Search Methods for Classical and Temporal Planning

Jussi Rintanen

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- Which actions to take to achieve your objectives?
- Number of agents
  - single agent, perfect information: s-t-reachability in succinct graphs
  - + nondeterminism/adversary: and-or tree search
  - + partial observability: and-or search in the space of beliefs
- Time
  - asynchronous or instantaneous actions (integer time, unit duration)
  - rational/real time, concurrency
- Objective
  - Reach a goal state.
  - Maximize probability of reaching a goal state.
  - Maximize (expected) rewards.
  - temporal goals (e.g. LTL)

Planning

What to do to achieve your objectives?

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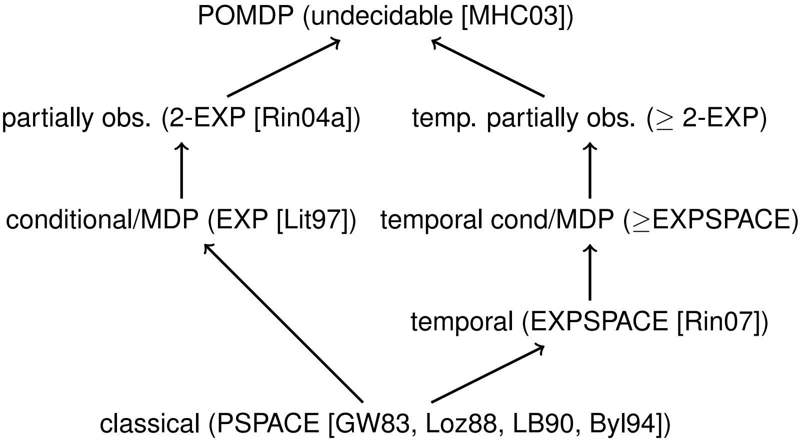
Hierarchy of Planning Problems



Classical (Deterministic, Sequential) Planning

- states and actions expressed in terms of state variables
- single initial state, that is known
- all actions deterministic
- actions taken sequentially, one at a time
- a goal state (expressed as a formula) reached in the end

Deciding whether a plan exists is PSPACE-complete [GW83, Loz88, LB90, Byl94].
With a polynomial bound on plan length, NP-complete [KS96].

Domain-Independent Planning

What is domain-independent?

- general language for representing problems (e.g. PDDL)
- general algorithms to solve problems expressed in it

Advantages and disadvantages:

+ Representation of problems at a high level
+ Fast prototyping
+ Often easy to modify and extend
- Often very high performance penalty w.r.t. specialized algorithms
- Trade-off between generality and efficiency

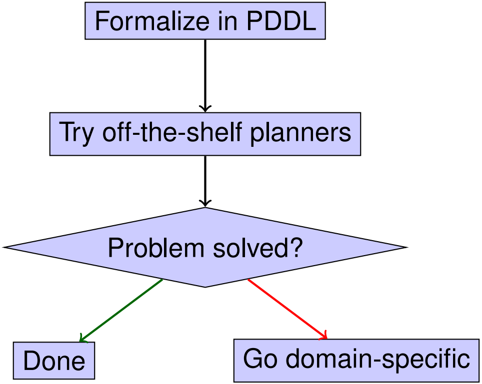
Domain-Specific Planning

What is domain-specific?

- application-specific representation
- application-specific constraints/propagators
- application-specific heuristics

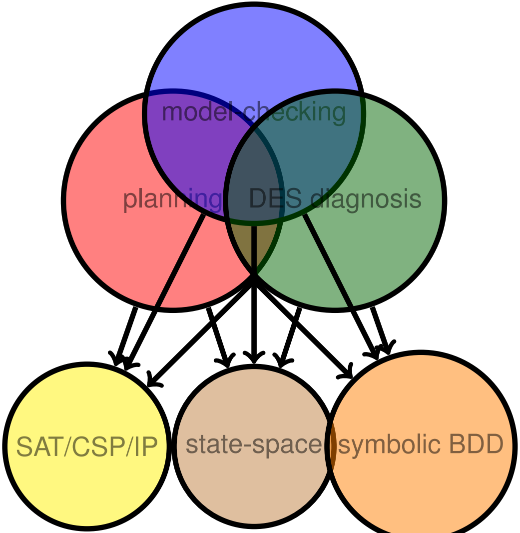
There are some planning systems that have aspects of these, but mostly this means: implement everything from scratch.

Domain-Dependent vs. -Independent Planning
Procedure

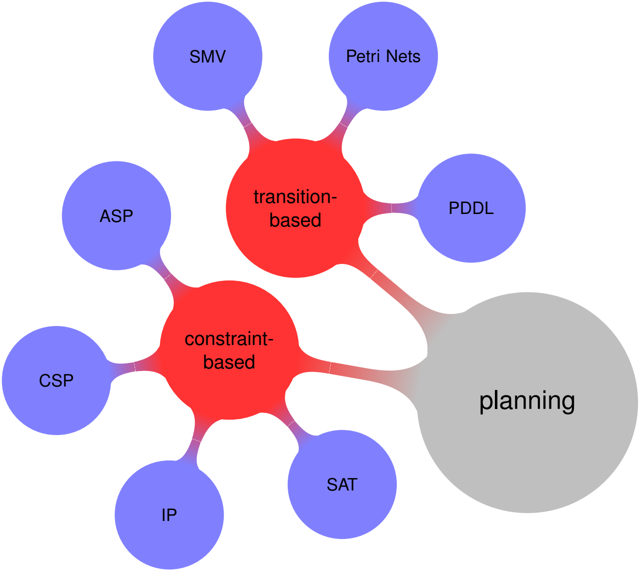


Related Problems, Reductions

planning, diagnosis [SSL+95], model-checking (verification)



How to Represent Planning Problems?



PDDL: Planning Domain Description Language

- Defined in 1998 [GHK+98], with several extensions later.
- Lisp-style syntax
- Widely used in the planning (competition) community.
- Most basic version with Boolean state variables only.
- Action sets expressed as schemata instantiated with objects.

```
(:action unload
    :parameters (?obj - obj ?airplane - vehicle ?loc - location)
    :precondition (and (in ?obj ?airplane) (at ?airplane ?loc))
    :effect (and (not (in ?obj ?airplane))))
```

States

States are valuations of state variables.

Example

State variables are

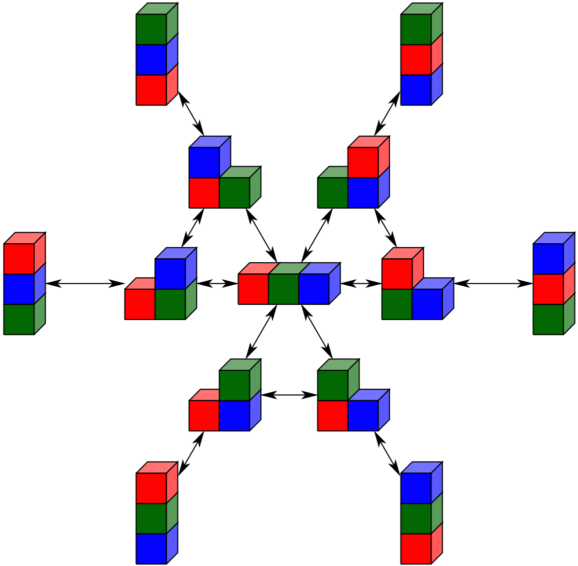
LOCATION:	$\{0,\ldots,1000\}$
GEAR:	$\{R,1,2,3,4,5\}$
FUEL:	$\{0,\ldots,60\}$
SPEED:	$\{-20,\ldots,200\}$
DIRECTION:	$\{0,\ldots,359\}$

One state is

LOCATION =	312
GEAR =	4
FUEL =	58
SPEED =	110
DIRECTION =	90

State-space transition graphs

Blocks world with three blocks



Actions

How values of state variables change

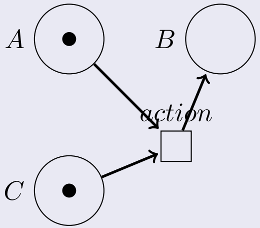
General form

precondition: A=1 $\wedge$ C=1
effect: A := 0; B := 1; C := 0;

STRIPS representation

PRE: A, C
ADD: B
DEL: A, C

Petri net



Weaknesses in Existing Languages

- High-level concepts not easily/efficiently expressible. Examples: graph connectivity, transitive closure, inductive definitions.
- Limited or no facilities to express domain-specific information (control, pruning, heuristics).
- The notion of classical planning is limited:
  - Real world rarely a single run of the sense-plan-act cycle.
  - Main issue often uncertainty, costs, or both.
  - Often rational time and concurrency are critical.

Formalization of Planning in This Tutorial

A problem instance in (classical) planning consists of the following.

- set X of state variables
- set A of actions $\langle p, e \rangle$ where
  - p is the precondition (a set of literals over X)
  - e is the effects (a set of literals over X)
- initial state $I : X \to \{0, 1\}$ (a valuation of X)
- goals G (a set of literals over X)

(We will later extend this with time and continuous change.)

The planning problem

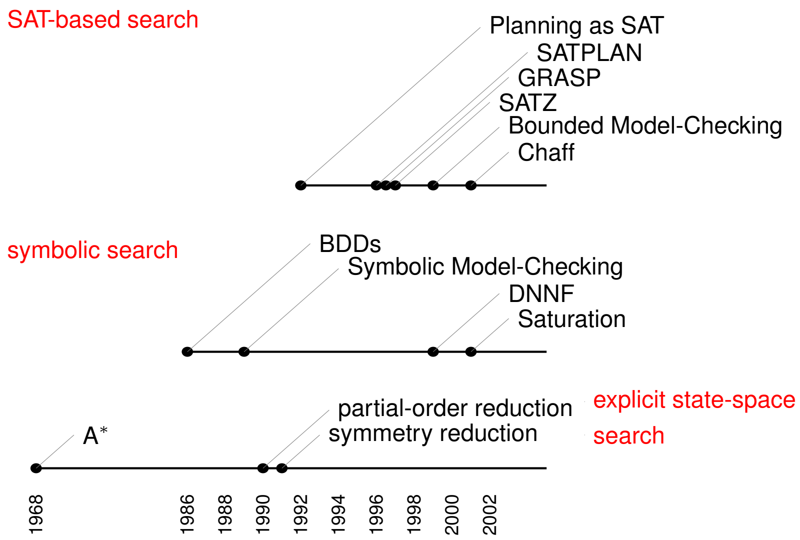
An action $a = \langle p, e \rangle$ is executable in state s iff $s \models p$.
The successor state $s' = \textit{exec}_a(s)$ is defined by

- $s' \models e$
- $s(x) = s'(x)$ for all $x \in X$ that don't occur in e.

Problem

Find $a_1, \ldots, a_n$ such that
$\textit{exec}_{a_n}(\textit{exec}_{a_{n-1}}(\cdots \textit{exec}_{a_2}(\textit{exec}_{a_1}(I)) \cdots)) \models G$?

Development of state-space search methods



Explicit State-Space Search

- The most basic search method for transition systems
- Very efficient for small state spaces (1 million states)
- Easy to implement
- Very well understood
- Also known as "forward search" (in contrast to "backward search" with regression [Rin08])
- Pruning methods:
  - symmetry reduction [Sta91, ES96]
  - partial-order reduction [God91, Val91]
  - lower-bounds / heuristics, for informed search [HNR68]

State Representation

Every state represented explicitly $\Rightarrow$ compact state representation important

- Boolean (0, 1) state variables represented by one bit
- Inter-variable dependencies enable further compaction:
  - ¬(at(A,L1)∧at(A,L2)) always true
  - automatic recognition of invariants [BF97, Rin98, Rin08]
  - n exclusive variables $x_1, \ldots, x_n$ represented by $1 + \lfloor \log_2(n-1) \rfloor$ bits

(See [GV03] for references to representative works on compact representations of state sets.)

Search Algorithms

- uninformed/blind search: depth-first, breadth-first, ...
- informed search: “best first” search (always expand best state so far)
- informed search: local search algorithms such as simulated annealing, tabu search and others [KGJV83, DS90, Glo89] (little used in planning)
- optimal algorithms: A^* [HNR68], IDA^* [Kor85]

Symmetry Reduction [Sta91, ES96]

Idea

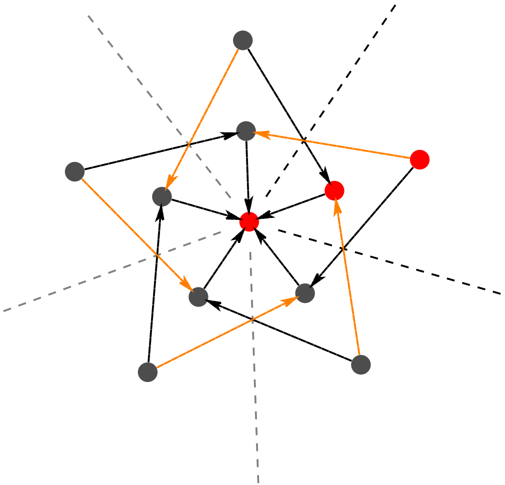
1. Define an equivalence relation $\sim$ on the set of all states: $s_1 \sim s_2$ means that state s_1 is symmetric with s_2.
2. Only one state s_C in each equivalence class $[s_C]$ needs to be considered.
3. If state $s \in [s_C]$ with $s \neq s_C$ is encountered, replace it with s_C.

Example

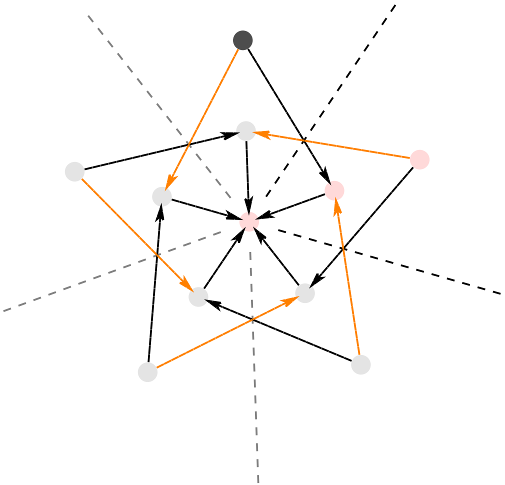
States $P(A) \wedge \neg P(B) \wedge P(C)$ and $\neg P(A) \wedge P(B) \wedge P(C)$ are symmetric because of the permutation $A \mapsto B, B \mapsto A, C \mapsto C$.

Symmetry Reduction

Example: 11 states, 3 equivalence classes

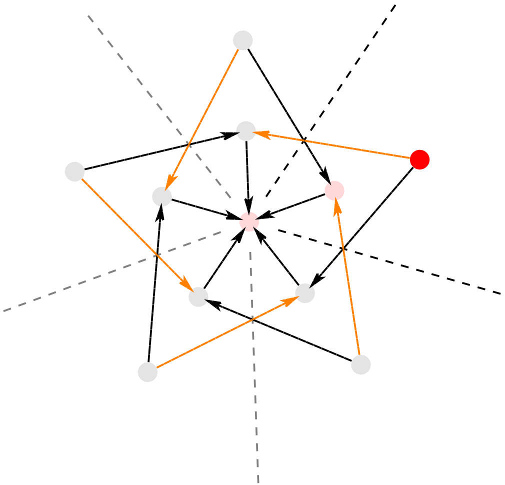


Symmetry Reduction
Example: 11 states, 3 equivalence classes



Symmetry Reduction

Example: 11 states, 3 equivalence classes



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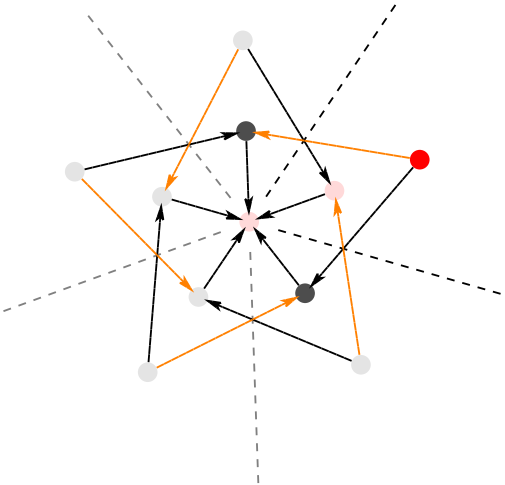

Symmetry Reduction

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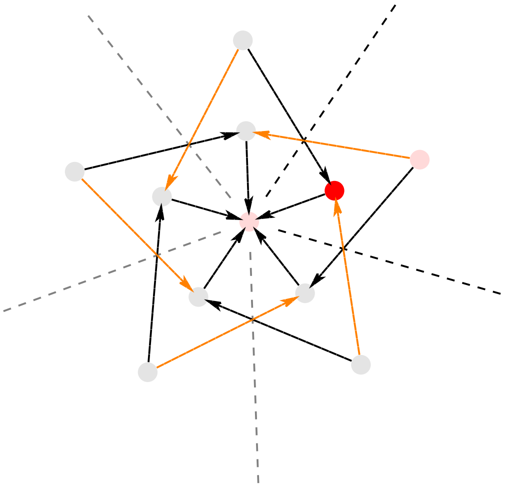

Symmetry Reduction

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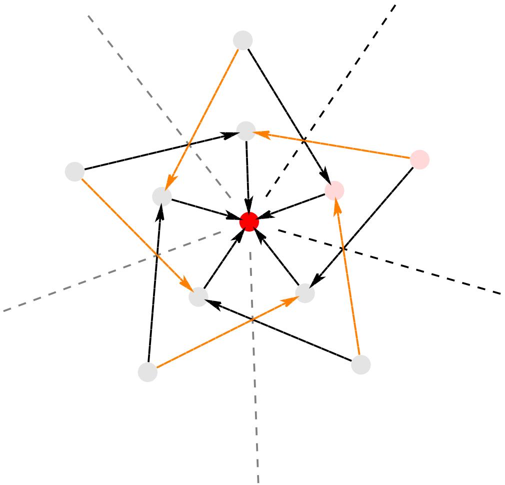


Idea [God91, Val91]

Independent actions unnecessary to consider in all orderings, e.g. A_1, A_2 and A_2, A_1.

Example

Let there be lamps $1, 2, \ldots, n$ which can be turned on. There are no other actions. One can restrict to plans in which lamps are turned on in the ascending order: switching lamp n after lamp $m > n$ unnecessary.[a]

[a] The same example is trivialized also by symmetry reduction!

Heuristics for Classical Planning

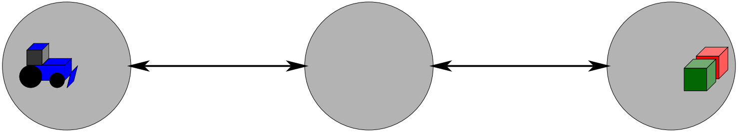
The most basic heuristics used for non-optimal *domain-independent* planning:

h^{max}	[BG01, McD96]	best-known admissible heuristic
h^{+}	[BG01]	still state-of-the-art
h^{relax}	[HN01]	often more accurate but performs like h^{+}

Definition of h^{max}, h^+ and h^{relax}

- Basic insight: estimate distances between possible state variable values, not states themselves.
- $g_s(l) = \begin{cases} 0 & \text{if } s \models l \\ \min_{a \text{ with effect } p} (1 + g_s(\text{prec}(a))) & \end{cases}$
- h^+ defines $g_s(L) = \sum_{l \in L} g_s(l)$ for sets S.
- h^{max} defines $g_s(L) = \max_{l \in L} g_s(l)$ for sets S.
- h^{relax} counts the number of actions in computation of h^{max}.

Computation of h^{max}
Tractor example



1. Tractor moves:
   - from 1 to 2: $T12 = \langle T1, \{T2, \neg T1\}\rangle$
   - from 2 to 1: $T21 = \langle T2, \{T1, \neg T2\}\rangle$
   - from 2 to 3: $T23 = \langle T2, \{T3, \neg T2\}\rangle$
   - from 3 to 2: $T32 = \langle T3, \{T2, \neg T3\}\rangle$
2. Tractor pushes A:
   - from 2 to 1: $A21 = \langle T2 \wedge A2, \{T1, A1, \neg T2, \neg A2\}\rangle$
   - from 3 to 2: $A32 = \langle T3 \wedge A3, \{T2, A2, \neg T3, \neg A3\}\rangle$
3. Tractor pushes B:
   - from 2 to 1: $B21 = \langle T2 \wedge B2, \{T1, B1, \neg T2, \neg B2\}\rangle$
   - from 3 to 2: $B32 = \langle T3 \wedge B3, \{T2, B2, \neg T3, \neg B3\}\rangle$

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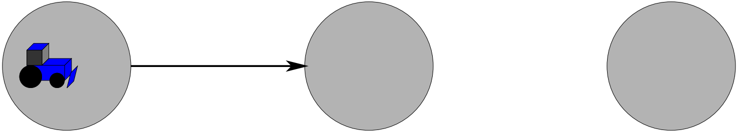
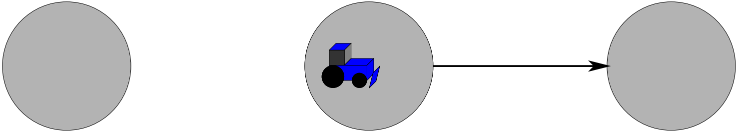


t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $T12 = \langle T1, \{T2, \neg T1\}\rangle$

Computation of h^{max}

Tractor example



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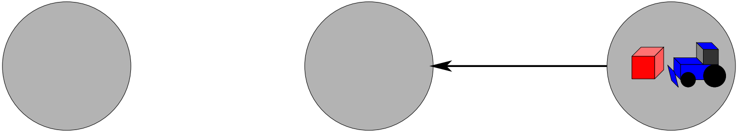
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $T23 = \langle T2, \{T3, \neg T2\}\rangle$

Computation of h^{max}
Tractor example



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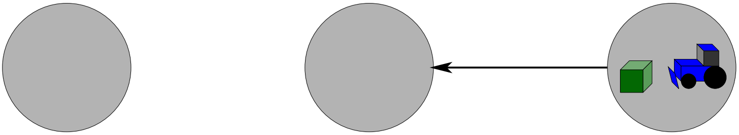
Apply $A32 = \langle T3 \wedge A3, \{T2, A2, \neg T3, \neg A3\}\rangle$

Computation of h^{max}
Tractor example



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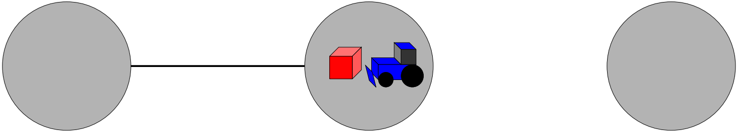
Computation of h^{max}
Tractor example



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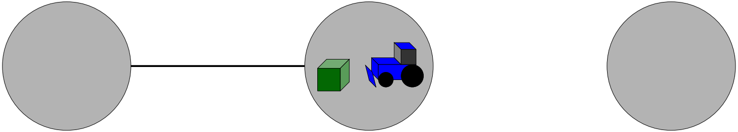
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $A21 = \langle T2 \wedge A2, \{T1, A1, \neg T2, \neg A2\}\rangle$

Computation of h^{max}

Tractor example



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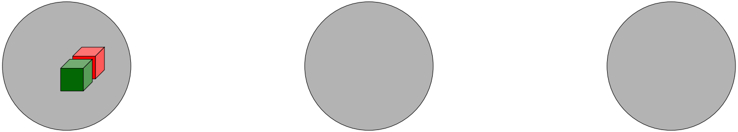
Apply $B21 = \langle T2 \wedge B2, \{T1, B1, \neg T2, \neg B2\}\rangle$

Computation of h^{max}

Tractor example



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3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	TF	TF	TF	TF	TF	TF

Distance of $A1 \wedge B1$ is 4.

h^{max} Underestimates

Example

Estimate for lamp1on $\wedge$ lamp2on $\wedge$ lamp3on with

$$\langle\top, \{\text{lamp1on}\}\rangle$$
$$\langle\top, \{\text{lamp2on}\}\rangle$$
$$\langle\top, \{\text{lamp3on}\}\rangle$$

is 1. Actual shortest plan has length 3.
By definition, $h^{max}(G_1 \wedge \cdots \wedge G_n)$ is the maximum of $h^{max}(G_1), \ldots, h^{max}(G_n)$.
If goals are independent, the sum of the estimates is more accurate.

Computation of h^+

Tractor example

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	F	TF	TF	F	TF	TF
5	TF	TF	TF	TF	TF	TF	TF	TF	TF

Apply $A21 = \langle T2 \wedge A2, \{T1, A1, \neg T2, \neg A2\}\rangle$.
$h^+(T2 \wedge A2)$ is 1+3.
$h^+(A1)$ is 1+3+1 = 5 (h^{max} gives 4.)

Computation of h^+
Tractor example

t	T1	T2	T3	A1	A2	A3	B1	B2	B3
0	T	F	F	F	F	T	F	F	T
1	TF	TF	F	F	F	T	F	F	T
2	TF	TF	TF	F	F	T	F	F	T
3	TF	TF	TF	F	TF	TF	F	TF	TF
4	TF	TF	TF	F	TF	TF	F	TF	TF
5	TF	TF	TF	TF	TF	TF	TF	TF	TF

h^+ of $A1 \wedge B1$ is $5 + 5 = 10$.

Comparison of the Heuristics

- For the Tractor example:
  - actions in the shortest plan: 8
  - h^{max} yields 4 (never overestimates).
  - h^{+} yields 10 (may under or overestimate).
- The sum-heuristic and its various extensions, including relaxed plan heuristics [HN01, KHH12, KHD13] are used in practice for non-optimal planners.

Heuristic State-space Planners

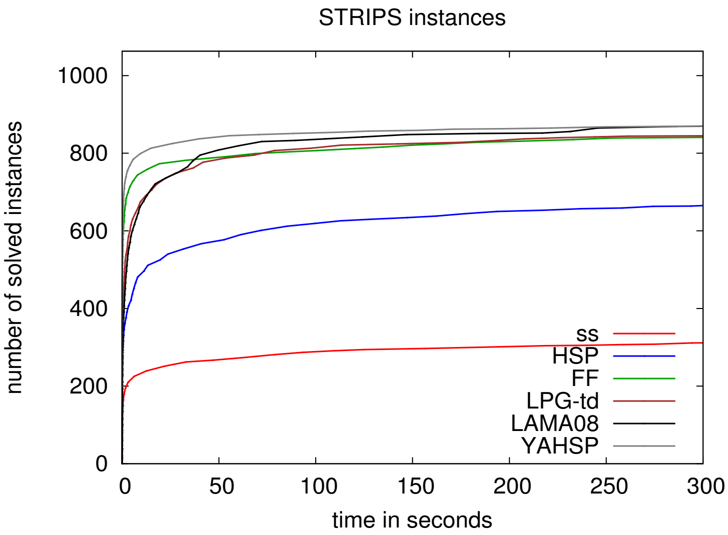
Some planners representing the current state of the art

$$
\begin{tikzcd}
 & \text{HSP [BLG97, BG01]} \arrow[dl] \arrow[d] \arrow[dr] & \\
\text{YAHSP3 [Vid04, Vid11]} & \text{LAMA [RW10]} & \text{PROBE [LG11]}
\end{tikzcd}
$$

- LAMA adds a preference for actions suggested by the computation of heuristic as good “first actions” towards goals [Vid04, RH09].
- YAHSP2/YAHSP3 and PROBE do – from each encountered state with a best-first search with h^+ – incomplete local searches to find shortcuts towards the goals.

Performance of State-Space Search Planners

Planning Competition Problems 2008-2011



Heuristics for Optimal Planning

Admissible heuristics are needed for finding optimal plans, e.g with A* [HNR68]. Scalability much poorer.

Pattern Databases [CS96, Ede00]

Abstract away many/most state variables, and use the length/cost of the optimal solution to the remaining problem as an estimate.

Generalized Abstraction (compose and abstract) [DFP09]

A generalization of pattern databases, allowing more complex aggregation of states (not just identification of ones agreeing on a subset of state variables.)
Planning people call it “merge and shrink”.

Landmark-cut [HD09] has worked well with standard benchmarks.

- Proposed by Kautz and Selman [KS92].
- Idea as in Cook's proof of NP-hardness of SAT [Coo71]: encode each step of a plan as a propositional formula.
- Intertranslatability of NP-complete problems $\Rightarrow$ reductions to many other problems possible, often simple.

Other NP-complete search frameworks

constraint satisfaction (CSP)	[vBC99, DK01]
NM logic programs / answer-set programs	[DNK97]
Mixed Integer Linear Programming (MILP)	[DG02]

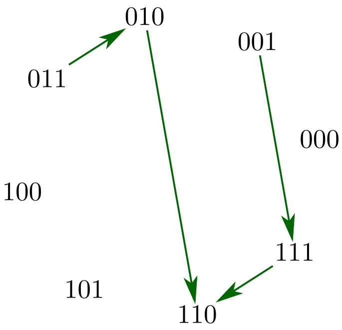
Transition relations in propositional logic

State variables are
$X = \{a, b, c\}$.

$$
\begin{array}{l}
(\neg a \wedge b \wedge c \wedge \neg a' \wedge b' \wedge \neg c') \vee \\
(\neg a \wedge b \wedge \neg c \wedge a' \wedge b' \wedge \neg c') \vee \\
(\neg a \wedge \neg b \wedge c \wedge a' \wedge b' \wedge c') \vee \\
(a \wedge b \wedge c \wedge a' \wedge b' \wedge \neg c')
\end{array}
$$

The corresponding matrix is

	000	001	010	011	100	101	110	111
000	0	0	0	0	0	0	0	0
001	0	0	0	0	0	0	0	1
010	0	0	0	0	0	0	1	0
011	0	0	1	0	0	0	0	0
100	0	0	0	0	0	0	0	0
101	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	1	0



Encoding of Actions as Formulas
for Sequential Plans

Actions as propositional formulas

New value of state variable x_i is a function of the old values of $x_1, \ldots, x_n$:
action j = conjunction of the precondition $P_j@t$ and

$$x_i@(t+1) \leftrightarrow F_i(x_1@t, \ldots, x_n@t)$$

for all $i \in \{1, \ldots, n\}$. Denote this by $E_j@t$.

Example (move-from-X-to-Y)

$$\overbrace{atX@t}^{\text{precond}} \wedge \overbrace{\begin{array}{l}(atX@(t+1) \leftrightarrow \bot) \wedge (atY@(t+1) \leftrightarrow \top) \\ \wedge (atZ@(t+1) \leftrightarrow atZ@t) \wedge (atU@(t+1) \leftrightarrow atU@t)\end{array}}^{\text{effects}}$$

Choice between actions $1, \ldots, m$ expressed by the formula

$$\mathcal{R}@t = E_1@t \vee \cdots \vee E_m@t.$$

Finding a Plan with SAT solvers

Let

- I be a formula expressing the initial state, and
- G be a formula expressing the goal states.

Then a plan of length T exists iff

$$I@0 \wedge \bigwedge_{t=0}^{T-1} \mathcal{R}@t \wedge G_T$$

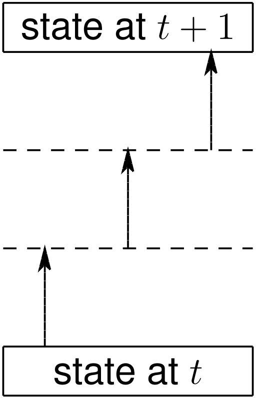
is satisfiable.

Remark

Most SAT solvers require formulas to be in CNF. There are efficient transformations to achieve this [Tse68, JS05, MV07].

Parallel Plans: Motivation

- Don't represent all intermediate states of a sequential plan.
- Don't represent the relative ordering of some consecutive actions.
- Reduced number of explicitly represented states $\Rightarrow$ smaller formulas



Parallel plans ($\forall$-step plans)

Blum and Furst [BF97], Kautz and Selman 1996 [KS96]

Allow actions $a_1 = \langle p_1, e_1 \rangle$ and $a_2 = \langle p_2, e_2 \rangle$ in parallel whenever they don't interfere, i.e.

- both $p_1 \cup p_2$ and $e_1 \cup e_2$ are consistent, and
- both $e_1 \cup p_2$ and $e_2 \cup p_1$ are consistent.

Theorem

If $a_1 = \langle p_1, e_1 \rangle$ and $a_2 = \langle p_1, e_1 \rangle$ don't interfere and s is a state such that $s \models p_1$ and $s \models p_2$, then $\mathit{exec}_{a_1}(\mathit{exec}_{a_2}(s)) = \mathit{exec}_{a_2}(\mathit{exec}_{a_1}(s))$.

$\forall$-step plans: encoding

Define $\mathcal{R}^{\forall}@t$ as the conjunction of

$$x@(t+1) \leftrightarrow ((x@t \wedge \neg a_1@t \wedge \cdots \wedge \neg a_k@t) \vee a'_1@t \vee \cdots \vee a'_{k'}@t)$$

for all $x \in X$, where $a_1, \ldots, a_k$ are all actions making x false, and $a'_1, \ldots, a'_{k'}$ are all actions making x true, and

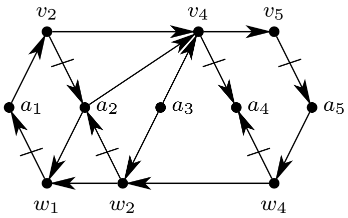
$$a@t \rightarrow l@t \text{ for all } l \text{ in the precondition of } a,$$

and

$$\neg(a@t \wedge a'@t) \text{ for all } a \text{ and } a' \text{ that interfere.}$$

This encoding is quadratic due to the interference clauses.

Action a with effect l disables all actions with precondition $\bar{l}$, except a itself.
This is done in two parts: disable actions with higher index, disable actions with lower index.



This is needed for every literal.

$\exists$-step plans
Dimopoulos et al. 1997 [DNK97]

Allow actions $\{a_1, \ldots, a_n\}$ in parallel if they can be executed in at least one order.

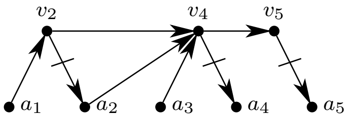
- $\bigcup_{i=1}^{n} p_i$ is consistent.
- $\bigcup_{i=1}^{n} e_i$ is consistent.
- There is a total ordering $a_1, \ldots, a_n$ such that $e_i \cup p_j$ is consistent whenever $i \leq j$: disabling an action earlier in the ordering is allowed.

Several compact encodings exist [RHN06].
Fewer time steps are needed than with $\forall$-step plans. Sometimes only half as many.

$\exists$-step plans: linear encoding
Rintanen et al. 2006 [RHN06]

Choose an arbitrary fixed ordering of all actions $a_1, \ldots, a_n$.

Action a with effect l disables all later actions with precondition $\bar{l}$.



This is needed for every literal.

Disabling graphs
Rintanen et al. 2006 [RHN06]

Define a disabling graph with actions as nodes and with an arc from a_1 to a_2 (a_1 disables a_2) if $p_1 \cup p_2$ and $e_1 \cup e_2$ are consistent and $e_1 \cup p_2$ is inconsistent.

The test for valid execution orderings can be limited to strongly connected components (SCC) of the disabling graph.

In many structured problems all SCCs are singleton sets.
$\Longrightarrow$ No tests for validity of orderings needed during SAT solving.

Summary of Notions of Plans

plan type	reference	comment
sequential	[KS92]	one action per time point
$\forall$-parallel	[BF97, KS96]	parallel actions independent
$\exists$-parallel	[DNK97, RHN06]	executable in at least one order

The last two expressible in terms of the relation disables restricted to applied actions:

- $\forall$-parallel plans: the disables relation is empty.
- $\exists$-parallel plans: the disables relation is acyclic.

Search through Horizon Lengths

The planning problem is reduced to the satisfiability tests for

$$
\begin{aligned}
\Phi_0 &= I@0 \wedge G@0 \\
\Phi_1 &= I@0 \wedge \mathcal{R}@0 \wedge G@1 \\
\Phi_2 &= I@0 \wedge \mathcal{R}@0 \wedge \mathcal{R}@1 \wedge G@2 \\
\Phi_3 &= I@0 \wedge \mathcal{R}@0 \wedge \mathcal{R}@1 \wedge \mathcal{R}@2 \wedge G@3 \\
&\vdots \\
\Phi_u &= I@0 \wedge \mathcal{R}@0 \wedge \mathcal{R}@1 \wedge \cdots \mathcal{R}@(u-1) \wedge G@u
\end{aligned}
$$

where u is the maximum possible plan length.

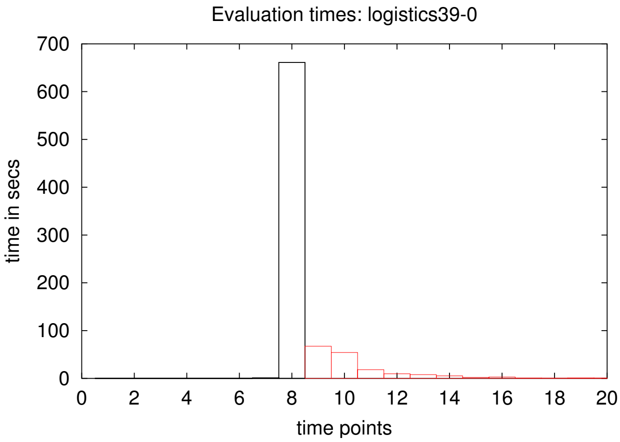
Q: How to schedule these satisfiability tests?

Search through Horizon Lengths

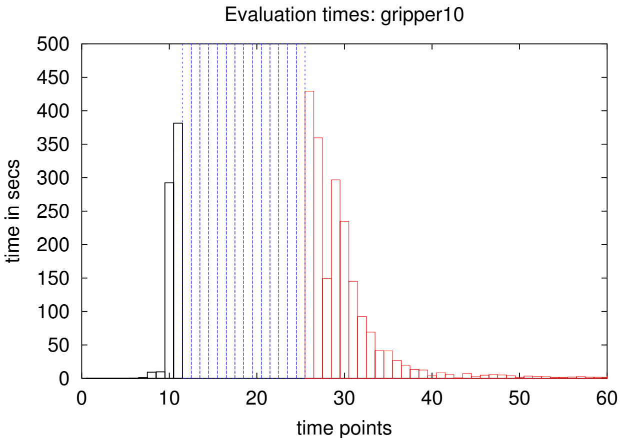
algorithm	reference	comment
sequential	[KS92, KS96]	slow, guarantees min. horizon
binary search	[SS07]	prerequisite: “tight” length UB
n processes	[Rin04b, Zar04]	fast, more memory needed
geometric	[Rin04b]	fast, more memory needed

- sequential: first test Φ_0, then Φ_1, then Φ_2, $\ldots$
  - This is breadth-first search / iterative deepening.
  - Guarantees shortest horizon length, but is slow.
- parallel strategies: solve several horizon lengths simultaneously
  - depth-first flavor
  - usually much faster
  - no guarantee of minimal horizon length

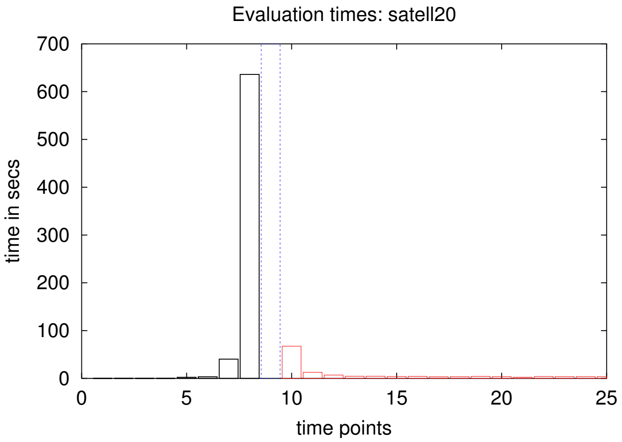
Some runtime profiles



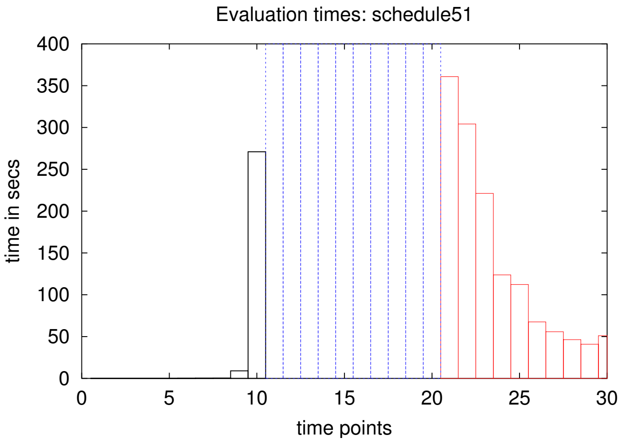
Some runtime profiles



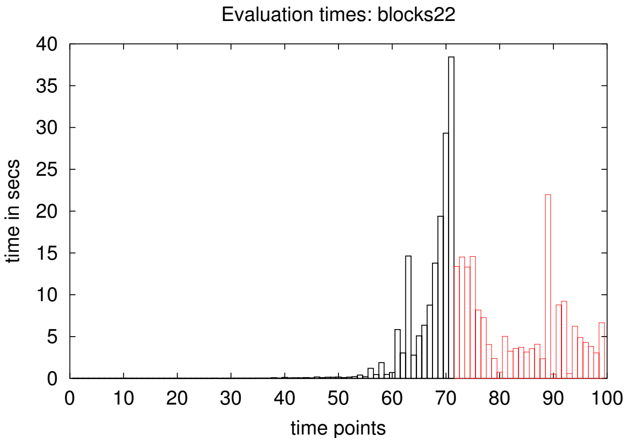
Some runtime profiles



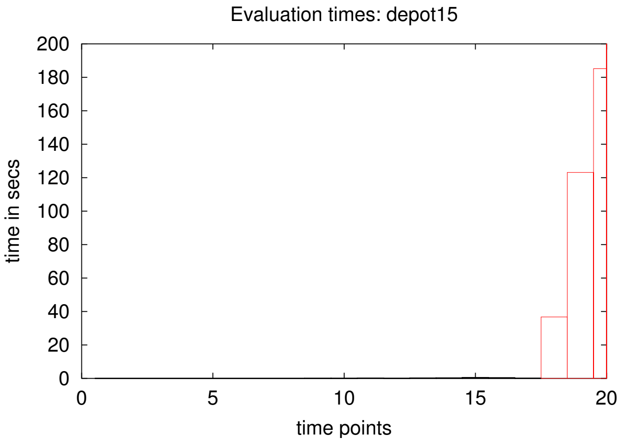
Some runtime profiles



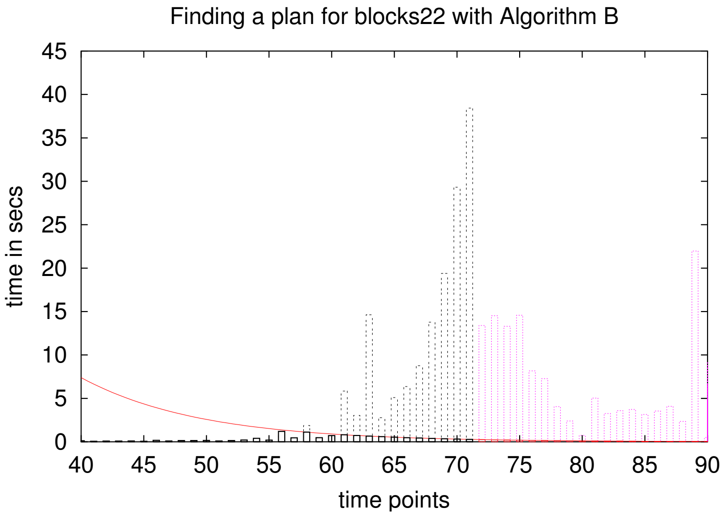
Some runtime profiles



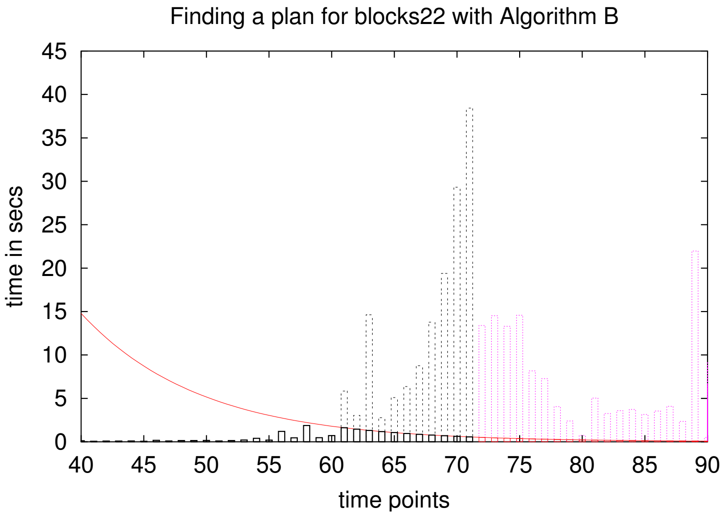
Some runtime profiles



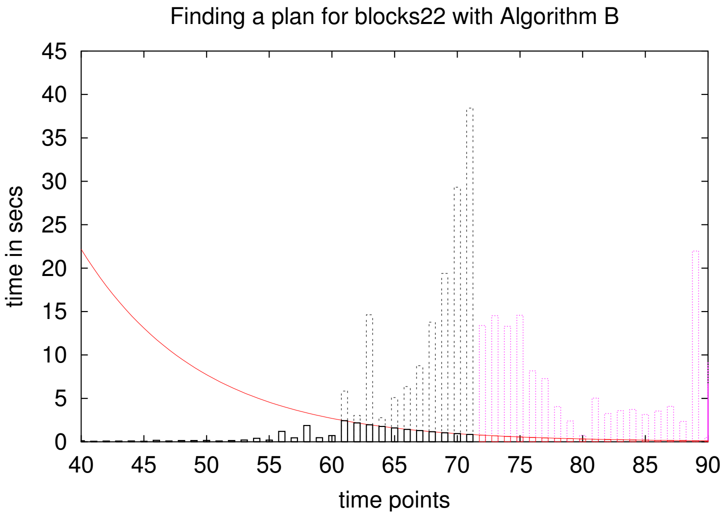
Geometric Evaluation



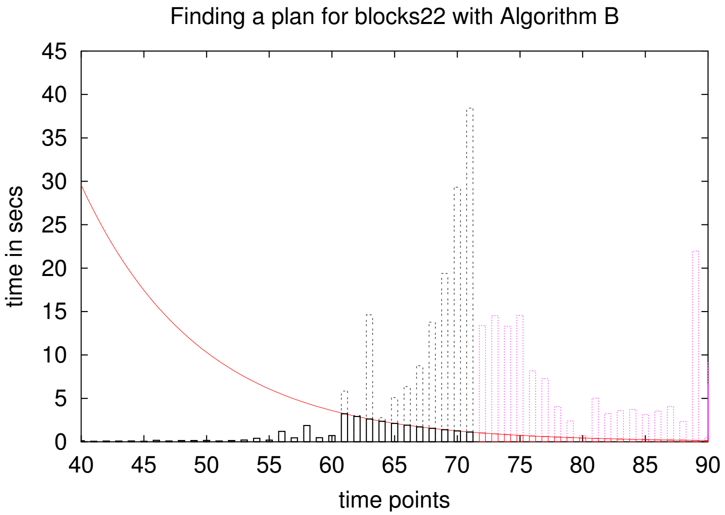
Geometric Evaluation



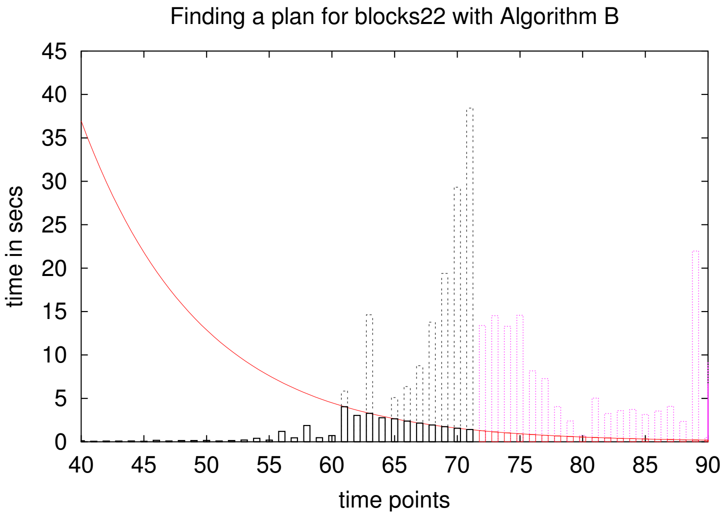
Geometric Evaluation



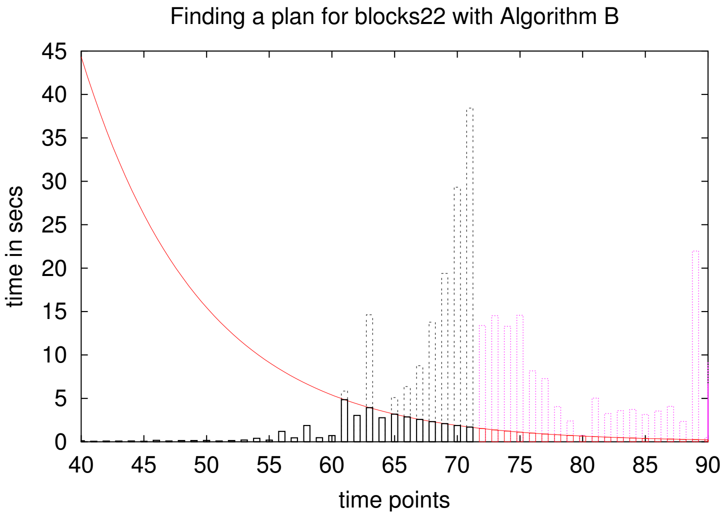
Geometric Evaluation



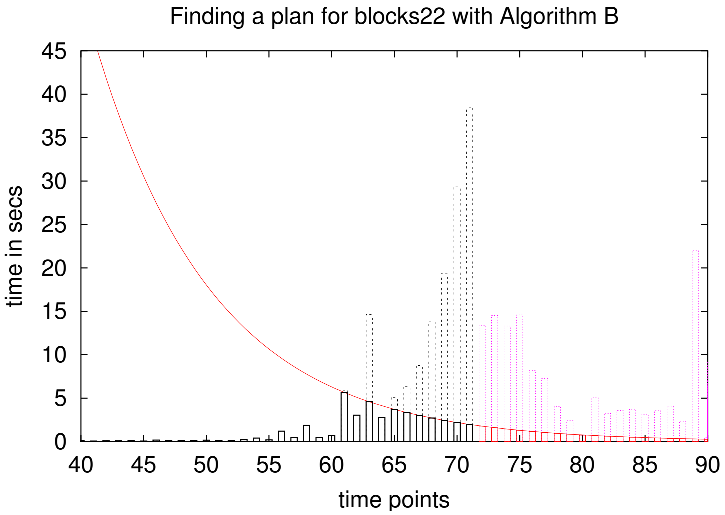
Geometric Evaluation



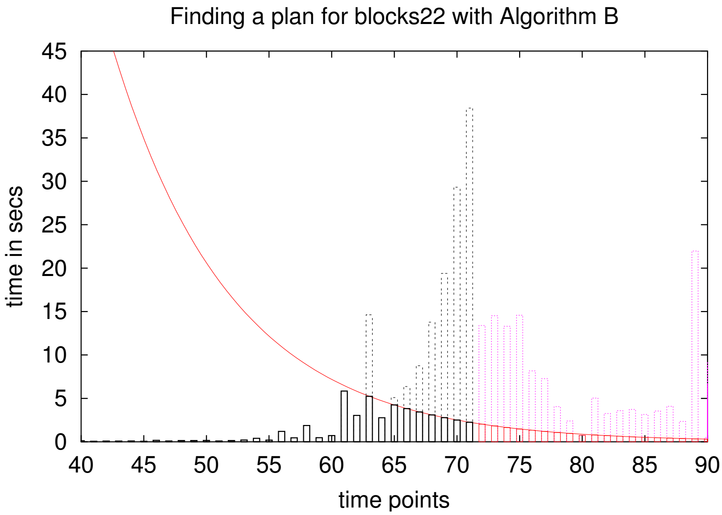
Geometric Evaluation



Geometric Evaluation



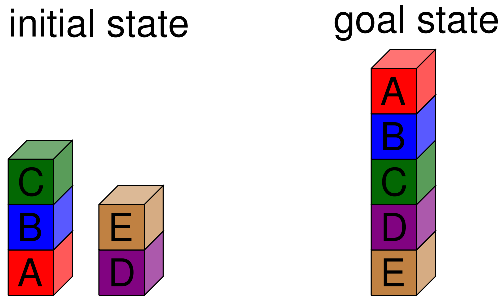
Geometric Evaluation



Solving the SAT Problem

SAT problems obtained from planning are solved by

- generic SAT solvers
  - Mostly based on Conflict-Driven Clause Learning (CDCL) [MMZ+01].
  - Very good on hard combinatorial planning problems.
  - Not designed for solving the extremely large but "easy" formulas (arising in some types of benchmark problems).
- specialized SAT solvers [Rin10, Rin12]
  - Replace standard CDCL heuristics with planning-specific ones.
  - For certain problem classes substantial improvement
  - New research topic: lots of unexploited potential



Problem solved almost without search:

- Formulas for lengths 1 to 4 shown unsatisfiable without any search.
- Formula for plan length 5 is satisfiable: 3 nodes in the search tree.
- Plans have 5 to 7 operators, optimal plan has 5.

Solving the SAT Problem

Example

	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
clear(a)	F	F					F	F	F		T	T	F	F	F	T	T	T
clear(b)	F					F	F	F		T	T	F	F	F	T	T	T	F
clear(c)	T	T			F	F	T	T	T	T	F	F	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
on(a,b)	F	F	F			T	F	F	F	F	F	T	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,a)	T	T			F	F	T	T	T		F	F	T	T	T	F	F	F
on(b,c)	F	F			T	T	F	F	F	F	T	T	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,b)	T			F	F	F	T	T		F	F	F	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T	F	F	T	T	T	T	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F	T	F	F	F	F	F	T	F	F	F	F	F
ontable(a)	T	T	T			F	T	T	T	T	T	F	T	T	T	T	T	F
ontable(b)	F	F			F	F	F	F	F		F	F	F	F	F	T	F	F
ontable(c)	F			F	F	F	F	F		F	F	F	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T	F	T	T	T	T	T	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: $\neg$clear(b)^1.
3. Branch: clear(a)^3.
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

Solving the SAT Problem
Example

	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
clear(a)	F	F					F	F	F		T	T	F	F	F	T	T	T
clear(b)	F					F	F	F		T	T	F	F	F	T	T	T	F
clear(c)	T	T			F	F	T	T	T	T	F	F	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
on(a,b)	F	F	F			T	F	F	F	F	F	T	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,a)	T	T			F	F	T	T	T		F	F	T	T	T	F	F	F
on(b,c)	F	F			T	T	F	F	F	F	T	T	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,b)	T			F	F	F	T	T		F	F	F	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T	F	F	T	T	T	T	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F	T	F	F	F	F	F	T	F	F	F	F	F
ontable(a)	T	T	T			F	T	T	T	T	T	F	T	T	T	T	T	F
ontable(b)	F	F			F	F	F	F	F		F	F	F	F	F	T	F	F
ontable(c)	F			F	F	F	F	F		F	F	F	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T	F	T	T	T	T	T	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: $\neg$clear(b)[1].
3. Branch: clear(a)[3].
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

Solving the SAT Problem
Example

	0	1	2	3	4	5
clear(a)	F	F				
clear(b)	F					F
clear(c)	T	T			F	F
clear(d)	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F
on(a,b)	F	F	F			T
on(a,c)	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F
on(b,a)	T	T			F	F
on(b,c)	F	F			T	T
on(b,d)	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F
on(c,b)	T			F	F	F
on(c,d)	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F
ontable(a)	T	T	T			F
ontable(b)	F	F			F	F
ontable(c)	F			F	F	F
ontable(d)	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T

	0	1	2	3	4	5
clear(a)	F	F	F		T	T
clear(b)	F	F		T	T	F
clear(c)	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F
on(a,b)	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F
on(b,a)	T	T	T		F	F
on(b,c)	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F
on(c,b)	T	T		F	F	F
on(c,d)	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F
ontable(a)	T	T	T	T	T	F
ontable(b)	F	F	F		F	F
ontable(c)	F	F		F	F	F
ontable(d)	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T

	0	1	2	3	4	5
clear(a)	F	F	F	T	T	T
clear(b)	F	F	T	T	T	F
clear(c)	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F
on(a,b)	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F
on(b,a)	T	T	T	F	F	F
on(b,c)	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F
on(c,b)	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F
ontable(a)	T	T	T	T	T	F
ontable(b)	F	F	F	T	F	F
ontable(c)	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: $\neg$clear(b)^1.
3. Branch: clear(a)^3.
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

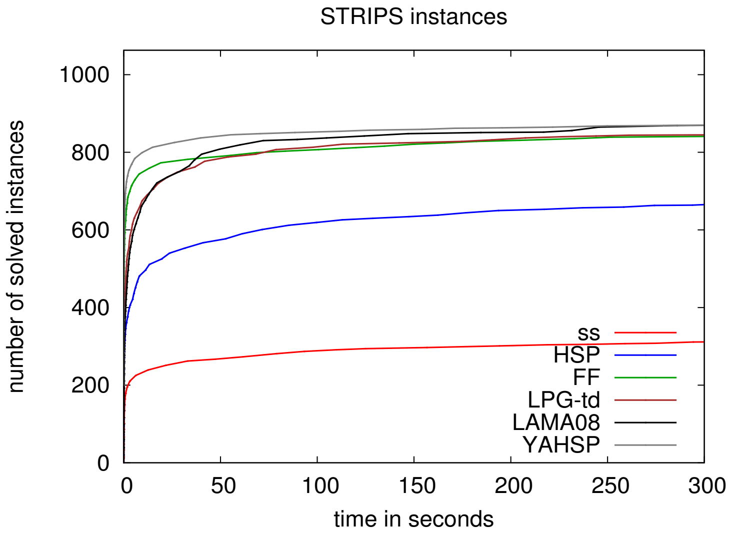
Solving the SAT Problem
Example

	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
clear(a)	F	F					F	F	F		T	T	F	F	F	T	T	T
clear(b)	F					F	F	F		T	T	F	F	F	T	T	T	F
clear(c)	T	T			F	F	T	T	T	T	F	F	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
on(a,b)	F	F	F			T	F	F	F	F	F	T	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,a)	T	T			F	F	T	T	T		F	F	T	T	T	F	F	F
on(b,c)	F	F			T	T	F	F	F	F	T	T	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,b)	T			F	F	F	T	T		F	F	F	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T	F	F	T	T	T	T	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F	T	F	F	F	F	F	T	F	F	F	F	F
ontable(a)	T	T	T			F	T	T	T	T	T	F	T	T	T	T	T	F
ontable(b)	F	F			F	F	F	F	F		F	F	F	F	F	T	F	F
ontable(c)	F			F	F	F	F	F		F	F	F	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T	F	T	T	T	T	T	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: $\neg\text{clear(b)}^1$.
3. Branch: clear(a)^3.
4. Plan found:

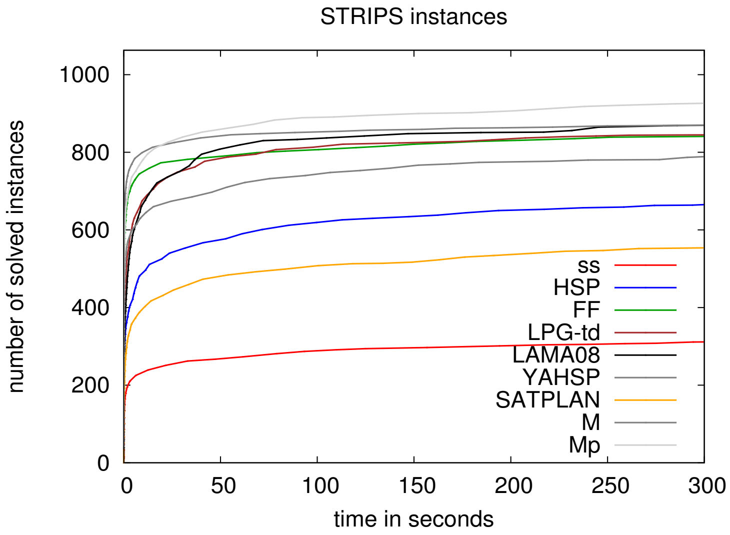
	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

Performance of SAT-Based Planners
Planning Competition Problems 1998-2008



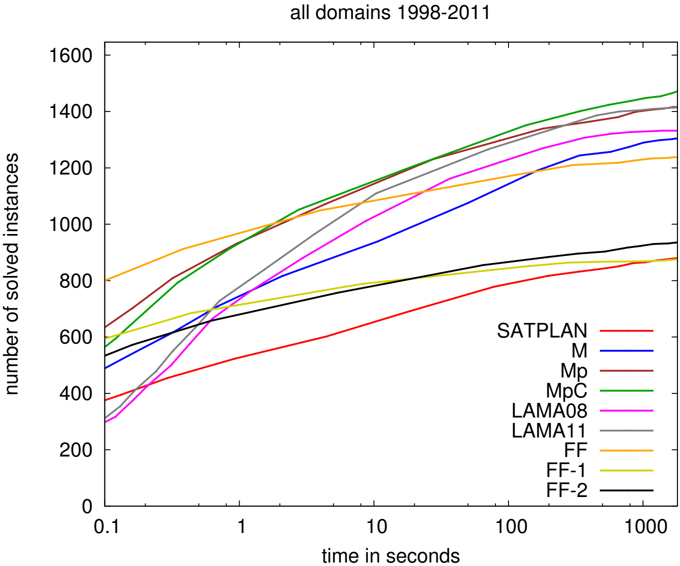
Performance of SAT-Based Planners

Planning Competition Problems 1998-2008



Performance of SAT-Based Planners

Planning Competition Problems 1998-2011 (revised)



Symbolic Search Methods
Motivation

- logical formulas as data structure for sets, relations
- state-space search (planning, model-checking, diagnosis, ...) in terms of set & relational operations
- Algorithms that can handle very large state sets, bypassing inherent limitations of enumerative methods.

- SAT and explicit state-space search: primary use finding one path from an initial state to a goal state
- "Symbolic" search methods can be used for more general problems:
  - Finding set of all reachable states
  - Distances/plans from the initial state to all states
  - Distances/plans to goal states from all states
- Competitive for optimal planning and detecting unsolvability.
- BDDs are a representation of belief states [BCRT01, Rin05].
- Algebraic Decision Diagrams (ADD) [FMY97, BFG+97] can represent value functions in probabilistic planning [HSAHB99].

Transition relations in propositional logic

State variables are
$X = \{a, b, c\}$.

$$
\begin{aligned}
&(\neg a \wedge b \wedge c \wedge \neg a' \wedge b' \wedge \neg c')\vee \\
&(\neg a \wedge b \wedge \neg c \wedge a' \wedge b' \wedge \neg c')\vee \\
&(\neg a \wedge \neg b \wedge c \wedge a' \wedge b' \wedge c')\vee \\
&(a \wedge b \wedge c \wedge a' \wedge b' \wedge \neg c')
\end{aligned}
$$

The corresponding matrix is

	000	001	010	011	100	101	110	111
000	0	0	0	0	0	0	0	0
001	0	0	0	0	0	0	0	1
010	0	0	0	0	0	0	1	0
011	0	0	1	0	0	0	0	0
100	0	0	0	0	0	0	0	0
101	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0
111	0	0	0	0	0	0	1	0

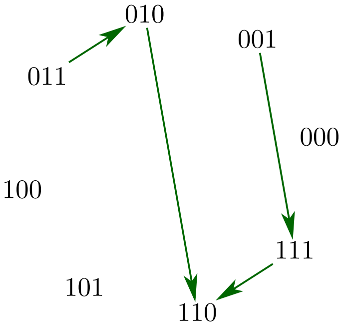


Image operations

The image of a set T of states w.r.t. action a is

$$\mathit{img}_a(T) = \{s' \in S | s \in T, sas'\}.$$

The pre-image of a set T of states w.r.t. action a is

$$\mathit{preimg}_a(T) = \{s \in S | s' \in T, sas'\}.$$

These operations reduce to the relational join and projection operations with a logic-representation of sets (unary relations) and binary relations.

(Pre-image corresponds to regression used with backward-search [Rin08].)

Finding All Plans with a Symbolic Algorithm
[BCL+94]

All reachable states with breadth-first search

$$S_0 = \{I\}$$
$$S_{i+1} = S_i \cup \bigcup_{a \in A} \mathit{img}_a(S_i)$$

If $S_i = S_{i+1}$, then $S_j = S_i$ for all $j \geq i$, and the computation can be terminated.

- $S_i, i \geq 0$ is the set of states with distance $\leq i$ from the initial state.
- $S_i \backslash S_{i-1}, i \geq 1$ is the set of states with distance i.
- If $G \cap S_i$ for some $i \geq 0$, then there is a plan.

Action sequence recovered from sets S_i by a sequence of backward-chaining steps (linear in plan length and number of state variables)
(Approximations of the above algorithm compute invariants [Rin08]).

Symbolic State-Space Search Algorithms

- Symbolic Breadth-First [BCL$^+$94]
- Symbolic (BDD) versions of A*:
  - BDDA* [ER98]
  - SetA* [JVB08]
  - ADDA* [HZF02]
- The Saturation algorithm [CLS01, CLM07, YCL09] trades optimality (as obtained with breadth-first) to far better scalability: find all reachable states, without accurate distance information.

Representation of Sets as Formulas

state sets	**formulas over X**
those $\frac{2^{\lvert X \rvert}}{2}$ states where x is true	$x \in X$
$\overline{E}$ (complement)	$\neg E$
$E \cup F$	$E \vee F$
$E \cap F$	$E \wedge F$
$E \setminus F$ (set difference)	$E \wedge \neg F$
the empty set $\emptyset$	$\bot$ (constant *false*)
the universal set	$\top$ (constant *true*)

question about sets	**question about formulas**
$E \subseteq F$?	$E \models F$?
$E \subset F$?	$E \models F$ and $F \not\models E$?
$E = F$?	$E \models F$ and $F \models E$?

Sets (of states) as formulas

Formulas over X represent sets

$a \vee b$ over $X = \{a, b, c\}$

represents the set $\{\overset{a\,b\,c}{010}, 011, 100, 101, 110, 111\}$.

Formulas over $X \cup X'$ represent binary relations

$a \wedge a' \wedge (b \leftrightarrow b')$ over $X \cup X'$ where $X = \{a, b\}$, $X' = \{a', b'\}$

represents the binary relation $\{(\overset{a\,b}{10}, \overset{a'b'}{1\,0}), (11, 11)\}$.

Valuations $\overset{a\,b\,a'b'}{1\,0\,1\,0}$ and 1111 of $X \cup X'$ can be viewed respectively as pairs of valuations $(\overset{a\,b}{10}, \overset{a'b'}{1\,0})$ and $(11, 11)$ of X.

Relation Operations

relation operation	**logical operation**
projection	abstraction
join	conjunction

Existential and Universal Abstraction

Definition

Existential abstraction of a formula ϕ with respect to $x \in X$:

$$\exists x.\phi = \phi[\top/x] \vee \phi[\bot/x].$$

Universal abstraction is defined analogously by using conjunction instead of disjunction.

Definition

Universal abstraction of a formula ϕ with respect to $x \in X$:

$$\forall x.\phi = \phi[\top/x] \wedge \phi[\bot/x].$$

∃-Abstraction

Example

$$
\begin{aligned}
&\exists b.((a \rightarrow b) \wedge (b \rightarrow c)) \\
&= ((a \rightarrow \top) \wedge (\top \rightarrow c)) \vee ((a \rightarrow \bot) \wedge (\bot \rightarrow c)) \\
&\equiv c \vee \neg a \\
&\equiv a \rightarrow c
\end{aligned}
$$

$$
\begin{aligned}
&\exists ab.(a \vee b) = \exists b.(\top \vee b) \vee (\bot \vee b) \\
&= ((\top \vee \top) \vee (\bot \vee \top)) \vee ((\top \vee \bot) \vee (\bot \vee \bot)) \\
&\equiv (\top \vee \top) \vee (\top \vee \bot) \equiv \top
\end{aligned}
$$

$\forall$ and $\exists$-Abstraction in Terms of Truth-Tables

$\forall c$ and $\exists c$ correspond to combining lines with the same valuation for variables other than c.

Example

$$\exists c.(a \vee (b \wedge c)) \equiv a \vee b \qquad \forall c.(a \vee (b \wedge c)) \equiv a$$

a	b	c	$a \vee (b \wedge c)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

a	b	$\exists c.(a \vee (b \wedge c))$
0	0	0
0	1	1
1	0	1
1	1	1

a	b	$\forall c.(a \vee (b \wedge c))$
0	0	0
0	1	0
1	0	1
1	1	1

Encoding of Actions as Formulas

Let X be the set of all state variables. An action a corresponds to the conjunction of the precondition P_j and

$$x' \leftrightarrow F_i(X)$$

for all $x \in X$. Denote this by $\tau_X(a)$.

Example (move-from-A-to-B)

$$atA \wedge (atA' \leftrightarrow \bot) \wedge (atB' \leftrightarrow \top) \wedge (atC' \leftrightarrow atC) \wedge (atD' \leftrightarrow atD)$$

This is exactly the same as in the SAT case, except that we have x and x' instead of $x@t$ and $x@(t+1)$.

Images as Relational Operations

$$\boxed{\begin{array}{c} s_0 \\ s_2 \end{array}} \bowtie \boxed{\begin{array}{cc} s_0 & s_1 \\ s_0 & s_2 \\ s_1 & s_0 \\ s_1 & s_2 \\ s_2 & s_0 \end{array}} = \boxed{\begin{array}{cc} s_0 & \mathbf{s_1} \\ s_0 & \mathbf{s_2} \\ s_2 & \mathbf{s_0} \end{array}}$$

Images as Relational Operations

$$\boxed{\begin{array}{c} s_0 \\ s_2 \end{array}} \bowtie \boxed{\begin{array}{cc} s_0 & s_1 \\ s_0 & s_2 \\ s_1 & s_0 \\ s_1 & s_2 \\ s_2 & s_0 \end{array}} = \boxed{\begin{array}{cc} s_0 & \mathbf{s_1} \\ s_0 & \mathbf{s_2} \\ s_2 & \mathbf{s_0} \end{array}}$$

$$\boxed{\begin{array}{c} s_0 00 \\ s_2 10 \end{array}} \bowtie \boxed{\begin{array}{c} s_0 s_1 00\ 01 \\ s_0 s_2 00\ 10 \\ s_1 s_0 01\ 00 \\ s_1 s_2 01\ 10 \\ s_2 s_0 10\ 00 \end{array}} = \boxed{\begin{array}{c} s_0 s_1 00\ 01 \\ s_0 s_2 00\ 10 \\ s_2 s_0 10\ 00 \end{array}}$$

Images as Relational Operations

$$
\boxed{\begin{array}{l} s_0 00 \\ s_2 10 \end{array}}
\bowtie
\boxed{\begin{array}{l} s_0 s_1 00\ 01 \\ s_0 s_2 00\ 10 \\ s_1 s_0 01\ 00 \\ s_1 s_2 01\ 10 \\ s_2 s_0 10\ 00 \end{array}}
=
\boxed{\begin{array}{l} s_0 s_1 00\ 01 \\ s_0 s_2 00\ 10 \\ s_2 s_0 10\ 00 \end{array}}
\quad
\begin{array}{l|l} x_0 x_1 & \\ \hline 00 & 1 \\ 01 & 0 \\ 10 & 1 \\ 11 & 0 \end{array}
\bowtie
\begin{array}{l|l} x_0 x_1 x_0' x_1' & \\ \hline 0000 & 0 \\ 0001 & 1 \\ 0010 & 1 \\ 0011 & 0 \\ 0100 & 1 \\ 0101 & 0 \\ 0110 & 1 \\ 0111 & 0 \\ 1000 & 1 \\ 1001 & 0 \\ 1010 & 0 \\ 1011 & 0 \\ 1100 & 0 \\ 1101 & 0 \\ 1110 & 0 \\ 1111 & 0 \end{array}
=
\begin{array}{l|l} x_0 x_1 x_0' x_1' & \\ \hline 0001 & 1 \\ 0010 & 1 \\ 1000 & 1 \end{array}
$$

Computation of Successor States

Let

- $X = \{x_1, \ldots, x_n\}$,
- $X' = \{x'_1, \ldots, x'_n\}$,
- ϕ be a formula over X that represents a set T of states.

Image Operation

The image $\{s' \in S | s \in T, sas'\}$ of T with respect to a is

$$\mathit{img}_a(\phi) = (\exists X.(\phi \wedge \tau_X(a)))[X/X'].$$

The renaming is necessary to obtain a formula over X.

Computation of Predecessor States

Let

- $X = \{x_1, \ldots, x_n\}$,
- $X' = \{x'_1, \ldots, x'_n\}$,
- ϕ be a formula over X that represents a set T of states.

Preimage Operation

The pre-image $\{s \in S | s' \in T, sas'\}$ of T with respect to a is

$$\textit{preimg}_a(\phi) = (\exists X'.(\phi[X'/X] \wedge \tau_X(a))).$$

The renaming of ϕ is necessary so that we can start with a formula over X.

Normal Forms

normal form	reference	comment
NNF Negation Normal Form		
DNF Disjunctive Normal Form		
CNF Conjunctive Normal Form		
BDD Binary Decision Diagram	[Bry92]	most popular
DNNF Decomposable NNF	[Dar01]	more compact
d-DNNF deterministic DNNF	[Dar02]	

Darwiche's terminology: knowledge compilation languages [DM02]

Trade-off

- more compact $\mapsto$ less efficient operations
- But, "more efficient" is in the size of a correspondingly inflated formula. (Also more efficient in terms of wall clock?) BDD-SAT is $\mathcal{O}(1)$, but e.g. translation into BDDs is (usually) far less efficient than testing SAT directly.

Complexity of Operations

	$\vee$	$\wedge$	$\neg$	TAUT	SAT	$\phi \equiv \phi'$?	#SAT
NNF	poly	poly	poly	co-NP	NP	co-NP	#P
DNF	poly	exp	exp	co-NP	P	co-NP	#P
CNF	exp	poly	exp	P	NP	co-NP	#P
BDD	exp	exp	poly	P	P	P	poly
DNNF	poly	exp	exp	co-NP	P	co-NP	#P
d-DNNF	poly	exp	exp	co-NP	P	co-NP	poly

Remark

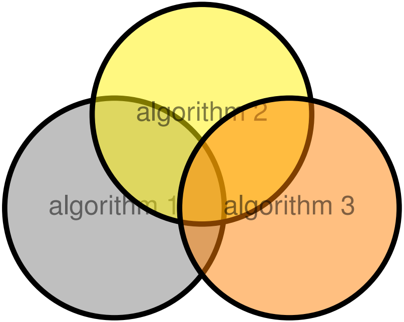
For BDDs one $\vee / \wedge$ is polynomial time/size (size is doubled) but repeated $\vee / \wedge$ lead to exponential size.

Engineering Efficient Planners

- Gap between Theory and Practice large: engineering details of implementation critical for performance in current planners.
- Few of the most efficient planners use textbook methods.
- Explanations for the observed differences between planners lacking: this is more art than science.

Algorithm Portfolios

- Algorithm portfolio = combination of two or more algorithms
- Useful if there is no single “strongest” algorithm.



Algorithm Portfolios
Composition methods

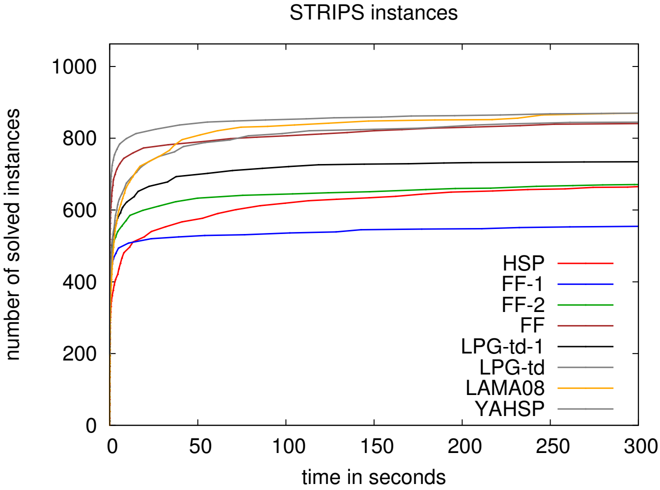
Methods for composing a portfolio

selection	choose one for current instance [XHHLB08]
parallel	run components in parallel [GS97, HLH97]
sequential	run consecutively, according to a schedule

Other variations of the above [HDH+00].

Early uses in planning: BLACKBOX [KS99] (manual configuration), FF [HN01] and LPG [GS02] (fixed configuration)

Many works on automated portofolio construction in the SAT area, directly applicable to planning as they are not specific to SAT or planning.



FF	=	FF-1 followed by FF-2 ($\sim$ HSP)
LPG-td	=	LPGT-td-1 followed by FF-2 ($\sim$ HSP)

Evaluation of Planners

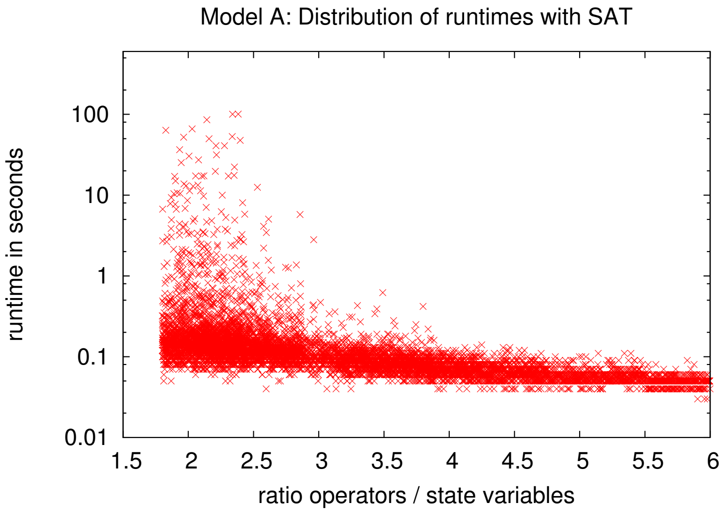
Evaluation of planning systems is based on

- Hand-crafted problems (from the planning competitions)
  - This is the most popular option.
  - \+ Problems with (at least moderately) different structure.
  - \- Real-world relevance mostly low.
  - \- Instance generation uncontrolled: not known if easy or difficult.
  - \- Many have a similar structure: objects moving in a network.
- Benchmark sets obtained by translation from other problems
  - graph-theoretic problems: cliques, colorability, ... [PMB11]
- Instances sampled from all instances [Byl96, Rin04c].
  - \+ Easy to control problem hardness.
  - \- No direct real-world relevance (but: core of any “hard” problem)

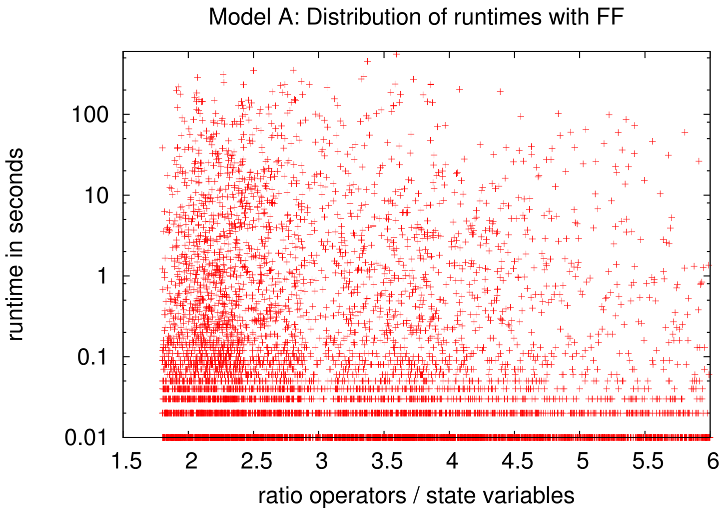
Sampling from the Set of All Instances
[Byl96, Rin04c]

- Generation:
  1. Fix number N of state variables, number M of actions.
  2. For each action, choose preconditions and effects randomly.
- Has a phase transition from unsolvable to solvable, similarly to SAT [MSL92] and connectivity of random graphs [Bol85].
- Exhibits an easy-hard-easy pattern, for a fixed N and an increasing M, analogously to SAT [MSL92].
- Hard instances roughly at the 50 per cent solvability point.
- Hardest instances are very hard: 20 state variables (2^{20} states) too difficult for many planners.

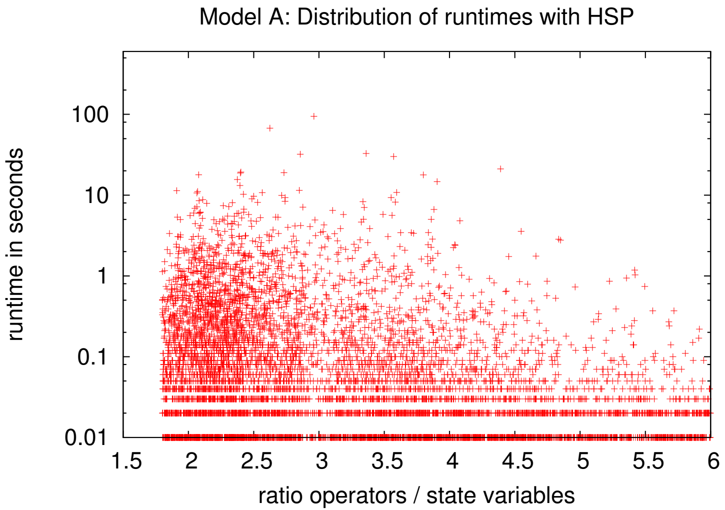
Sampling from the Set of All Instances
Experiments with planners



Sampling from the Set of All Instances
Experiments with planners



Sampling from the Set of All Instances
Experiments with planners



Minimization of the duration of the execution phase:

- Two short actions may be better than one long one.
- Actions can be taken in parallel.
- Connection to scheduling problems [SFJ00].

This is a core consideration in most mixed planning+scheduling problems. (Duration and especially concurrency ignored in classical planning and basic state-space search methods.)

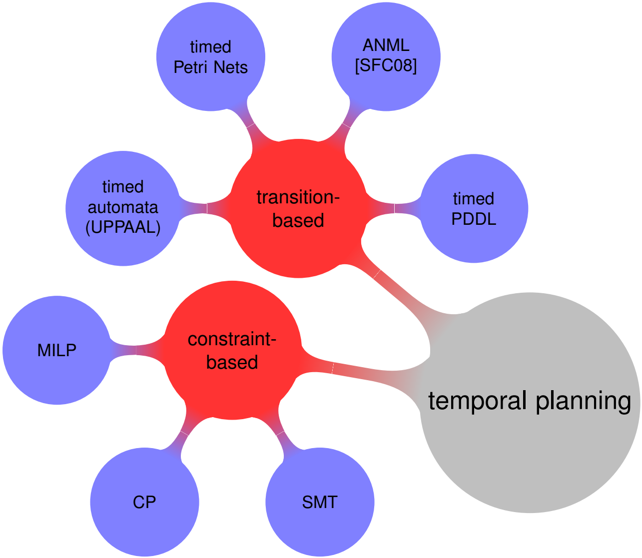
Inherent concurrency of actions

- Taking an action may require other concurrent actions.
- Some effects may only be achieved as joint effects of multiple actions.

Less important in practice: can often (always?) be avoided by modelling problem differently.

- Actions that must be used concurrently can be combined.
- Replace one complex action by several simpler ones: go to Paris = go to airport, board plane, fly, exit, take train to city

How to Represent Temporal Planning Problems?



Basic Modelling Concepts

Actions	Taken at a given time point t
Precondition	Must be satisfied at t.
Effects	Assignments $x := v$ at time points $t' > t$.
Dependencies	If action 1 taken at t, action 2 cannot be at $[t_1, t_2]$.

Action Dependencies through Resources

- n-ary resources
  Simultaneous use of resource can be at most n units.
  If each action needs 1 unit of the resource, no more than n actions can be using it simultaneously.
  Example: n identical tools or machines
- state resources
  A resource is in at most one state at a time.
  Multiple actions can use the resource in the same state.
  Example: generator that can produce 110V,60Hz or 220V,50Hz

Relation to scheduling

- Planning = action selection + scheduling.
- Scheduling = assignment of starting times to tasks/actions, respecting resource constraints
- Expressive languages for temporal planning include scheduling and hence support the representation of resources.
- Resources and ordering constraints are the mechanism for guaranteeing that plans are executable.

Complexity

Most important scheduling problems are NP-complete [GJ79].
Temporal planning complete for PSPACE or EXPSPACE [Rin07].
Action selection is the main difference between them.

Embedding of Scheduling in Temporal Planning

Representation of a simple job-shop scheduling problem in temporal planning.

1. For each job j = a sequence of tasks $t_1^j, \ldots, t_{n_j}^j$, introduce state variable $p_j : \{1, \ldots, n+1\}$.
2. Each task is mapped to action a_i^j with
   - precondition $p_j = i$,
   - effect $p_j = i + 1$ after the duration of t_i^j,
   - resource requirements as in the scheduling problem.
3. In the initial state $p_j = 1$ for every job j.
4. In the goal we have $p_j = n_{j+1}$.

Tasks and their ordering inside the job are fixed. Remaining problem is scheduling the tasks/actions for different jobs relative to other jobs' tasks/actions and minimizing the makespan. Solutions of the temporal planning problem are exactly the solutions to the job-shop scheduling problem.

Timed State-Space

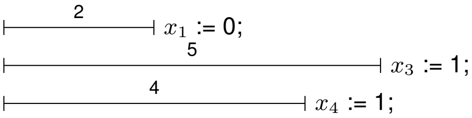
- state = values of state variables + values of clocks
- Clocks induce a schedule of future events.
- Actions initialize clocks.
- Time progresses, affecting all clocks.
- Reaching a critical clock value triggers scheduled events:
  - effects taking place later than the action's "starting" time point
  - resources allocated and later freed

This is the model behind all search methods.

Seemingly simple route to temporal planning with explicit state-space search.

Updates to the timed state
Scheduling (taking) an action

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = 0$$
$$x_4 = 0$$

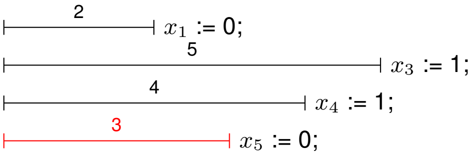


Updates to the timed state

Scheduling (taking) an action

Take action with precondition $x_2 = 1$ and effect $x_5 := 0$ at time 3.

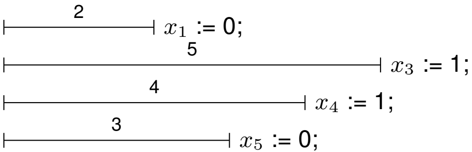
$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$



Updates to the timed state

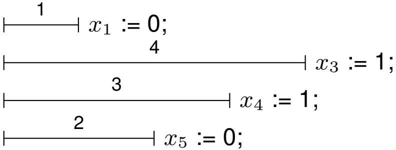
Advancing time

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$



Updates to the timed state
Advancing time

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$



Updates to the timed state

Advancing time

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$

x_1 := 0;

3 x_3 := 1;

2 x_4 := 1;

1 x_5 := 0;

Updates to the timed state
Advancing time

$$
\begin{aligned}
x_1 &= 0 \\
x_2 &= 1 \\
x_3 &= 0 \\
x_4 &= 0 \\
x_5 &= 1
\end{aligned}
$$

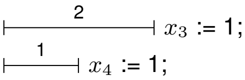
2
x_3 := 1;

1
x_4 := 1;

x_5 := 0;

Updates to the timed state
Advancing time

$$x_1 = 0$$
$$x_2 = 1$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 0$$



Completeness of Timed State-Space Search

- Since time is continuous, an action can be started at any of an infinite number of time points. $\Longrightarrow$ search space and branching factor infinite
- Simplistic policies for advancing time lead to incompleteness [MW06]. Most early temporal planners are incomplete. Few temporal planners have been proved to be complete.
- region abstraction [AD94] abstracts an infinite number of timed states to finitely many behaviorally equivalent regions.

Separation of planning and scheduling
CPT planner [VG06]

- Separate two problems
  1. selection of actions (only ordering, no timing)
  2. scheduling of these actions

  and interleave their solution.
- Action selection induces temporal constraints [DMP91]
- These temporal constraints can be solved separately.
- Completeness regained.

Systems for Temporal Planning

- Probably the most powerful verification tool based on explicit state-space search in the state-space induced by timed automata and their extension hybrid automata is UPPAAL [BLL+96].
  UPPAAL has been used in modelling and solving planning scenarios for example in robotics [QBZ04] and autonomous embedded systems [AAG+07, KMH01].
- CPT [VG06]
- Temporal Fast-Downward, based on the Fast-Downward planner for classical planning

Temporal Planning by Constraint Satisfaction

- Temporal planning can be encoded in
  - SAT modulo Theories (SMT) [WW99, ABC+02].
  - Constraint Programming [RvBW06]
  - Mixed Integer Linear Programming [DG02]

  (Similarly to scheduling [ABP+11].)
- The encoding methods for all are essentially the same. Differences in surface structure of the encoding, especially the types of constraints that can be encoded directly.
- In this tutorial we focus on SMT, due to its closeness to SAT.
- Differences in performance and pragmatic differences:
  - CP: support for customized search (heuristics, propagators, ...)
  - SMT: fully automatic, powerful handling of Boolean constraints.
  - MILP: for problems with intensive linear optimization

Each SMT instance fixes the number of steps i analogously to untimed (asynchoronous) state-space problems in SAT.

variables in SMT encoding

var	type	description
Δ_i	real	time between steps $i-1$ and i
$a@i$	bool	Is action a taken at step i?
$c_a@i$	real	Value of clock for action a at step i
$x@i$	bool	Value of Boolean state variable at step i

Executability of an action

Action cannot be taken if it is already active:

$$a@i \rightarrow (c_a@(i-1) \geq \textit{dur}(a)) \tag{1}$$

($\textit{dur}(a)$ denotes the duration a).

If actions actions a_1 and a_2 use the same unary resource respectively at $[t_1, t'_1]$ and at $[t_2, t'_2]$ then we have

$$t_2 + t'_2 - c_{a_1}@i \leq t_1 \tag{2}$$

$$t_1 + t'_1 \leq t_2 - c_{a_1} \tag{3}$$

Additionally, if $[t_1, t'_1]$ and $[t_2, t'_2]$ overlap, we have

$$\neg a_1@i \vee \neg a_2@i \tag{4}$$

Encodings of Timed Problems in SMT

Formula ϕ with every variable x replaced by $x@i$ is denoted by $\phi@i$.

Action with precondition p:

$$a@i \rightarrow p@i \tag{5}$$

If action is taken, its clock is initialized to 0:

$$a@i \rightarrow (c_a@i = 0) \tag{6}$$

If action is not taken, its clock advances:

$$\neg a@i \rightarrow (c_a@i = c_a@(i-1) + \Delta_i) \tag{7}$$

Encodings of Timed Problems in SMT

Effects of an action

An effect l scheduled at relative time t:

$$(c_a@i = t) \rightarrow l@i \tag{8}$$

Encodings of Timed Problems in SMT

Passage of time

Time may not pass a scheduled effect at relative time t:

$$c_a@(i-1) < t \rightarrow c_a@i \leq t \quad (9)$$

Time always passes by a non-zero amount:

$$\Delta_i > 0 \quad (10)$$

Let $(a_1, t_1), \ldots, (a_k, t_k)$ be all actions and times such that action a_i makes x true at time t relative to its start.

$$(\neg x@(i-1) \wedge x@i) \rightarrow ((c_{a_1}@i = t_1) \vee \cdots \vee (c_{a_k}@i = t_k)) \tag{11}$$

The frame axiom for x becoming false is analogous.

Encodings of Timed Problems in SMT

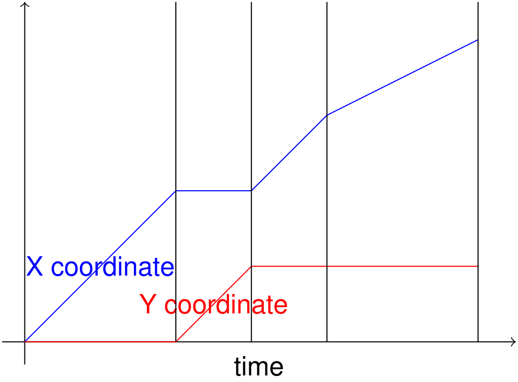
- Real variables in SMT incur a performance penalty.
- The encoding we gave is very general. In many practical cases (e.g. unit durations, small integer durations) more efficient encodings possible (SAT rather than SMT), similarly to scheduling problems.

Planning with Continuous Change

Hybrid systems = discrete change + continuous change

- Physical systems have continuous change.
  - movement of physical objects, substances, liquids (velocity, acceleration)
  - chemical and biological processes
  - light, electromagnetic radiation
  - electricity: voltage, charge, AC frequency, AC phase
- Discrete parts make the overall system piecewise continuous:
  - Discrete changes triggered by continuous change.
  - Continuous change controlled by discrete changes.
- Inherent issues with physical systems: lack of predictability, inaccuracy of control actions
- Problems primarily researched in control theory: Hybrid Systems Control, Model Predictive Control ("Planning" with continuous change not a separate research problem!)

Planning with Continuous Change
Example



actions: 2 east, 1 north, 1 east, $\frac{1}{2}$ east half speed

Hybrid Systems Modeling

- Continuous change a function of time.
- Type of change determined by discrete parts of the system.
- Example: heater on, heater off, temperature $f(w_0, \Delta)$
- Example: object in free fall, on ground, altitude $f(h_0, \Delta)$
- Both actions and continuous values trigger discrete change.
- Example: Falling object reaches ground.
- Example: Container becomes full of liquid.

Hybrid Systems with SMT

- Basic framework exactly as in the discrete timed case.
- Value of continuous variables directly a function of Δ.

law	explanation
$f(x, \Delta) = x + c\Delta$	linear change proportional to Δ
$f(x, \Delta) = x \cdot r^{c\Delta}$	exponential change
$f(x, \Delta) = c$	new constant value
$f(x, \Delta) = x$	no change, previous value

- Other forms of change require a clock variable and an initial value. For example polynomials $c + x^n$.

Hybrid systems: computational properties

- Simple decision problems about hybrid systems undecidable [HKPV95, CL00, PC07]: complete algorithms only for narrow problem classes.
- decidable cases for reachability: rectangular automata [HKPV95], 2-d PCD [AMP95], planar multi-polynomial systems [ČV96]
- semi-decision procedures: no termination when plans don't exist.
- stability: sensitivity to small inaccuracies in control [YMH98]

Hybrid systems: reasoning and analysis

- Main approaches generalize those for discrete timed systems.
    - explicit state-space search (e.g. HyTech [HHWT97])
    - SAT, constraints [SD05]
- Linear systems handled by efficient standard methods (MILP, linear arithmetics) in tools like MILP solvers and SAT modulo Theories solvers [SD05, ABCS05].
- Challenge: non-linear change
    - non-linear programming a very wide subarea of mathematical optimization. mixed integer nonlinear programming solvers (MINLP):
        - AIMMS
        - MAPLE
        - Mathematica
        - MATLAB
    - SMT solvers with non-linear arithmetic [JDM12, GKC13].

Model Predictive Control
Inaccuracy of control, uncertainty, unpredictability

Model Predictive Control [GPM89] (“Dynamical Matrix Control”, “Generalized Predictive Control”, “Receding Horizon Control”)

- Physical systems often not predictable enough for deterministic control.
- Continuous observation - prediction - control cycle.
- Predictions over a finite receding horizon
- Hybrid Model Predictive Control, integrating discrete variables.

Mixed Logical Dynamical (MLD) systems [BM99]

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