Lecture Note – COSC3410 JCH010919

Canonical Forms (Normal Forms)

Any Boolean function can be rewritten into its *disjunctive normal form (sum of minterms)* or *conjunctive normal form (product of maxterms)*.

Sum of minters:

$f(x) = f(0)x' + f(1)x$
$f(x, y) = f(0, 0)x'y' + f(0, 1)x'y + f(1, 0)xy' + f(1, 1)xy$
$f(x, y, z) = f(0, 0, 0)x'y'z' + f(0, 0, 1)x'y'z + f(0, 1, 0)x'yz' + f(0, 1, 1)x'yz$
$\quad + f(1, 0, 0)xy'z' + f(1, 0, 1)xy'z + f(1, 1, 0)xyz' + f(1, 1, 1)xyz$
and so on and so forth.

Product of maxterms:

$f(x) = (f(0) + x)(f(1) + x')$
$f(x, y) = (f(0, 0) + x + y)(f(0, 1) + x + y')(f(1, 0) + x' + y)(f(1, 1) + x' + y')$
$f(x, y, z) = (f(0,0,0)+x+y+z)(f(0,0,1)+x+y+z')(f(0,1,0)+x+y'+z)(f(0,1,1)+x+y'+z')$
$\quad (f(1,0,0)+x'+y+z)(f(1,0,1)+x'+y+z')(f(1,1,0)+x'+y'+z)(f(1,1,1)+x'+y'+z')$
and so on and so forth.

A Boolean function can be represented by a Karnaugh map in which each cell corresponds to a minterm. The cells are arranged in such a way that any two immediately adjancent cells correspond to two minterms of distance 1. There is more than one way to construct a map with this property. The maps shown in the next page is considered to be best in many ways. You can adopt any one with which you feel most comfortable.

Note that the ordering of variables involved is important. That is to say, the map for f(x, y, z) is not the same as that for f(y, x, z), for example. Do you know why?

Some notations used in the textbook are somewhat misleading. For example, the general form of the truth table of a Boolean function of two variables should be

x	y	f
0	0	f(0, 0)
0	1	f(0, 1)
1	0	f(1, 0)
1	1	f(1, 1)

instead of

x	y	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

Many inter-related concepts are depicted in these tables and maps. Make sure you understand clearly and correctly.

Karnaugh Maps

For a function of two variables, say, f(x, y),

	x'	x
y'	f(0,0)	f(1,0)
y	f(0,1)	f(1,1)

For a function of three variables, say, f(x, y, z)

	x'y'	x'y	xy	xy'
z'	f(0,0,0)	f(0,1,0)	f(1,1,0)	f(1,0,0)
z	f(0,0,1)	f(0,1,1)	f(1,1,1)	f(1,0,1)

For a function of four variables: f(w, x, y, z)

	w'x'	w'x	wx	wx'
y'z'	0	4	12	8
y'z	1	5	13	9
yz	3	7	15	11
yz'	2	6	14	10

Here an integer is used as the short-hand notation for a particular valuation of f(w, x, y, z). For example, the binary representation of 13 is 1101. Thus 13 in the map represents f(1, 1, 0, 1).

For a function of five variables: f(v, w, x, y, z)

	v'				v			
	w'x'	w'x	wx	wx'	wx'	wx	w'x	w'x'
y'z'	0	4	12	8	24	28	20	16
y'z	1	5	13	9	25	29	21	17
yz	3	7	15	11	27	31	23	19
yz'	2	6	14	10	26	30	22	18

For a function of six variables: f(u, v, w, x, y, z)

		u'				u			
		w'x'	w'x	wx	wx'	wx'	wx	w'x	w'x'
v'	y'z'	0	4	12	8	40	44	36	32
	y'z	1	5	13	9	41	45	37	33
	yz	3	7	15	11	43	47	39	35
	yz'	2	6	14	10	42	46	38	34
v	yz'	18	22	30	26	58	62	54	50
	yz	19	23	31	27	59	63	55	51
	y'z	17	21	29	25	57	61	53	49
	y'z'	16	20	28	24	56	60	52	48

Not only that a Karnaugh map can be used to visualize the possibilities for simplification, it can also be used to visualize the possibilities for functional decomposition as well. For instance, the functional decomposition illustrated in Example 4.7 of the text can be visualized as shown below.

1							
	1	1	1	1	1	1	1
1							
	1	1	1	1	1	1	1

=

	1	1	1	1	1	1	1
	1	1	1	1	1	1	1

+

1							
1							

=

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

•

	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1

+

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

•

1							
1							
1							
1							