On Pancyclicity Properties of OTIS Networks

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Abstract. The OTIS-Network (also referred to as two-level swapped network) is composed of *n* clones of an *n*-node original network constituting its clusters. It has received much attention due to its many favorable properties such as high degree of scalability, regularity, modularity, package-ability and high degree of algorithmic efficiency. In this paper, using the construction method, we show that the OTIS-Network is Pancyclic if its basic network is Hamiltonian-connected. The study of cycle embeddings with different sizes arises naturally in the implementation of a number of either computational or graph problems such as those used for finding storage schemes for logical data structures, layout of circuits in VLSI, etc. Our result is resolving an open question posed in [6] and generalizing a number of proofs in the literature for specific Hamiltonian properties of similar networks.

1 Introduction

The Optical Transpose Interconnection System (OTIS), which was proposed in [1, 2, 3], generates a wide class of high-performance scalable interconnection networks. It offers a new optoelectronic computer architecture that takes benefits from both optical and electronic technologies. In this architecture, processors are divided into groups where electronic interconnects are used to connect processors within each group, while optical interconnects are used for inter-group communication [3]. In such an OTIS system, an optical link connects processor *p* of group *g* to processor *g* of group *p*. It has been shown in [4] that when the number of processors in a group equals the number of groups, the bandwidth and the power consumption in OTIS-Networks are optimized and system area and volume are minimized. Thus, in an N^2-processor OTIS network, processors are partitioned into N groups of N processors. Besides the electronic connections between the processors in each group, processor i in group j is optically connected to processor j of group i. The OTIS-hypercube and OTIS-mesh are two of the most widely studied instances of the OTIS architecture [5-17]. A number of algorithms have been developed for these networks, such as routing, selection, and sorting [5, 8, and 11], data rearrangement [8], matrix multiplication [13], and broadcasting [6]. Many of topological properties of these systems, such as node degree, diameter, β-cut, and bisection width are addressed in previous studies [6, 8]. Also, in [9, 10], it was proved that if G is a Hamiltonian-connected graph, so is the OTIS-*G*. Moreover, the fault tolerance of OTIS-Networks has been addressed and the

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fault diameter of OTIS-*G* is derived with respect to the fault diameter of G [9]. In [15, 17] the performance merits of the OTIS-*hypercube* and the effect of different structural and workload parameters on the overall performance are investigated. Their result reveals that the OTIS multi-computers are good candidates for interconnection networks of future generation parallel computers.

In this paper, we prove the conjecture proposed in [6]. According to this conjecture if G contains a cycle of length *L*, then there exists a cycle of length L^2 in OTIS-*G*. We also generalize this result and demonstrate that if there exists a Hamiltonian path between every two arbitrary nodes of graph *G* (or if *G* is Hamiltonian-connected), then all cycles with length 7, 8 … $|V(G)|^2$ can be constructed in OTIS-*G*.

The organization of the paper is as follows. In Section 2, OTIS-Network is formally defined and some basic properties of this network are reported. In Section 3, the Pancyclicity property of the OTIS-*G* with regard to the Hamiltonian property of the basic network G is proved. Finally, Section 4 concludes this paper.

2 The OTIS Network: Definition and Basic Properties

The reader is referred to [6] for an in-depth account of basic concepts and properties of the OTIS networks such as topology, routing algorithms, broadcasting, embedding of graphs, etc. In this section, more specific concepts are described.

Definition 1. Let $G = (V_G, E_G)$ be an undirected graph. The OTIS-$G = (V_{OT}, E_{OT})$ network is an undirected graph defined by:

$V_{OT} = \{<g, p> \mid g, p \in V_G\}$ and

$E_{OT} = \{(<g, p_1>, <g, p_2>) \mid g \in V_G, (p_1, p_2) \in V_G\} \cup \{(<g, p>, <p, g>) \mid g, p \in V_G, g \neq p\}$.

The graph *G* is called the factor (or basic) network of OTIS-*G*. If *G* has N nodes, the OTIS-*G* is composed of *N* node-disjoint sub-networks $G_1, G_2, \ldots, G_N$, called groups. Each of these groups is isomorphic to the factor graph *G*. A node <g, p> in OTIS-*G* corresponds to the node of address p in group G_g. An intra-group edge of the form $(<g, p_1>, <g, p_2>)$ corresponds to an electronic link, while an inter-group edge of the form (<g, p>, <p, g>) corresponds to an optical link and passing over such a link is referred to as an OTIS movement [6]. Figure 1 displays some examples of the OTIS-*G* where *G* is either a Q_3 (3-dimensional hypercube) or a K_5 (complete graph of 5 nodes) [15].

A generalization of the OTIS-Networks is an *l*-level hierarchical swapped network [9], denoted *SW(G, l)*, which is based on a nucleus (factor) graph *G*, too. To build an *l*-level swapped network, *SW(G, l)*, we use $N_{l-1} = |SW(G, l-1)|_V$ identical copies (clusters) of *SW(G, l-1)*. Each copy of *SW(G, l-1)* is viewed as a level-*l* cluster. Node *i* in cluster *j* is connected to node *j* in cluster *i* for all $i \neq j$, $0 \leq i, j \leq N_{l-1}$. It is clear that *SW(G, 2)* is topologically equivalent to the OTIS-*G* network. The *SW(G, 3)* is topologically isomorphic to the OTIS-OTIS-*G* network and this holds for higher level networks.

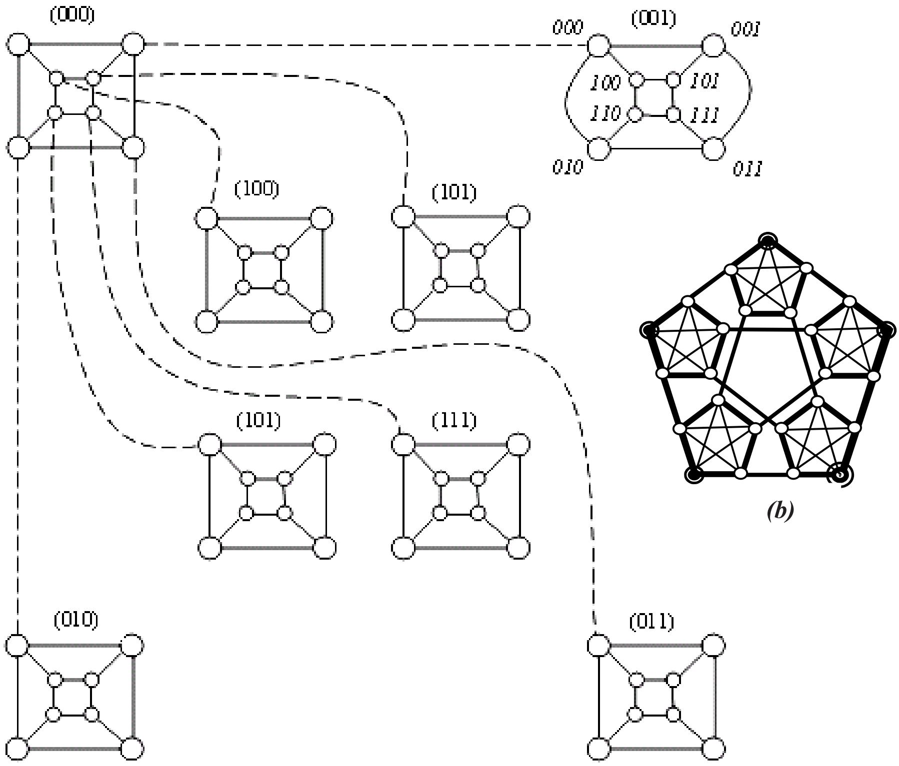

Fig. 1. (a) A 3-dimensional OTIS-hypercube with the optical connections exiting one of the sub-graphs (numbers inside parenthesis are the group numbers) (b) The topologies of OTIS-K_5

Theorem 1. [9] If G has node degree d and diameter D, the degree and diameter of $SW(G, l)$ are $d_{sw} = d + l$ and $D_{sw} = 2^{l-1}(D + 1) - 1$, respectively.

3 Pancyclicity of OTIS-Networks

In this section, we propose a solution to the open question stated in [6]. Let G be a graph of $n \geq 3$ vertices. A k-cycle is a cycle of length k. A Hamiltonian cycle (path) of G is a cycle (path) containing every vertex of G. A Hamiltonian graph is a graph containing a Hamiltonian cycle. We prove that if there is an L-cycle in the factor graph G, this would give a cycle of length L^2 in OTIS-G. In a special case, it would imply that if G is Hamiltonian, so is OTIS-G. Studying Hamiltonicity and related properties of interconnection networks has attracted much attention [5, 9, 14, 18-20].

Theorem 2. [6] If there exists an L-cycle (a cycle of length L) in G, then there exists a cycle of length L^2-L in OTIS-G.

Theorem 3. If graph *G* contains an *L*-cycle, then there exists an L^2-cycle in OTIS-*G*.

Proof. We prove this by induction on *L*, the length of the cycle. By considering the parity of *L*, we regard 2 separate cases.

Case 1. *L is even*: Let *L* = 2*S* and $\gamma_0, \gamma_1, \gamma_2, ...\gamma_{2S-2}, \gamma_{2S-1}, \gamma_{2S} = \gamma_0$ be the circular sequence of node addresses corresponding to an *L*-cycle in *G*, $L \le |V(G)|$. We obtain such a cycle in each of 2*S* groups namely $G_{\gamma_0}, G_{\gamma_1}, G_{\gamma_2}, ..., G_{\gamma_{2S-2}}, G_{\gamma_{2S-1}}$ of OTIS-*G*. When *L* is equal to 4, one can easily check the correctness of the base of induction.

To achieve the L^2-cycle within OTIS-*G,* for *L>4*, we remove from each cycle $\gamma_0, \gamma_1, \gamma_2, ...\gamma_{2S-2}, \gamma_{2S-1}, \gamma_{2S} = \gamma_0$ of sub-graph $G_{\gamma_i}, 5 \le i \le 2S-1$, the two edges $(<\gamma_i, \gamma_0>, <\gamma_i, \gamma_1>)$ and $(<\gamma_i, \gamma_2>, <\gamma_i, \gamma_3>)$, and replace them by the optical inter-group edges $(<\gamma_i, \gamma_j>, <\gamma_j, \gamma_i>)$ for 5≤*i*≤2*S*-1, 0≤*j*≤3. As well, within the sub-graphs G_{γ_0} and G_{γ_1}, we remove all edges of the form $(<\gamma_0, \gamma_{2t}>, <\gamma_0, \gamma_{2t+1}>)$ and $(<\gamma_1, \gamma_{2t}>, <\gamma_1, \gamma_{2t+1}>)$ for 2≤*t*≤*S*-1. In a similar way, within the sub-graphs G_{γ_2} and G_{γ_3}, we remove all links of the form $(<\gamma_2, \gamma_{2t-1}>, <\gamma_2, \gamma_{2t}>)$ and $(<\gamma_3, \gamma_{2t-1}>, <\gamma_3, \gamma_{2t}>)$ for 3≤*t*≤*S*. In addition, let us remove edges $(<\gamma_0, \gamma_2>, <\gamma_0, \gamma_3>)$ and $(<\gamma_4, \gamma_0>, <\gamma_4, \gamma_1>)$ from the corresponding subgroup and replace them by the inter-group edges $(<\gamma_0, \gamma_2>, <\gamma_2, \gamma_0>)$, $(<\gamma_0, \gamma_3>, <\gamma_3, \gamma_0>)$, $(<\gamma_0, \gamma_4>, <\gamma_4, \gamma_0>)$, and $(<\gamma_1, \gamma_4>, <\gamma_4, \gamma_1>)$. Therefore, all of L^2 nodes which have already belonged to separate cycles within separate groups now create a unique L^2-node cycles within the OTIS-*G*.

Case 2. *L is odd:* Let *L* = 2*S*-1 and $\gamma_0, \gamma_1, \gamma_2, ...\gamma_{2S-2}, \gamma_{2S-2}, \gamma_{2S-1} = \gamma_0$ be the L-cycle in G, $L \le |V(G)|$. We have such a cycle in each of 2*S*-1 groups namely $G_{\gamma_0}, G_{\gamma_1}, G_{\gamma_2}, ..., G_{\gamma_{2S-2}}, G_{\gamma_{2S-2}}$ in the OTIS-*G*. The following construction method can build all cycles of length *L*≥3: Remove edge $(<\gamma_i, \gamma_0>, <\gamma_i, \gamma_1>)$ from each sub-graph G_{γ_i}, 2≤*i*≤2*S*-2, and replace it with an inter-group edges of the form $(<\gamma_0, \gamma_i>, <\gamma_i, \gamma_0>)$ and $(<\gamma_1, \gamma_i>, <\gamma_i, \gamma_1>)$ for 2≤*i*≤2*S*-2. Also, remove all edges of form $(<\gamma_0, \gamma_{2t-1}>, <\gamma_0, \gamma_{2t}>)$ and $(<\gamma_1, \gamma_{2t}>, <\gamma_1, \gamma_{2t+1}>)$, for 1≤*t*≤*S*-1, within sub-graphs G_{γ_0} and G_{γ_1}. Finally, add the edge $(<\gamma_0, \gamma_1>, <\gamma_1, \gamma_0>)$ to close the cycle. □

Theorem 4. If *G* is Hamiltonian, then OTIS-*G* is also Hamiltonian.

Proof. Theorem 3 implies that if *G* has a cycle of length $|V(G)|$, then OTIS-*G* has a cycle of length $|V(G)|^2 = |V(OTIS-G)|$. □

In the rest of this section, the relation of two important properties of Hamiltonian-connectedness of the factor graph *G* and pancyclicity of the OTIS-*G* is studied.

Precisely, it will be proved that the OTIS-*G* network possesses all cycles of length 7, 8, …, and $\left|V(G)\right|^2$, provided that the factor graph *G* is Hamiltonian-connected.

Definition 2. For a given network *G*=(*V*, *E*), *G* is said to be Hamiltonian-connected if it contains a Hamiltonian path starting from any node $x\in V$ and ending at any node $y\in V-\{x\}$, i.e. a path that leads from *x* to *y* and traverses every node of *G* exactly once.

Definition 3. For a given network *G*=(*V*, *E*) and a given set $\Sigma \subseteq \{3, 4, ..., \left|V(G)\right|\}$, *G* is called to be $\Sigma - pancyclic$ provided that *G* contains all cycles of length $\delta \in \Sigma$. Comparably, *G* is said to be $\overline{\Sigma} - pancyclic$ if *G* contains all cycles of length $\delta \in$ {3,4 … |*V(G)*|}- Σ.

Definition 4. A graph *G* is said to be *pancyclic*, if it is $\Sigma - pancyclic$ where Σ ={3,4 … |*V(G)*|}. Graph *G* is said to be *weakly pancyclic* [20] if it contains cycles of all lengths between a minimum and a maximum cycle length that can be embedded within *G*.

Pancyclicity is an important property determining if the topology of a network is suitable for an application in which mapping rings of different lengths into the host network is required. Such cycle embedding in various networks has attracted much attention and been a challenging research issue in recent years [18- 20]. For OTIS-networks, the only result about its Hamiltonian property is the one in [9, 10]. There, authors showed that if the factor graph *G* is Hamiltonian, so is the OTIS-*G*. However, the following theorem presents a remarkable relation between Hamiltonian-connectedness of the factor graph *G* and pancyclicity of the OTIS-*G*.

Theorem 5. The OTIS-*G* network is $\overline{\Sigma} - pancyclic$, for Σ ={3,4,5,6}, provided that the factor graph *G* is Hamiltonian-connected (i.e. OTIS-*G* is weakly pancyclic).

Proof. To construct an *L*-cycle within the OTIS-*G* graph, for $7 \le L \le \left|V(G)\right|^2$, we consider two different cases. First, we deal with the problem of finding the cycles of length L, $7 \le L \le 2\times\left|V(G)\right|+1$, and then we consider other remaining cycle lengths.

Case 1. Since *G* is Hamiltonian-connected, we are able to find a Hamiltonian path between two arbitrary neighboring nodes *u* and *v* in *G*. Let $\gamma_u = \gamma_0, \gamma_1, ...\gamma_{\left|V(G)\right|-2}, \gamma_{\left|V(G)\right|-1} = \gamma_v$ be the nodes address sequence corresponding to this $\left|V(G)\right|-$cycle in G. Two following methods construct all cycles of length $7 \le L \le 2\times\left|V(G)\right|+1$ within OTIS-*G*.

When L is odd: Let $L = 2\xi + 1$, for $3 \le \xi \le L$. We build the *L*-cycle within OTIS-*G* graph as follows. (Note that these cycles only use three different groups in OTIS-*G*)

$C_{2\xi+1}$ **:** $(<\gamma_0,\gamma_1>,<\gamma_1,\gamma_0>)$ || $(<\gamma_1,\gamma_0>,<\gamma_1,\gamma_1>)$ || $(<\gamma_1,\gamma_1>,<\gamma_1,\gamma_2>)$ || … || $(<\gamma_1,\gamma_{\xi-2}>,<\gamma_1,\gamma_{\xi-1}>)$ || $(<\gamma_{\xi-1},\gamma_1>,<\gamma_{\xi-1},\gamma_0>)$ || $(<\gamma_0,\gamma_{\xi-1}>,<\gamma_0,\gamma_{\xi-2}>)$ || … || $(<\gamma_0,\gamma_2>,<\gamma_0,\gamma_1>)$.

When L is even: Let $L = 2\xi + 4$, for $2 \le \xi \le L - 2$. We build the cycle of length L within OTIS-G as follows (we assume $|V(G)| > 3$):

$$C_{2\xi+4}: (<\gamma_0,\gamma_2>,<\gamma_2,\gamma_0>) \parallel (<\gamma_2,\gamma_0>,<\gamma_2,\gamma_1>) \parallel (<\gamma_1,\gamma_2>,<\gamma_1,\gamma_3>) \parallel \ldots \parallel (<\gamma_1,\gamma_\xi>,<\gamma_1,\gamma_{\xi+1}>) \parallel (<\gamma_{\xi+1},\gamma_1>,<\gamma_{\xi+1},\gamma_0>) \parallel (<\gamma_0,\gamma_{\xi+1}>,<\gamma_0,\gamma_\xi>) \parallel \ldots \parallel (<\gamma_0,\gamma_3>,<\gamma_0,\gamma_2>).$$

The above-mentioned cycle consists of joining two paths of length ξ within the groups g_0 and g_1, and two path of length 2 within groups g_2 and g_ξ, respectively. For the case of $|V(G)| = 3$, we compose the cycle of length 8 as follows:

$$C_8: (<\gamma_0,\gamma_2>,<\gamma_2,\gamma_0>) \parallel (<\gamma_2,\gamma_0>,<\gamma_2,\gamma_2>) \parallel (<\gamma_2,\gamma_2>,<\gamma_2,\gamma_1>) \parallel (<\gamma_2,\gamma_1>,<\gamma_1,\gamma_2>) \parallel (<\gamma_1,\gamma_2>,<\gamma_1,\gamma_1>) \parallel (<\gamma_1,\gamma_1>,<\gamma_1,\gamma_0>) \parallel (<\gamma_1,\gamma_0>,<\gamma_0,\gamma_1>) \parallel (<\gamma_0,\gamma_1>,<\gamma_0,\gamma_2>).$$

Case 2. To build a cycle of length L, for $2 \times |V(G)| + 1 \le L \le |V(G)|^2$, L could be written as $L = K|V(G)| + \xi$, where $2 < \xi < |V(G)|$ and $3 \le K < |V(G)|$ (case of $K = |V(G)|$ could be treated according to theorem 3). The following method generates a cycle of length $K|V(G)| + \xi$ within OTIS-G. The sequence $\gamma_u = \gamma_0, \gamma_1, \ldots \gamma_{|V(G)|-2}, \gamma_{|V(G)|-1} = \gamma_v$ forms a Hamiltonian path between two arbitrary nodes of u and v in G showed by HP(u, v). Furthermore, we choose group G_m to construct a path of length ξ within it, where $m = max(K, \xi)$.

$$C_{K|V(G)|+\xi}: (<\gamma_m,\gamma_{m-1}>,<\gamma_m,\gamma_{m-2}>) \parallel (<\gamma_m,\gamma_{m-2}>,<\gamma_m,\gamma_{m-3}>) \parallel \ldots \parallel (<\gamma_m,\gamma_{m-\xi+1}>,<\gamma_m,\gamma_{m-\xi}>) \parallel (<\gamma_m,\gamma_{m-\xi}>,<\gamma_{m-\xi},\gamma_m>) \parallel HP(<\gamma_{m-\xi},\gamma_m>,<\gamma_{m-\xi},\gamma_{m-\xi+1}>) \parallel (<\gamma_{m-\xi},\gamma_{m-\xi+1}>,<\gamma_{m-\xi+1},\gamma_{m-\xi}>) \parallel HP(<\gamma_{m-\xi+1},\gamma_{m-\xi}>,<\gamma_{m-\xi+1},\gamma_{m-\xi+2}>) \parallel (<\gamma_{m-\xi+1},\gamma_{m-\xi+2}>,<\gamma_{m-\xi+2},\gamma_{m-\xi+1}>) \parallel \ldots \parallel HP(<\gamma_{m-\xi+\zeta+1},\gamma_{m-\xi+\zeta}>,<\gamma_{m-\xi+\zeta+1},\gamma_{m-\xi+\zeta+2}>) \parallel (<\gamma_{m-\xi+\zeta+1},\gamma_{m-\xi+\zeta+2}>,<\gamma_{m-\xi+\zeta+2},\gamma_{m-\xi+\zeta+1}>) \parallel \ldots \parallel HP(<\gamma_{m-\xi+k-2},\gamma_{m-\xi+k-3}>,<\gamma_{m-\xi+k-2},\gamma_{m-1}>) \parallel (<\gamma_{m-\xi+k-2},\gamma_{m-1}>,<\gamma_{m-1},\gamma_{m-\xi+k-2}>) \parallel HP(<\gamma_{m-1},\gamma_{m-\xi+k-2}>,<\gamma_{m-1},\gamma_m>) \parallel (<\gamma_{m-1},\gamma_m>,<\gamma_m,\gamma_{m-1}>).$$

In the above cycle, variable ζ changes between 0 and k-3.

For the special case of $L = K|V(G)|+1$, $3 \le K < |V(G)|$, the following construction method could be used. Let $BackP(<\gamma_i,\gamma_j>,<\gamma_i,\gamma_{j'}>)$, for $0 \le j < j' \le |V(G)|-1$ denote a path within group G_i which starts from node $<\gamma_i,\gamma_j>$, ends at node $<\gamma_i,\gamma_{j'}>$, and does not include any node of the form $<\gamma_i,\gamma_l>$ for all $j < l < j'$. This path can be easily constructed as follows:

$$BackP(<\gamma_i,\gamma_j>,<\gamma_i,\gamma_{j'}>): \; <\gamma_i,\gamma_j> \| <\gamma_i,\gamma_{j-1}> \| \ldots \| <\gamma_i,\gamma_0> \| <\gamma_i,\gamma_{|V(G)-1|}> \| \ldots \| <\gamma_i,\gamma_{j'+1}> \| <\gamma_i,\gamma_{j'}>.$$

Now, we can easily state our method to build a cycle of length $L = K|V(G)|+1$:

$$C_{K|V(G)|+1}: \; BackP(<\gamma_0,\gamma_{|V(G)|-1}>,<\gamma_0,\gamma_1>) \| (<\gamma_0,\gamma_1>,<\gamma_1,\gamma_0>) \| BackP(<\gamma_1,\gamma_0>,<\gamma_1,\gamma_2>) \| \ldots \| BackP(<\gamma_i,\gamma_{i-1}>,<\gamma_i,\gamma_{i+1}>) \| (<\gamma_i,\gamma_{i+1}>,<\gamma_{i+1},\gamma_i>) \| \ldots \| BackP(<\gamma_{K-2},\gamma_{K-3}>,<\gamma_{K-2},\gamma_{K-1}>) \| (<\gamma_{K-2},\gamma_{K-1}>,<\gamma_{K-1},\gamma_{K-2}>) \| BackP(<\gamma_{K-1},\gamma_{K-2}>,<\gamma_{K-1},\gamma_{|V(G)|-1}>) \| (<\gamma_{K-1},\gamma_{|V(G)|-1}>,<\gamma_{|V(G)|-1},\gamma_{K-1}>) \| HP(<\gamma_{|V(G)|-1},\gamma_{K-1}>,<\gamma_{|V(G)|-1},\gamma_0>) \| (<\gamma_{|V(G)|-1},\gamma_0>,<\gamma_0,\gamma_{|V(G)|-1}>).$$

The abovementioned cycle consists of concatenating K-1 paths of length $|V(G)|$-1, one path of length K, and one Hamiltonian path of length $|V(G)|$, contributing in a cycle of length $K|V(G)|+1$ in the OTIS-G. □

The above cases show that the Hamiltonian-connectedness of factor graph G implies the weak Pancyclicity of the OTIS-G for $\Sigma = \{7,8 \ldots |V(G)|^2\}$.

4 Conclusions and Future Work

The OTIS structure is an attractive inter-node communication network for large multiprocessor systems as it offers benefits from both optical and electrical technologies. A number of suitable properties of OTIS-G networks including basic topological properties, embedding, routing, broadcasting, and fault tolerance have been studied in the past. As well, a number of parallel algorithms like parallel sorting and Lagrange interpolation algorithm have been studied [5-14]. In this paper, we addressed an open question stated in [6]; it says that the OTIS-G possesses an L^2-cycle provided that the factor graph G contains an L-cycle. Moreover, embedding of different size cycles into networks is an important issue for development of some algorithms of control/data flow for distributed computation, which is also known as Pancyclicity. We have expressed in mathematical terms a general definition for Pancyclicity of a graph which can cover all the different cases previously stated in the literature. In this connection, we have proved the Weak Pancyclicity of OTIS-*G* when the factor graph *G* is Hamiltonian-connected.

The future work, in this line, includes identifying the necessary and sufficient conditions of factor graph *G* for the Pancyclicity of the OTIS-*G*.

References

1. Marsden, G., Marchand, P., Harvey, P., Esener, S.: Optical transpose interconnection system architectures. Optics Lett. 18, 1083–1085 (1993)
2. Yeh, C.-H., Parhami, B.: Swapped networks: unifying the architectures and algorithms of a wide class of hierarchical parallel processors. In: Proc. Int'l Conf. Parallel and Distributed Systems, pp. 230–237 (1996)
3. Zane, F., Marchand, P., Paturi, R., Esener, S.: Scalable network architectures using the optical transpose interconnection system. J. Parallel Distribute Computing 60, 521–538 (2000)
4. Krishnamoorthy, P., Marchand, F., Kiamilev, F., Esener, S.: Grain–size considerations for optoelectronic multistage interconnection networks. Applied Optics 31(26), 5480–5507 (1992)
5. Wang, C., Sahni, S.: Basic operations on the OTIS-mesh optoelectronic computer. IEEE Transactions on Parallel and Distributed Systems 9(12), 1226–1236 (1998)
6. Day, K., Al-Ayyoub, A.: Topological properties of OTIS-networks. IEEE Trans. Parallel Distrib. Systems 13, 359–366 (2002)
7. HoseinyFarahabady, M., Sarbazi-Azad, H.: The grid-pyramid: A generalized pyramid network. Journal of Supercomputing 37(1), 23–45 (2006)
8. Rajasekaran, S., Sahni, S.: Randomized routing, selection, and sorting on the OTIS-mesh. IEEE Trans. on Parallel and Distributed Systems 9(9), 833–840 (1998)
9. Parhami, B.: Swapped interconnection networks: Topological, performance, and robustness attributes. J. Parallel Distrib. Comput. 65(11), 1443–1452 (2005)
10. Hoseiny Farahabady, M., Sarbazi-Azad, H.: Algorithmic Construction of Hamiltonian Cycles in OTIS-Networks. In: IASTED 2005 Networks and Communication Systems, pp. 464–110 (2005)
11. Osterloh, A.: Sorting on the OTIS-mesh. In: IPDPS 2000, pp. 269–274 (2000)
12. Prasanta, K.: Polynomial interpolation and polynomial root finding on OTIS-mesh. Parallel Computing 32, 301–312 (2006)
13. Wang, C.-F., Sahni, S.: Matrix multiplication on the OTIS-mesh optoelectronic computer. In: MPPOI 1999, pp. 131–138 (1999)
14. Hoseiny Farahabady, M., Imani, N., Sarbazi-Azad, H.: Some Topological and Combinatorial Properties of WK-Recursive Mesh and WK-Pyramid Interconnection Networks. Technical Report, IPM School of Computer Science, Tehran, Iran (2007)
15. Hashemi Najaf-abadi, H., Sarbazi-Azad, H.: Comparative Evaluation of Adaptive and Deterministic Routing in the OTIS-Hypercube. In: Yew, P.-C., Xue, J. (eds.) ACSAC 2004. LNCS, vol. 3189, pp. 349–362. Springer, Heidelberg (2004)
16. Hashemi Najaf-abadi, H., Sarbazi-Azad, H.: Multicast communication in OTIS-hypercube multi-computer systems. International Journal of High Performance Computing and Networking 4(3-4), 161–173 (2006)
17. Hashemi Najaf-abadi, H., Sarbazi-azad, H.: On routing capabilities and performance of cube-based OTIS-systems. In: SPECTS 2004, pp. 161–166 (2004)

18. Xu, J., Ma, M.: Cycles in folded hypercubes. Applied Mathematics Letters 19, 140–145 (2006)
19. Araki, T.: Edge-pancyclicity of recursive circulants. Information Processing Letters 88(6), 287–292 (2003)
20. Hsieh, S., Chang, N.: Hamiltonian Path Embedding and Pancyclicity on the Mobius Cube with Faulty Nodes and Faulty Edges. IEEE Transactions on Computers 55(7), 854–863 (2006)
21. Flandrin, E., Li, H., Marczyk, L., Schiermeyer, I., Wo´zniak, M.: Chvtal–Erdos condition and pancyclism. Disc. Math. Graph Theory 26, 335–342 (2006)
22. Li, T.: Cycle embedding in star graphs with edge faults. Applied Mathematics and Computation 167, 891–900 (2005)
23. Fan, J., Lin, X., Jia, X.: Node-Pancyclicity and edge-Pancyclicity of crossed cubes. Information Processing Letters 93, 133–138 (2005)