Assignment 1

COSC 6320

[09/24/2021] This is the final version of the assignment. I cannot rule out the possibility of making some minor mistakes. So, check back often.

Unless otherwise stated, you must do the assignment on your own. Copying from any source (person, website, books, etc.) is a violation of the honor code. Please read the academic honesty policy of the university.

1. (Recurrence Relation, 10 points) Solve the recurrence relation by using the characteristic root method discussed in class. This is a non-homogenous case, so you have to use the annihilator method to solve the recurrence.

   $a_0 = 2,$

   $a_1 = 5,$

   $a_n = 3a_{n-1} - 2a_{n-2} + 2^{n-1} + 3,$ for n ≥ 2.

2. (Generating Function, 10 points) Solve the following recurrence relation using generating function method. Consult the table in the slides for conversion between generating functions and the recurrence sequences (power series).

   $a_0 = 1$

   $a_1 = 9$

   $a_n - 6a_{n-1} + 9a_{n-2} = 0,$ for n ≥ 2.

3. (Induction Proof, 10 points) Use the Principle of Induction to prove the following.

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6},$$

   for n is a natural number ≥ 1.

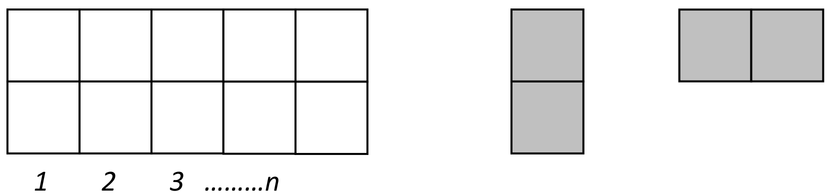
4. (Maximum Sum Problem, 20 points) We have presented four different solutions for the Maximum Sum problem in the lecture. We would like to extend these approaches to the two-dimension version of the problem. Given a 2-D matrix of real numbers A[1..m, 1..n], find a sub-matrix S[i..j, s..t] which produces the maximum sum of all elements in S. The generalization of Solution 1 is trivial, so we will skip it.

   (a) Solution 2X. Discuss how you can generalize the idea to the 2D version of the problem. What will the complexity of the algorithm? Please include a program for the solution in a separate file. The program may be in Python or C++.

   (b) Solution 3X. Repeat the process for Solution 3 (Divide-and-Conquer).

5. (Problem Solving, 10 points) (a) Find a recurrence relation for the number of ways (A_n) to completely cover a $2 \times n$ chessboard with 1×2 dominos. You must explain (justify, prove) how you derived the recurrence relation.

   (b) Solve the recurrence relation (the solution can be found in the textbook, just quote the well-known recurrence) and then give your best estimation of the growth rate of A_n.



6. (Prove by contradiction, 10 points) If a, b, and c are integers and $a^2 + b^2 = c^2$ then at least one of a or b is even.