COSC 3362: Project #1 **Name:** _____

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1. The nth order Hilbert matrix $H_n = (h_{ij})_{i,j=1}^n$ is defined by

$$h_{ij} = \frac{1}{i+j-1} \qquad 1 \le i, j \le n.$$

This matrix is often used for test purposes because its condition number increases exponentially with n.

a) Show that H_n is symmetric and positive definite.
(*Hint:* Use the fact that $\frac{1}{i+j+1} = \int_0^1 \xi^i \xi^j d\xi, \quad 0 \le i, j.$)

b) Calculate the condition number $\kappa(H_n)$, for $n = 1, 2, \ldots, 10$, and plot it versus n using a semi-log scale plot. (In MATLAB you can use `cond(hilb(n))` to calculate $\kappa(H_n)$, and `semilogy()` function for ploting. If you don't use MATLAB you are on your own for these calculations.) What can you conclude about the growth of $\kappa(H_n)$ with n from the graph?

c) Solve $H_{10}x = b$, where H_{10} is a 10th order Hilbert matrix, and b is such that $x = (1, 1, \ldots, 1)$ using simple Gaussian elimination and Gaussian elimination with scaled row pivoting. Compute the errors in each solution. Which method is more accurate? Why?

2. Program the Gauss-Seidel and Jacobi methods and test their convergence on tridiagonal systems of the form

$$Ax = b$$

where

$$A = \begin{bmatrix} a & b & & & & \\ b & a & b & & & \\ & \ddots & \ddots & \ddots & & \\ & & & b & a & b \\ & & & & b & a \end{bmatrix}.$$

Select the right-hand-side vectors such that you know the exact solution and solve the following problems.

a) $a = 2, b = -1, n = 2, 4, 8, 16, 100, 1000$

b) $a = 4, b = -1, n = 2, 4, 8, 16, 100, 1000$

Print out the norms of the errors. (Do not print the solutions!) What can you conclude about the above systems?

3. Computer problem 2, pp. 245 in [1].

References:

1. D. Kincaid and W. Cheney, *Numerical Analysis*, 3rd Ed., Brooks/Cole, 2002.