R_ε^A.

Let $f \in L^2(\mathbb{R}^d)$ be a pdf

$X_1, \dots X_n$ be samples of f.

Let $k_\sigma(x, x')$ be smoothing kernel

$$\bar{f}_\sigma^n = \frac{1}{n}\sum_1^n k_\sigma(\cdot, X_i) \Leftarrow \text{kernel density est}$$

In practice ε is not known. so $\beta = \frac{1}{1-\varepsilon}$

$$Q_\beta(f) \triangleq \max(\beta f(\cdot) - \alpha, 0).$$

$$\alpha = \alpha(\beta).$$

Let D_σ^n be convex hull of $\{k_\sigma(\cdot, X_1), \dots k_\sigma(\cdot, X_n)\}$

Δ^n probability simplex

$$f_{\sigma,\beta}^n := \arg\min \|\beta \bar{f}_\sigma^n - g\|_{L^2}$$ (hull)

$\Downarrow$ represent

① $f_{\sigma,\beta}^n = \sum_1^n a_i k_\sigma(\cdot, X_i)$

② vector $a = [a_1, \dots a_n]^T$

③ Gram matrix $G_{ij} = \langle k_\sigma(\cdot, X_i), k_\sigma(\cdot, X_j)\rangle_L$

④ define $b = G\mathbf{1}\frac{\beta}{n}$.

⑤ Solve $\min_{a \in \Delta^n} a^T G a - 2b^T a$.