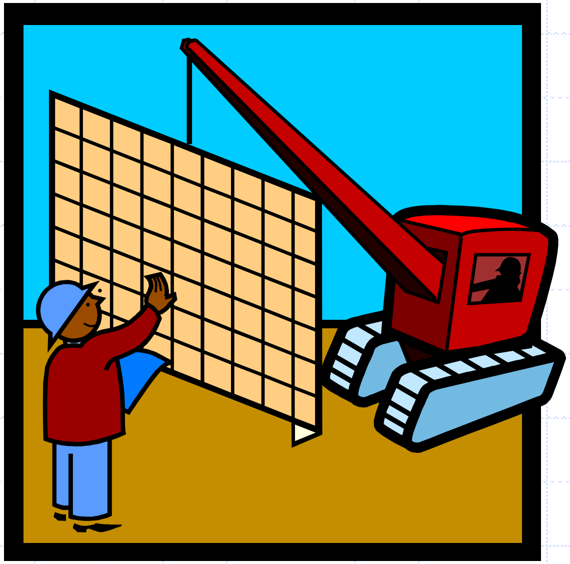
Array Lists



The Array List ADT

- The Vector or Array List ADT extends the notion of array by storing a sequence of objects
- An element can be accessed, inserted or removed by specifying its index (number of elements preceding it)
- An exception is thrown if an incorrect index is given (e.g., a negative index)

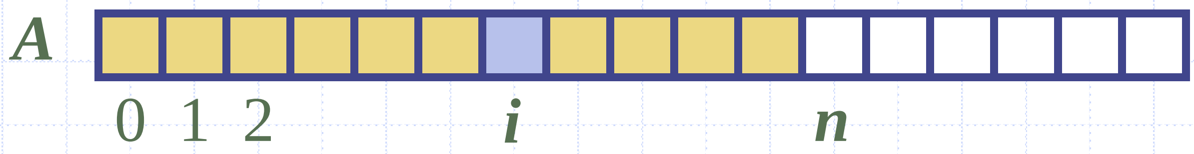
- Main methods:
  - at(integer i): returns the element at index i without removing it
  - set(integer i, object o): replace the element at index i with o
  - insert(integer i, object o): insert a new element o to have index i
  - erase(integer i): removes element at index i
- Additional methods:
  - size()
  - empty()

Applications of Array Lists

- Direct applications
  - Sorted collection of objects (elementary database)
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

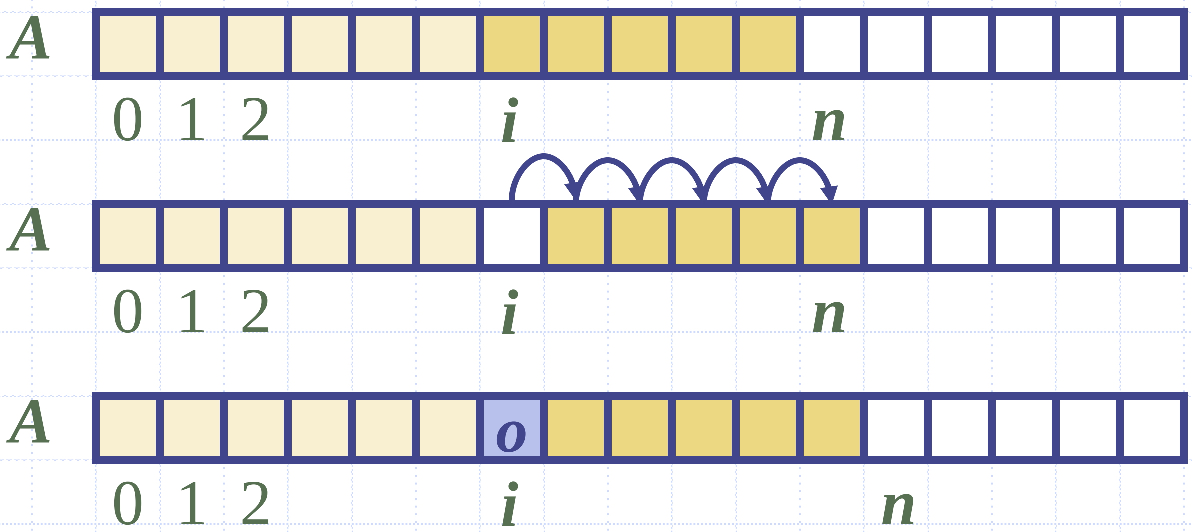
Array-based Implementation

- Use an array A of size N
- A variable n keeps track of the size of the array list (number of elements stored)
- Operation ***at***(i) is implemented in $O(1)$ time by returning $A[i]$
- Operation ***set***(i,o) is implemented in $O(1)$ time by performing $A[i] = o$



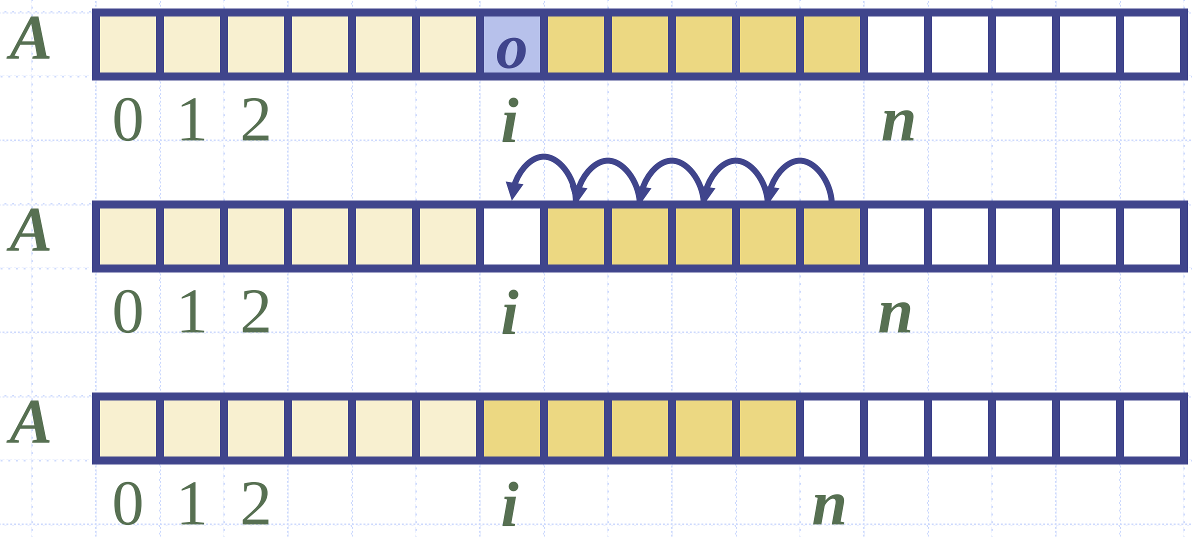
Insertion

- In operation ***insert***(i, o), we need to make room for the new element by shifting forward the $n - i$ elements $A[i], \ldots, A[n-1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Element Removal

- In operation ***erase***(i), we need to fill the hole left by the removed element by shifting backward the $n - i - 1$ elements $A[i + 1], \ldots, A[n - 1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Performance

- In the array based implementation of an array list:
  - The space used by the data structure is $O(n)$
  - ***size***, ***empty***, ***at*** and ***set*** run in $O(1)$ time
  - ***insert*** and ***erase*** run in $O(n)$ time in worst case
- If we use the array in a circular fashion, operations ***insert***$(0, x)$ and ***erase***$(0, x)$ run in $O(1)$ time
- In an ***insert*** operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one

Growable Array-based Array List

- In an insert(o) operation (without an index), we always insert at the end
- When the array is full, we replace the array with a larger one
- How large should the new array be?
  - Incremental strategy: increase the size by a constant c
  - Doubling strategy: double the size

Algorithm *insert*(o)
  if $t = S.length - 1$ **then**
    $A \leftarrow$ **new array of size …**
    for $i \leftarrow 0$ **to** $n-1$ **do**
      $A[i] \leftarrow S[i]$
    $S \leftarrow A$
  $n \leftarrow n + 1$
  $S[n-1] \leftarrow o$

Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time $T(n)$ needed to perform a series of n insert(o) operations
- We assume that we start with an empty stack represented by an array of size 1
- We call amortized time of an insert operation the average time taken by an insert over the series of operations, i.e., $T(n)/n$

Incremental Strategy Analysis

- We replace the array $k = n/c$ times
- The total time $T(n)$ of a series of n insert operations is proportional to

$$n + c + 2c + 3c + 4c + \ldots + kc =$$

$$n + c(1 + 2 + 3 + \ldots + k) =$$

$$n + ck(k + 1)/2$$

- Since c is a constant, $T(n)$ is $O(n + k^2)$, i.e., $O(n^2)$
- The amortized time of an insert operation is $O(n)$

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time $T(n)$ of a series of n insert operations is proportional to

  $$n + 1 + 2 + 4 + 8 + \ldots + 2^k =$$
  $$n + 2^{k+1} - 1 =$$
  $$3n - 1$$

- $T(n)$ is $O(n)$
- The amortized time of an insert operation is $O(1)$

geometric series