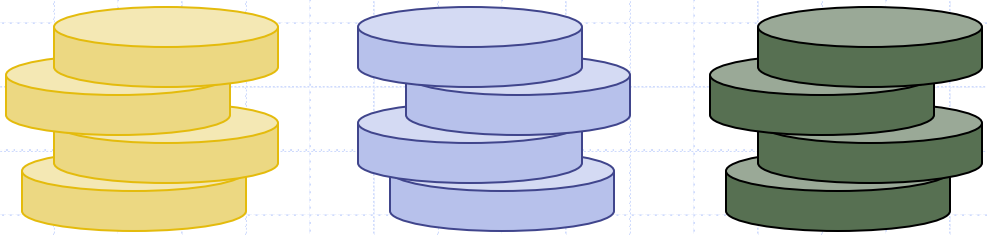
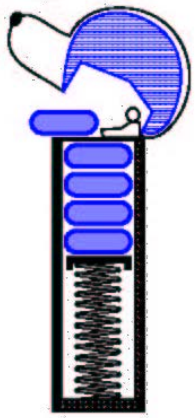
Stacks



Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations
- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT



- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean empty(): indicates whether no elements are stored

Stack Interface in C++

- C++ interface corresponding to our Stack ADT
- Uses an exception class StackEmpty
- Different from the built-in C++ STL class stack

```
template <typename E>
class Stack  {
public:
   int size() const;
   bool empty() const;
   const E& top() const
         throw(StackEmpty);
   void push(const E& e);
   void pop() throw(StackEmpty);
}
```

Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception
- Exceptions are said to be “thrown” by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty
- Attempting pop or top on an empty stack throws a StackEmpty exception

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the C++ run-time system
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

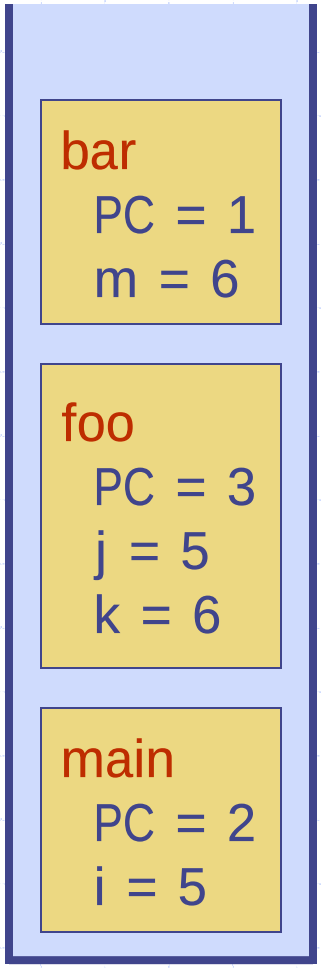
C++ Run-Time Stack

- The C++ run-time system keeps track of the chain of active functions with a stack
- When a function is called, the system pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When the function ends, its frame is popped from the stack and control is passed to the function on top of the stack
- Allows for recursion

```
main() {
  int i = 5;
  foo(i);
  }

foo(int j) {
  int k;
  k = j+1;
  bar(k);
  }

bar(int m) {
  ...
  }
```

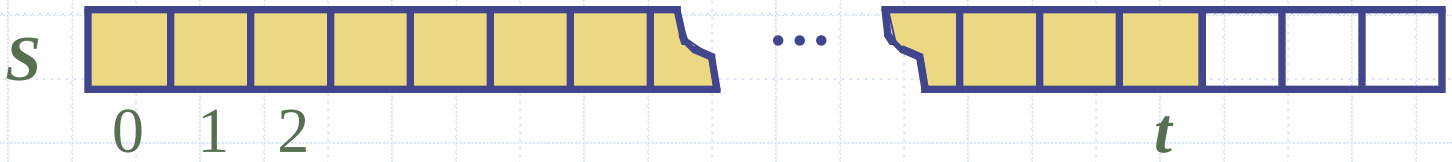


Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Algorithm ***size***()
  return $t + 1$

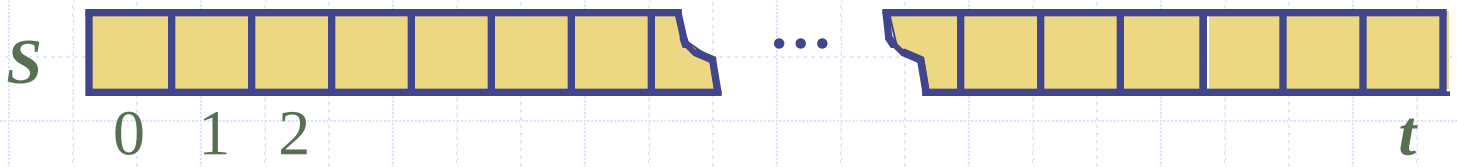
Algorithm ***pop***()
  if ***empty***() **then**
    throw ***StackEmpty***
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$



Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a StackFull exception
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Algorithm *push*(*o*)
  if $t = S.size() - 1$ **then**
    throw *StackFull*
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$



Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

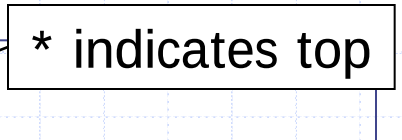
Array-based Stack in C++

```
template <typename E>
class ArrayStack {
private:
   E* S; // array holding the stack
   int cap; // capacity
   int t; // index of top element
public:
   // constructor given capacity
   ArrayStack( int c) :
       S(new E[c]), cap(c), t(-1) { }
```

```
   void pop() {
     if (empty()) throw StackEmpty
            ("Pop from empty stack");
      t--;
    }
   void push(const E& e) {
      if (size() == cap) throw
          StackFull("Push to full stack");
      S[++ t] = e;
}
…  (other methods of Stack interface)
```

Example use in C++

```
ArrayStack<int> A;                          // A = [ ], size = 0
A.push(7);                                  // A = [7*], size = 1
A.push(13);                                 // A = [7, 13*], size = 2
cout << A.top() << endl; A.pop();           // A = [7*], outputs: 13
A.push(9);                                  // A = [7, 9*], size = 2
cout << A.top() << endl;                    // A = [7, 9*], outputs: 9
cout << A.top() << endl; A.pop();           // A = [7*], outputs: 9
ArrayStack<string> B(10);                   // B = [ ], size = 0
B.push("Bob");                              // B = [Bob*], size = 1
B.push("Alice");                            // B = [Bob, Alice*], size = 2
cout << B.top() << endl; B.pop();           // B = [Bob*], outputs: Alice
B.push("Eve");                              // B = [Bob, Eve*], size = 2
```



Parentheses Matching

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parentheses Matching Algorithm

Algorithm ParenMatch(*X*,*n*):
Input: An array *X* of *n* tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number
Output: **true** if and only if all the grouping symbols in *X* match
Let *S* be an empty stack
for *i*=0 to *n*-1 **do**
    if *X*[*i*] is an opening grouping symbol **then**
        S.push(*X*[*i*])
    else if *X*[*i*] is a closing grouping symbol **then**
        if *S*.empty() **then**
            return false {nothing to match with}
        if *S*.pop() does not match the type of *X*[*i*] **then**
            return false {wrong type}
if *S*.empty() **then**
    return true {every symbol matched}
else return false {some symbols were never matched}

Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

14 – 3 * 2 + 7 = (14 – (3 * 2)) + 7

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right

Example: (x – y) + z rather than x – (y + z)

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

Algorithm for Evaluating Expressions

Slide by Matt Stallmann included with permission.

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push( y op x )
```

Algorithm repeatOps(refOp):

```
while ( valStk.size() > 1 ∧
        prec(refOp) ≤
        prec(opStk.top()))
    doOp()
```

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

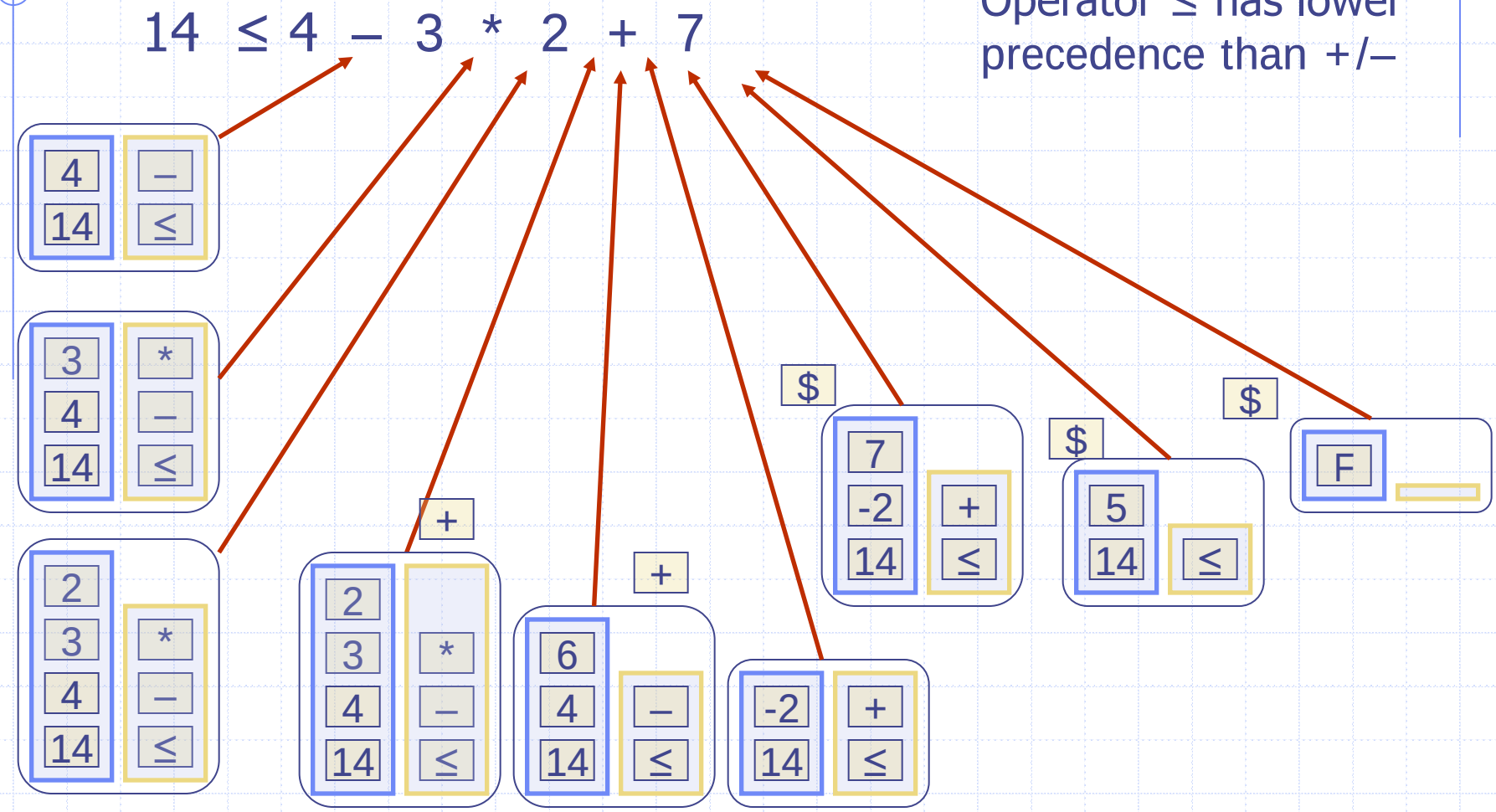
Output: the value of the expression

```
while there's another token z
    if isNumber(z) then
        valStk.push(z)
    else
        repeatOps(z);
        opStk.push(z)
repeatOps($);
return valStk.top()
```

Algorithm on an Example Expression

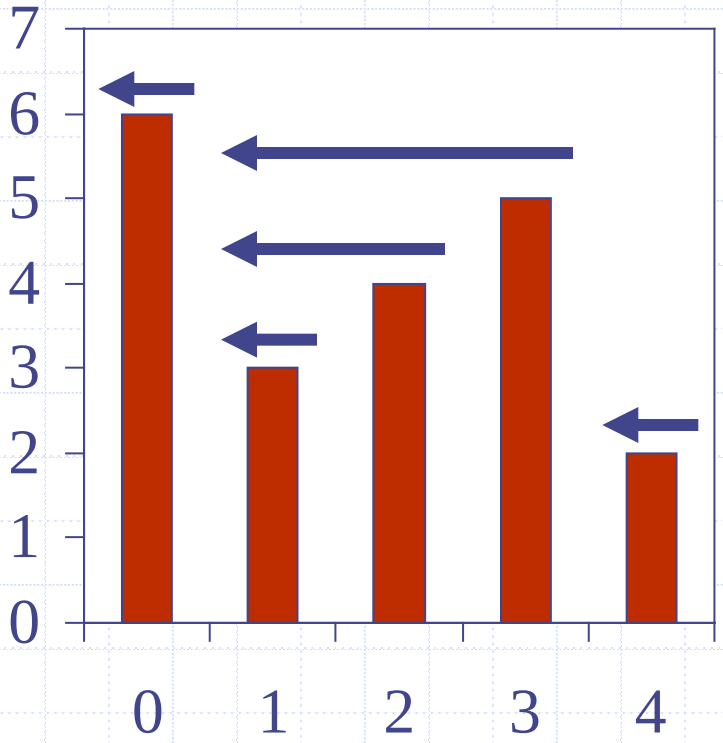
Slide by Matt Stallmann included with permission.

Operator ≤ has lower precedence than +/–



Computing Spans (not in book)

- Using a stack as an auxiliary data structure in an algorithm
- Given an an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

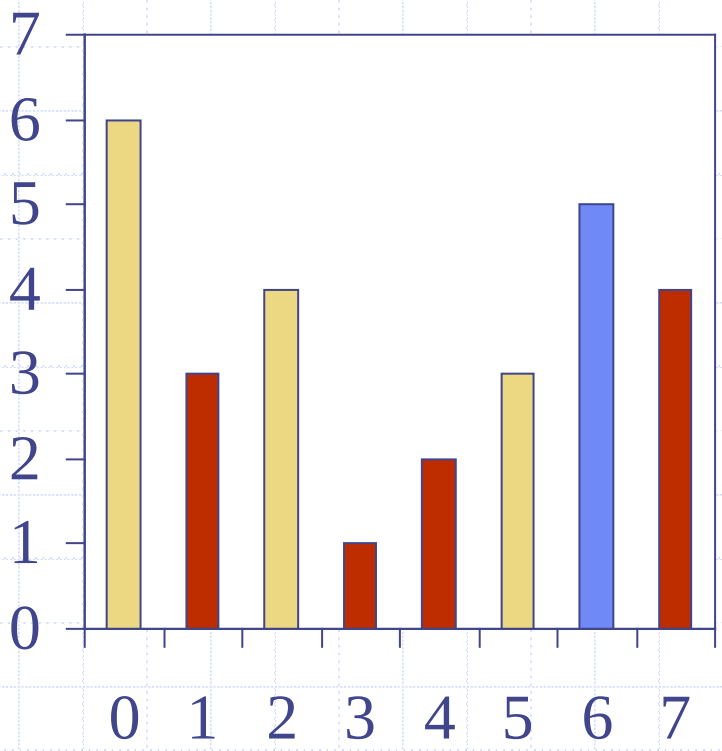
Quadratic Algorithm

Algorithm *spans1*(X, n)	
Input array X of n integers	
Output array S of spans of X	#
$S \leftarrow$ new array of n integers	n
for $i \leftarrow 0$ **to** $n - 1$ **do**	n
$s \leftarrow 1$	n
while $s \leq i \wedge X[i - s] \leq X[i]$	$1 + 2 + \ldots + (n - 1)$
$s \leftarrow s + 1$	$1 + 2 + \ldots + (n - 1)$
$S[i] \leftarrow s$	n
return S	1

- Algorithm *spans1* runs in $O(n^2)$ time

Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when “looking back”
- We scan the array from left to right
  - Let i be the current index
  - We pop indices from the stack until we find index j such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push x onto the stack



Linear Algorithm

- Each index of the array
  - Is pushed into the stack exactly one
  - Is popped from the stack at most once
- The statements in the while-loop are executed at most n times
- Algorithm *spans2* runs in $O(n)$ time

```
Algorithm spans2(X, n)                                  #
    S ← new array of n integers                         n
    A ← new empty stack                                 1
    for i ← 0 to n – 1 do                               n
        while (¬A.empty() ∧
                X[A.top()] ≤ X[i] ) do                  n
            A.pop()                                     n
        if A.empty() then                               n
            S[i] ← i + 1                                n
        else
            S[i] ← i – A.top()                          n
        A.push(i)                                       n
    return S                                            1
```