8) Translating Natural Language into FOPL [12]

a) Fred sends an e-mail to each student enrolled in COSC 6342 excluding him.
   \[\forall x \ [\text{student}(x) \land \text{enrolled}(x, \text{COSC}6342) \land \neg (x == \text{Fred}) \Rightarrow \text{Email} (\text{Fred}, x)]\]

b) Politician can fool some people all the time, and they can fool all the people some of the time, but they can’t fool all the people all the time.
   \[\forall p \ [\text{politician}(p) \Rightarrow ((\exists t \forall x \ [\text{person}(x) \land \text{time}(t) \land \text{fool}(p, x, t)]) \land
   \quad (\forall t \exists x \ [\text{person}(x) \land \text{time}(t) \land \text{fool}(p, x, t)]) \land
   \quad \neg (\forall t \forall x \ [\text{person}(x) \land \text{time}(t) \land \text{fool}(p, x, t)))]\]

c) Persons cannot have multiple social security numbers.
   \[\forall x \forall y \forall z \ [\ [\text{person}(x) \land \text{SSN}(y) \land \text{Is-SSN}(y, x) \land \text{SSN}(z) \land \text{Is-SSN}(z, x)] \Rightarrow (y==z) \]

d) Fred has at least 2 sisters.
   \[\exists x \exists y \ [\text{Sister}(\text{Fred}, x) \land \text{Sister}(\text{Fred}, y) \land \neg (x == y)]\]

e) If block A is on the top of block B and block B is on the top of block C, then block A is on the top of block C.
   \[\forall A \forall B \forall C \ [\text{Block}(A) \land \text{Block}(B) \land \text{Block}(C) \land \text{top}(A, B) \land \text{top}(B, C) \Rightarrow \text{top}(A, C)]\]

f) For any two real numbers r1 and r2 with r1 < r2 there always exists a real number r such that: r1 < r < r2
   \[\forall x \forall y (x < y \Rightarrow \exists r \ (x < r < y))\]

9) Resolution Proofs [14]

a) Textbooks mention that “resolution for first order predicate logic is semi-decidable”. What does this mean? [3]

The theorem prover finds the proof if it exists but runs forever if there is no proof

b) Show using Resolution (and not by using other methods!) [11]:
   (1) \(\forall x \forall y \forall z \ (Q(x,y) \land P(x,z)) \Rightarrow S(x,z)\)
(2) \( \forall a \forall b \ (S(a,b) \rightarrow R(a,b)) \)
(3) \( \forall c \forall d \ (P(c,d)) \)
(4) \( \forall s \forall t \ (Q(s,t) \rightarrow P(s,s)) \)
(X) \( \forall e \forall f \ (Q(e,f) \rightarrow R(e,g)) \)

(1) \( \neg Q(x,y) \lor \neg P(x,x) \lor S(x,Z(x,y)) \)
(2) \( \neg S(a,b) \lor R(a,b) \)
(3) \( P(C,d) \)
(4) \( \neg Q(s,t) \lor P(s,s) \)
(Xa) \( Q(E,f) \)
(Xb) \( \neg R(E,g) \)
(5) \( P(E,E) \) using Xa and 4 with \((s E)\)
(6) \( \neg S(E,g) \) using Xb and 2 with \((a E)(g b)\)
(7) \( \neg Q(E,y) \lor \neg P(E,E) \) using 6 and 1 with \((x E) (g ...))\)
(8) \( \neg Q(E,y) \) using 5 and 7
(9) empty using \((Xa)\)

10) More on Resolution [14]

a) Prove using resolution (and not using other methods) [10]

(A1) Every man loves at least one woman.

\[ \forall x \ (\text{man}(x) \rightarrow \exists y \ (\text{woman}(y) \land \text{loves}(x, y))) \]

(A2) Every woman loves at least one man.

\[ \forall y \ (\text{woman}(y) \rightarrow \exists x \ (\text{man}(x) \land \text{loves}(y, x))) \]

(A3) Women do not love men that are not intelligent.

\[ \forall x \forall y \ ((\text{man}(x) \land \text{woman}(y) \land \text{loves}(y,x)) \rightarrow \text{intelligent}(x)) \]

(A4) Vanessa is an intelligent woman.

\[ \text{woman} (\text{Vanessa}) \land \text{intelligent}(\text{Vanessa}) \]

(A) There is at least one intelligent man in the world.

\[ \exists x \ (\text{man}(x) \land \text{intelligent}(x)) \]
\[ \rightarrow (A) \ \forall x \ (\neg \text{man}(x) \lor \neg \text{intelligent}(x)) \]
Prove:

(A1) \( \text{man}(x) \rightarrow (\text{woman}(Y(x)) \land \text{loves}(x, Y(x))) \)

(A2) \( \text{woman}(y) \rightarrow (\text{man}(X(y)) \land \text{loves}(y, X(y))) \)

(A3) \( (\text{man}(x) \land \text{woman}(y) \land \text{loves}(y, x)) \rightarrow \text{intelligent}(x) \)

(A4) \( \text{woman}(\text{Vanessa}) \land \text{intelligent}(\text{Vanessa}) \)

(A5) \( \neg \text{man}(x) \lor \neg \text{intelligent}(x) \)

(1) \( \neg \text{man}(x) \lor \text{woman}(Y(x)) \)

(2) \( \neg \text{man}(x) \lor \text{loves}(x, Y(x)) \)

(3) \( \neg \text{woman}(y) \lor \text{man}(X(y)) \)

(4) \( \neg \text{woman}(y) \lor \text{loves}(y, X(y)) \)

(5) \( \neg \text{man}(x) \lor \neg \text{woman}(y) \lor \neg \text{loves}(y, x) \lor \text{intelligent}(x) \)

(6) \( \text{woman}(\text{Vanessa}) \)

(7) \( \text{intelligent}(\text{Vanessa}) \)

(8) \( \neg \text{man}(x) \lor \neg \text{intelligent}(x) \)

(9) \( \text{man}(X(\text{Vanessa})) \)

(10) \( \text{loves}(\text{Vanessa}, X(\text{Vanessa})) \)

(11) \( \neg \text{intelligent}(X(\text{Vanessa})) \)

(12) \( \text{intelligent}(X(\text{Vanessa})) \)

(13) []

b) Is it possible that a theorem prover runs forever, when trying to prove statement (A) in problem 10. c) If your answer is no, give reasons for your answer! If your answer is yes, explain under which circumstances the theorem prover might generate clauses “forever”? What are the practical implications of your answer for resolution theorem provers in general? [4]

Answer:

Yes. This is because there is no hard and fast rule on the theorem prover that requires it to unify two different statements. So the theorem prover may try to unify the same statements again and again and that may make the theorem prover to run forever.

For example:

(1) \( \neg \text{man}(x) \lor \text{woman}(Y(x)) \)

(2) \( \neg \text{woman}(y) \lor \text{man}(X(y)) \)

(3) \( \text{woman}(\text{Vanessa}) \)

(4) \( \text{man}(X(\text{Vanessa})) \) from 2 and 3

(5) \( \text{woman}(Y(X(\text{Vanessa}))) \) from 1 and 4

(6) \( \text{man}(X(Y(X(\text{Vanessa})))) \) from 2 and 5

This loop is caused because it executes the same rules (1 and 2) repeatedly.