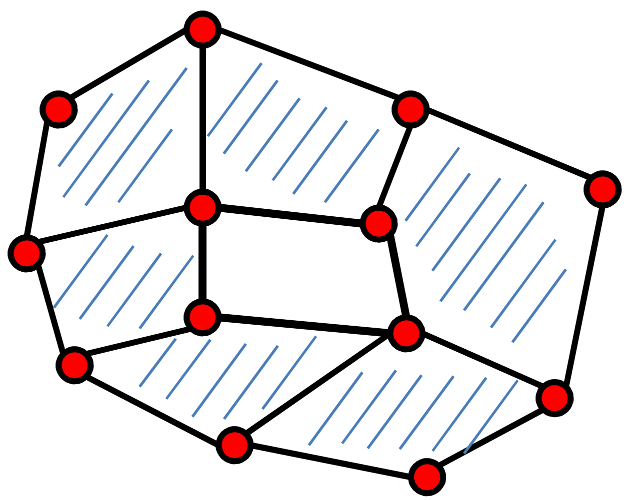


Dual of a Hexahedron



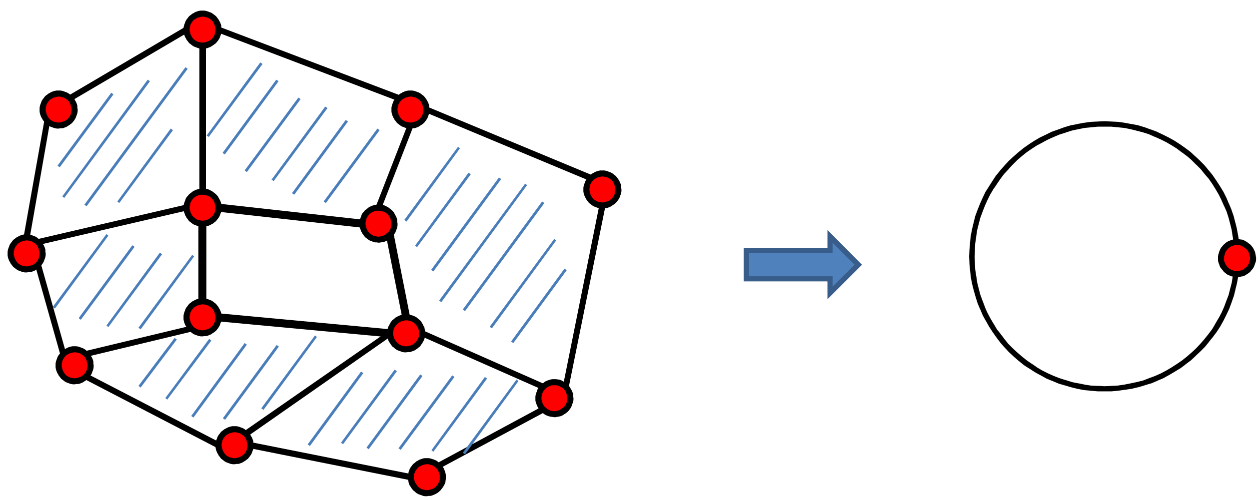
Does this dual change the Euler characteristic?

What Do Different Euler Characteristics Tell us?



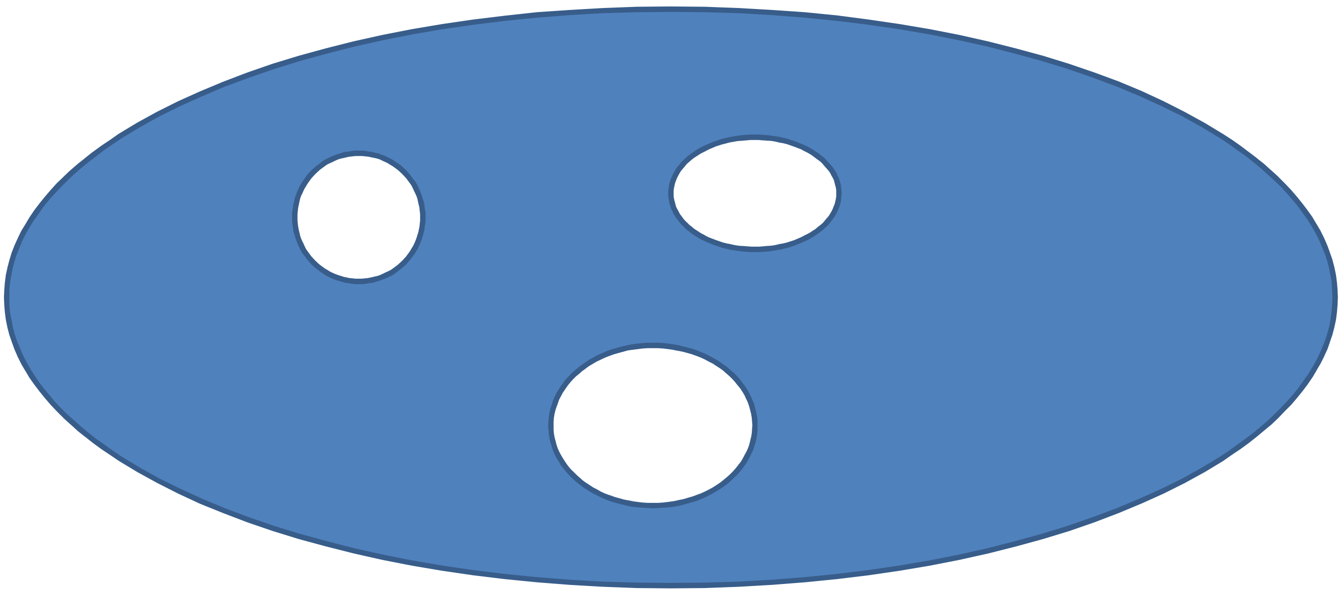
L=?

What Do Different Euler Characteristics Tell us?



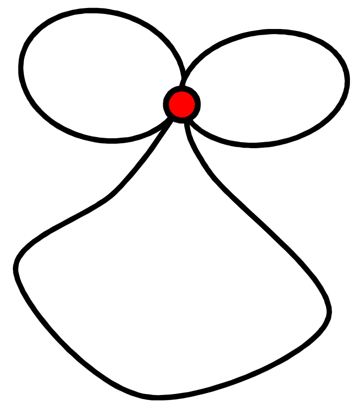
L=0

What Do Different Euler Characteristics Tell us?



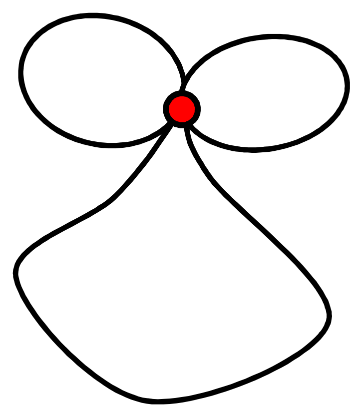
What is its Euler characteristic?

What Do Different Euler Characteristics Tell us?



What is its Euler characteristic?

What Do Different Euler Characteristics Tell us?

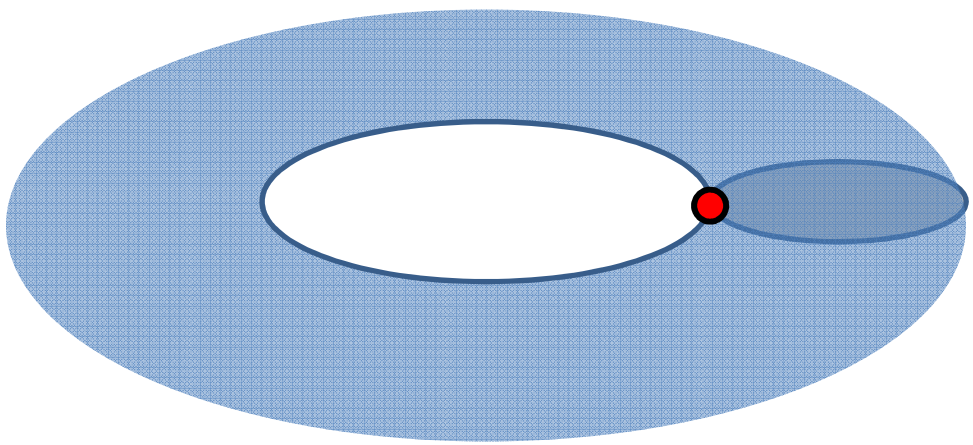


L = -2!

What about 3D surfaces?

What Do Different Euler Characteristics Tell us?

For 3D surfaces, L=V-E+F=?



Acknowledge

- Part of the materials are provided by
  - Prof. Eugene Zhang at Oregon State University