Bayesian networks

Chapter 14.1–3

Bayesian networks

A simple, graphical notation for conditional independence assertions and hence for compact specification of full joint distributions

Syntax:

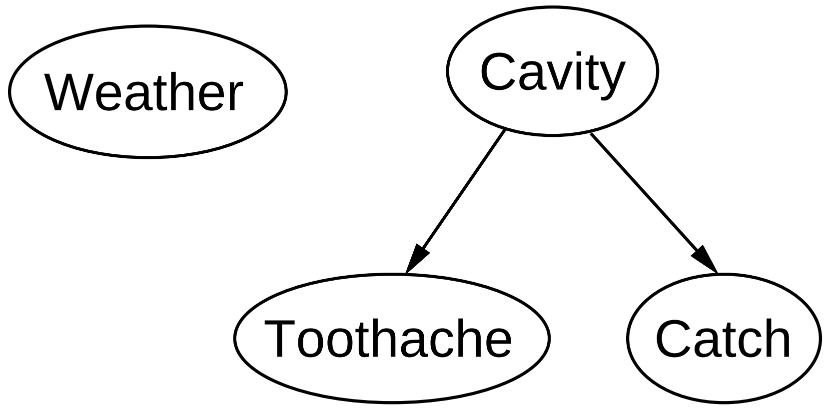
a set of nodes, one per variable
a directed, acyclic graph (link $\approx$ "directly influences")
a conditional distribution for each node given its parents:

$$\mathbf{P}(X_i|Parents(X_i))$$

In the simplest case, conditional distribution represented as a conditional probability table (CPT) giving the distribution over X_i for each combination of parent values

Example

Topology of network encodes conditional independence assertions:



$Weather$ is independent of the other variables

$Toothache$ and $Catch$ are conditionally independent given $Cavity$

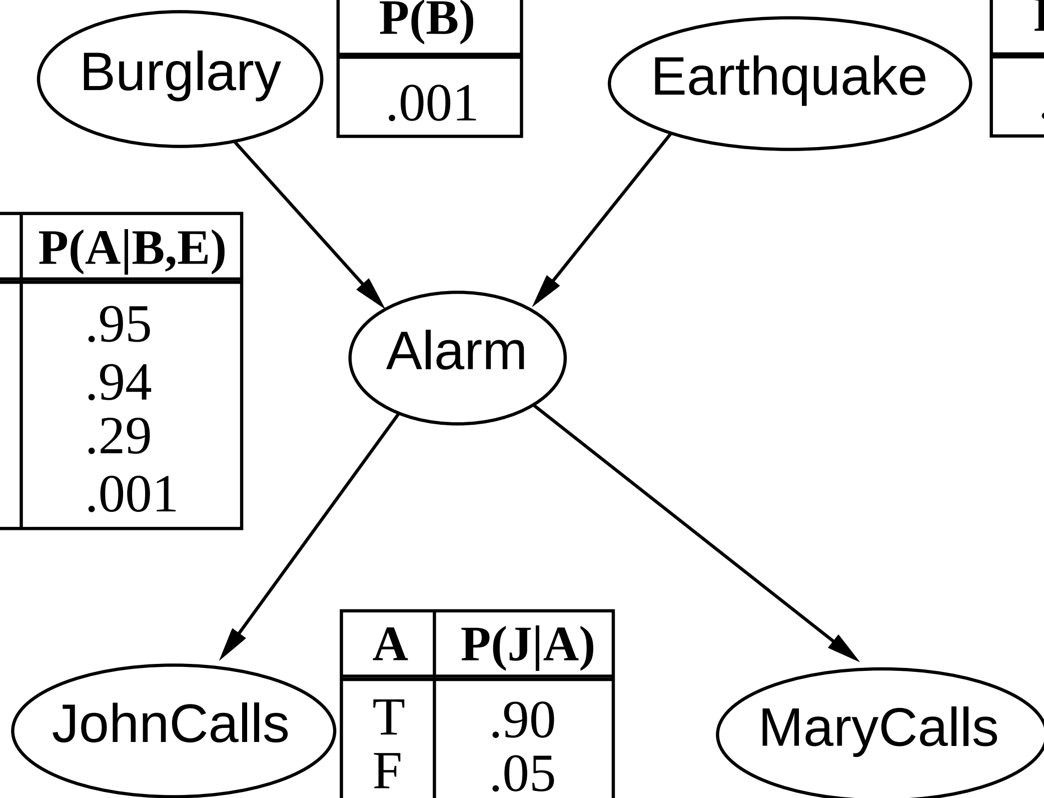
Example

I'm at work, neighbor John calls to say my alarm is ringing, but neighbor Mary doesn't call. Sometimes it's set off by minor earthquakes. Is there a burglar?

Variables: $Burglar$, $Earthquake$, $Alarm$, $JohnCalls$, $MaryCalls$
Network topology reflects "causal" knowledge:

- A burglar can set the alarm off
- An earthquake can set the alarm off
- The alarm can cause Mary to call
- The alarm can cause John to call

Example contd.



P(B)
.001

P(E)
.002

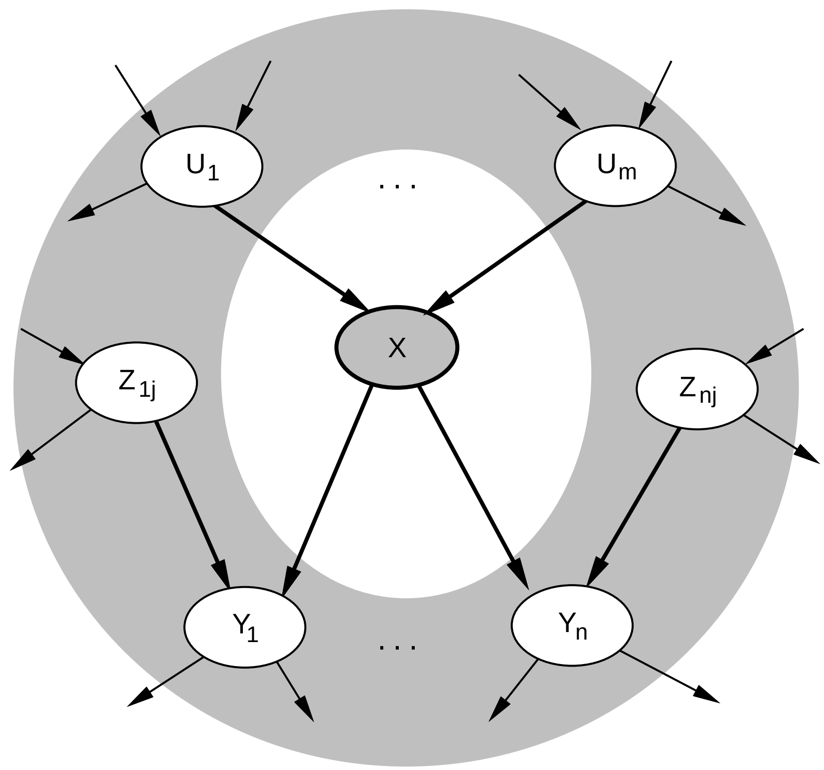
B	E	P(A|B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.05

A	P(M|A)
T	.70
F	.01

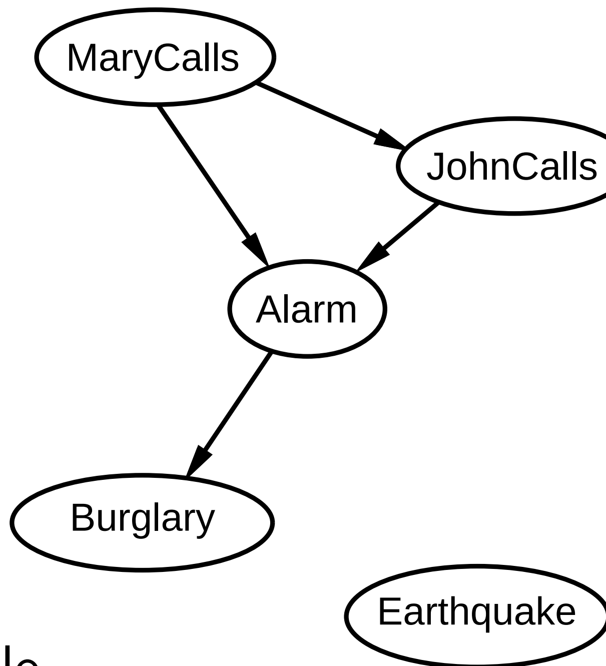
Markov blanket

Each node is conditionally independent of all others given its Markov blanket: parents + children + children's parents



Example

Suppose we choose the ordering M, J, A, B, E



$P(J|M) = P(J)$? No
$P(A|J, M) = P(A|J)$? $P(A|J, M) = P(A)$? No
$P(B|A, J, M) = P(B|A)$? Yes
$P(B|A, J, M) = P(B)$? No
$P(E|B, A, J, M) = P(E|A)$?
$P(E|B, A, J, M) = P(E|A, B)$?

Summary

Bayes nets provide a natural representation for (causally induced) conditional independence

Topology + CPTs = compact representation of joint distribution

Generally easy for (non)experts to construct

Canonical distributions (e.g., noisy-OR) = compact representation of CPTs

Continuous variables $\Rightarrow$ parameterized distributions (e.g., linear Gaussian)