Time-Dependent (Unsteady) Flow Analysis
What is Different?

Steady (time-independent) flows:
• flow itself constant over time
• \( \mathbf{v}(\mathbf{x}) \), e.g., laminar flows
• simpler case for visualization

Time-dependent (unsteady) flows:
• flow itself changes over time
• \( \mathbf{v}(\mathbf{x},t) \), e.g., turbulent flow
• more complex case
Mathematical Framework

• An unsteady **vector field**
  
  – is a continuous vector-valued function $\mathbf{v}(x, t)$ on a manifold $X$

  – can be expressed as a system of ODE $\frac{dx}{dt} = \mathbf{v}(x, t)$

  – is a map $\varphi : \mathbb{R} \times X \rightarrow X$

  – with initial condition $x(t_0) = x_0$, its solution is called an integral curve, trajectory, or orbit.
Stream, Path, and Streaklines

Terminology:

• **Streamline**: a curve that is everywhere tangent to the steady flow (release 1 massless particle)
  \[ s(t) = s_0 + \int_{0 \leq u \leq t} v(s(u)) \, du \]

• **Pathline**: a curve that is everywhere tangent to an unsteady flow field (release 1 massless particle)
  \[ s(t) = s_0 + \int_{0 \leq u \leq t} v(s(u), u) \, du \]

• **Streakline**: a curve traced by the continues release of particles in unsteady flow from the same position in space (release infinitely many massless particles)

• They are identical under the steady flow
Difference Between Streamlines and Pathlines

A moving center over time: (left) streamlines; (right) path lines
Source: Weinkauf et al. Vis2010
Streaklines

- Not tangent curves to the vector fields
- Union of the current positions of particles released at the same point in space

Timelines

- Union of the current positions of particles released at the same time in space
Time-Dependent Vector Field Analysis
Different Points of View

- Streamline-based
  - Extract the so-called “instantaneous” topology at each time step and keep track of the “evolution” of this topology over time
  - Feature tracking

- Pathline-based
  - Classify the behaviors of different pathlines

- Streakline-based
  - Extension of the pathline-based approach
Track Topology Evolution

• Parameter dependent topology:
  • “Fixed points” (no more fixed) move, appear, vanish, transform
  • Topological graph connectivity changes

• Structural stability: topology is stable w.r.t. small but arbitrary changes of parameter(s) if and only if
  • 1) Number of fixed points and closed orbits is finite and all are hyperbolic
  • 2) No saddle-saddle connection (unstable)
Bifurcations

• Transition from one stable structure to another through *unstable state*
• Bifurcation value: parameter value inducing the transition
• Local vs. global bifurcations
Local Bifurcations

- Transformation affects local region
- **Fold bifurcation**: saddle + sink/source

1D equivalent:

- source
- sink
- unstable
- no fixed point
Local Bifurcations

- Transformation affects local region
- **Hopf bifurcation**: sink/source + closed orbit
Global Bifurcations

- Affects overall topological connectivity
- Basin bifurcation

Saddle-saddle connection
Global Bifurcations

• Modifies overall topological connectivity
• Homoclinic bifurcation

Saddle-saddle connection

repelling cycle (source)
Global Bifurcations

- Modifies overall topological connectivity
- Periodic blue sky

![Diagram showing bifurcations with labels: sink, center, Homoclinic connection, source]
2+1D Topology

- Time-wise interpolation
- Cell-wise tracking over 2+1D grid
- Detect local bifurcations

- Track associated separatrices (surfaces)
2+1D Topology
2+1D Topology
Pathline-based Topology

• Issue of the instantaneous topology tracking
  – It is not easy to compute and has to be computed for each time step
  – To see the result, animation must be used
  – Segmentation does not reveal the true segmentation for coherent flow over time

• Any solutions?
  – Pathline-oriented
  – Finite-Time Lyapunov Exponent
Finite-Time Lyapunov Exponent

• FTLE Overview:
  – Provide a single scalar field that accounts for the integration over time
  – Provide an average that indicates the actual transport
  – Ridges separate regions of coherent flow (ideally no flow across ridges), called Lagrangian Coherent Structures (LCS)
Finite-Time Lyapunov Exponent

• Obtaining the scalar field
  – Observe particle trajectories
  – Measure the divergence between trajectories, i.e. how much flow stretch

[Shadden]
Finite-Time Lyapunov Exponent

• Description
  – Lyapunov exponents describe rate of separation or stretching of two infinitesimally close points over time in a dynamical system
  – FTLE refers to the largest Lyapunov exponent for only a **limited time** and is measured **locally**
  – *Largest exponent is governing the behavior of the system, smaller ones can be neglected*
Finite-Time Lyapunov Exponent

• Example: unsteady double-gyre

http://mmae.iit.edu/shadden/LCS-tutorial/examples.html

[Shadden]
Finite-Time Lyapunov Exponent

• Other examples

  Ocean transport  http://mmae.iit.edu/shadden/mbay

  Biotransport  http://mmae.iit.edu/shadden/jellyfish

[Shadden]
Finite-Time Lyapunov Exponent

• Further Applications
  – Detection of crowd flow in videos

Finite-Time Lyapunov Exponent

- FTLE ridges are approximately material structures
- Non-zero cross-flux across FTLE ridges
- Accuracy increases with integration time
- Problem: data sets often bounded with time
Additional Readings


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Final Project Topic Overview
1. Curvature Tensor Estimation and Shape Illustrative Rendering

See the additional reading list of Lecture 4
2. Medial Axis Estimation For 2D and 3D Shapes

- Distance transform
- Medial axis transform
- Scale axis transform
3. 1D Skeleton Extraction from 2D and 3D Shapes

- Straight line skeleton
- Curve skeleton extraction using mesh contraction
4. Ridges and Valleys Detection

- Ridges and valleys on surfaces

- Ridges and valleys in images
  - One example is the apparent ridges
5. Compute Reeb Graphs of 3D Shapes
6. Compute Morse-Smale Complex

Applications?
7. Compute Differential Topology of Higher Dimensional Vector Fields

Vector field topology and its applications?
8. Compute Morse Decomposition

Applications?
9. Compute Combinatorial Topology
10. Edge-Map
11. Time Dependent Vector Field Analysis

- Feature tracking
- FTLE computation
- Streak line/surface based topology
12. Vortex Detection
13. Symmetric Tensor Field Analysis
14. Asymmetric Tensor Field Analysis
15. Large Dynamic Graph Visual Analytic

- Cluster identification
- Connectivity evaluation
- Event identification

Open source:  [https://gephi.org/](https://gephi.org/)
http://www.caida.org/tools/visualization/walrus/
16. Text Visual Analytics

- Classifying documents
- Content extraction
- Relation identification
17. Higher-Dimensional Data Analysis

What is a person’s life span given gender, annual income, living area, marriage status, medical care, smoking or not, ...

What will determine/affect the temperature? Location (x, y, z), time in a day, humidity, pressure, cloudy or not, windy or not, human activities, ...
What is your idea?